# Foreign Reserves Management and Original Sin

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### Abstract

This paper studies the interaction between foreign exchange reserves and the currency composition of sovereign debt in emerging countries. Focusing on inflation targeting countries, we find that large holdings of foreign reserves are associated with higher local currency sovereign debt portfolios, an exchange rate which is less sensitive to global shocks, and a lower exchange rate risk premium in local currency sovereign spreads. We rationalize these findings within a financially constrained model of a small open economy. The Sovereign values local currency debt as a hedge against endowment risk, but since the exchange rate tends to depreciate in times of global downturns, risk averse international investors charge an additional currency risk premium on this debt. But when a country uses foreign reserves to lean against the wind in response to global shocks, this dampens the response of the exchange rate, providing insurance for the global investor. By reducing the investor ex-ante risk premium on local currency debt, foreign exchange reserves therefore facilitate a higher share of local currency debt in the sovereign portfolio. The key feature of the analysis highlights the role of reserves in stabilizing the component of exchange rate changes that respond to global shocks, and not the response to local shocks.

# 1 Introduction

Since the early 2000s, emerging country (EM) Sovereigns have been able to gradually escape from the "original sin" syndrome whereby global investors would hold EM debt only in international currencies such as the US dollar. In the last two decades, the data show that emerging Sovereigns have increasingly been able to issue more debt in local currency. At the same time, perhaps surprisingly, many of these countries have continued to accumulate foreign reserves. Simultaneously, we have seen an explicit move towards inflation targeting as a monetary policy framework in emerging economies.

This paper documents the empirical relationship between foreign reserves, local currency debt and inflation targeting. The key empirical findings are that inflation targeting emerging countries that hold substantial reserves tend simultaneously to have more local currency sovereign debt as well as lower local currency sovereign spreads. Furthermore, we find that sovereign spreads are lower not due to lower credit risk but rather due to a lower exchange rate risk component. In addition, such countries tend to have exchange rates much less sensitive to global factors.

Motivated by these empirical findings, we build a small open economy model with an endogenous currency composition of sovereign debt, endogenous foreign exchange management, and risk averse international investors. Ceteris paribus, local currency debt is preferred for the domestic Sovereign, as it is a good hedge for domestic endowment shocks. However, international investors charge a premium on local currency debt, due to exchange rate risk. This is because the exchange rate tends to depreciate in response to negative global shocks, making local currency EM debt riskier for risk averse international investors. Foreign exchange rate reserves can be used to lean against the wind, stabilizing exchange rate movements in the international investors' bad times.

Foreign exchange intervention is desirable from a domestic perspective. In our model, private households face a collateral constraint on borrowing. A real exchange rate depreciation in response to a global negative shock can tighten the collateral constraint so much that it precipitates a "sudden stop". An optimal foreign exchange intervention policy mitigates the impact of global shocks on the real exchange rate. An implication of this is that the management of foreign exchange reserves lowers the risk premium charged by international investors on local currency debt, therefore making the local currency bond more favourable.

**Literature review.** Our paper is to recent literature on overcoming "original sin", including Engel and Park (2018), Ottonello and Perez (2018) and Du et al. (2020).[1] These papers study the optimal currency composition and its interplay with monetary credibility (inflation policy). This paper advances the literature by studying the relationship between currency composition and foreign reserves.[2] We argue that foreign reserves management changes the risk profile of the currency, therefore pricing and sovereign currency portfolio.

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[1] Sunder-Plassmann (2020) also studies the interplay of inflation policy and local currency while taking the currency portfolio as given.

[2] In addition, Engel and Park (2018), Ottonello and Perez (2018) and Sunder-Plassmann (2020) considers only a risk-neutral investor

The paper is also related to the literature of reserves management. Papers by Bianchi et al. (2018), Bianchi and Sosa-Padilla (2020) and Hur and Kondo (2016) focus on the role of foreign reserves in reducing sovereign default risk. We show both empirically and theoretically the relevance of reserves on the currency composition of sovereign debt. Reserves management is also motivated as a tool to correct for different forms of externality, as studied in Arce et al. (2019), Davis et al. (2020), Fanelli and Straub (2021) and Kim and Zhang (2020). Finally, reserves management is also viewed as a form of exchange rate management. Hassan et al. (2022) proposes a risk-based theory of stabilizing currency movements. Amador et al. (2020) studies exchange rate managment at the zero lower bound.

This paper also contributes to the literature on "original sin redux" (Carstens and Shin 2019), which documents the rise of local currency sovereign debt and studies implications to international investor currency mismatch problem.[3] We provide a theoretical setup to these empirical findings and study the role of reserves management in dampening the response to global factors (Bruno et al. (2021)).

The model of sudden stops in the paper is closely related to Schmitt-Grohe and Uribe (2020), who show the existence of 'underborrowing' equilibria, where investor's deleveraging can generate large real exchange rate depreciation and current account reversals. The use of foreign exchange reserves to lean against the wind, as in Davis et al. (2020), could potentially improve interest rate terms of an emerging economy.

## 2    Empirical findings

In this section we document a number of novel empirical findings. First, we show a positive relationship between reserves and local currency debt in EMs. Second, we show that the exchange rates in EMs with higher reserves display lower sensitivity to global factors (proxied by the VIX). Third, we show countries with high reserves tends to have lower local currency spreads. Furthermore, the reduction in spreads can be attributed to lower exchange rate risk rather than to lower credit risk.

We focus on 24 emerging countries as in Arslanalp and Tsuda (2014). Arslanalp and Tsuda (2014) build and maintain a panel dataset of currency composition of emerging market sovereign using various data sources.[4] The dataset covers all the large and commonly studied emerging countries. The countries are listed in the table below.

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[3]See for example, Bertaut et al. (2021), Hofmann et al. (2020) and Hofmann et al. (2022)

[4]Engel and Park (2018), Ottonello and Perez (2018), Du et al. (2020) and Sunder-Plassmann (2020) use this dataset as well.

Table 1: Sample countries

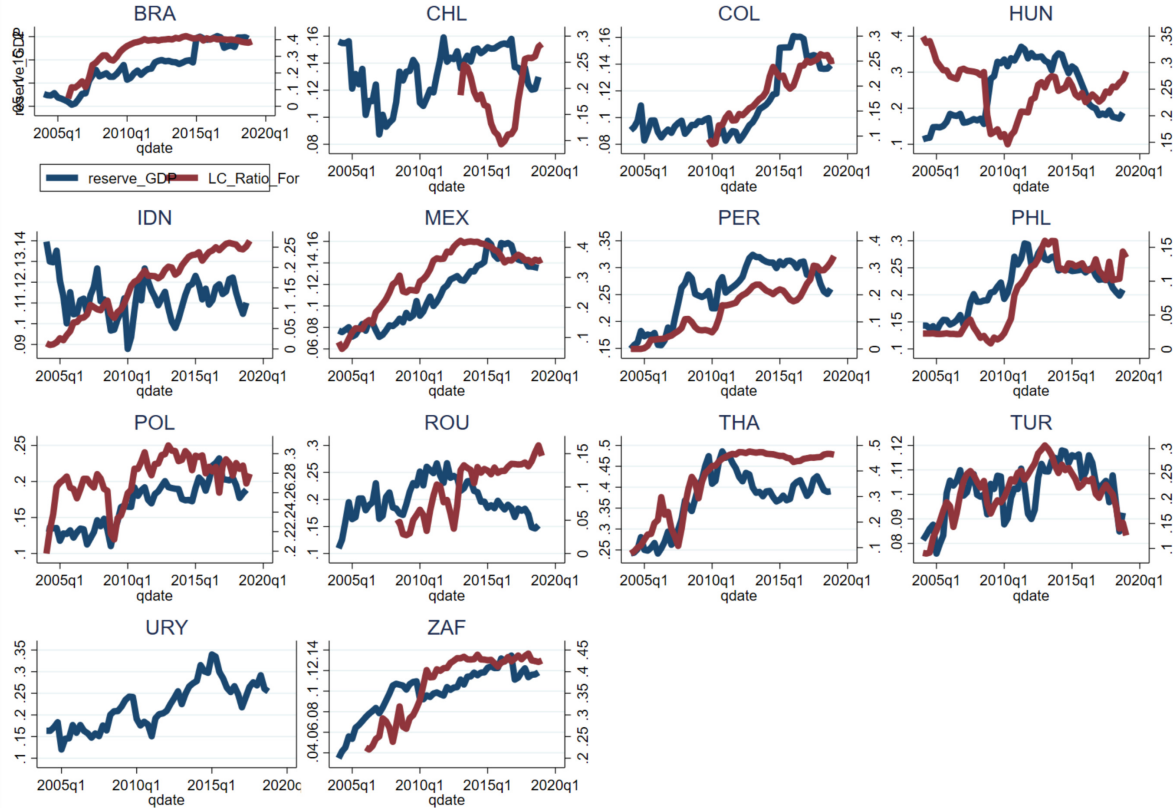| Asia | Latin America | European Union | Europe, Middle East, Africa |
|---|---|---|---|
| China (DS) (IT=0) | Argentina (IT=0) | Bulgaria (IT=0) | Egypt (IT=0) |
| India (DS) (IT=0) | Brazil (DS) | Hungary (DS) | Russia (DS) (IT=0) |
| Indonesia (DS) | Chile (DS) | Latvia (IT=0) | South Africa (DS) |
| Malaysia (DS) (IT=0) | Colombia (DS) | Lithuania (IT=0) | Turkey (DS) |
| Philippines (DS) | Mexico (DS) | Poland (DS) | Ukraine (IT=0) |
| Thailand (DS) | Peru (DS) | Romania | |
| | Uruguay | | |

Notes : DS denotes countries that are available in the Du and Schreger dataset for pricing data. IT=0 denotes countries that are not inflation targeter in our entire sample period. Note that Latvia and Lithuania joined the Eurozone in the later part of the sample period, but all the empirical results are not affected if we exclude these two.

**Foreign reserves and local currency debt.**

Figure 2 illustrates the unconditional relationship between the local currency share of sovereign debt and the ratio of reserves to GDP for a sub-sample of our countries who adopted inflation targeting. It seems apparent that there is a positive relationship for most countries. In order to explore this in more detail however, we estimate a panel regression.

Figure 1: Time series plot of reserves to GDP and LC ratio for inflation targeters



We define the variable as follows: $LCratio = \frac{\text{foreign held local currency sovereign debt}}{\text{foreign held total sovereign debt}}$. We follow most

of the literature in focusing on external debt. Following the studies cited above that focus on inflation commitment, we also control for whether a country is an inflation targeter. We use the definition by Ogrokhina and Rodriguez (2018) that documents the explicit inflation targeting dates for these countries.[5] The annual panel fixed effect regression takes the form:

$$LCratio_{i,t} = \alpha_i + \beta_1 IT_{i,t} + \beta_2 (IT_{i,t} = 0) \times ln(\frac{reserves}{GDP}) + \beta_3 (IT_{i,t} = 1) \times ln(\frac{reserves}{GDP}) + \gamma GC_t + \delta DC_{i,t} + \varepsilon_{i,t}$$
(1)

where *IT* is a dummy for whether the country is an inflation targeter, *GC* is a vector of global factor controls (VIX, US Treasury 5Y, US GDP growth) and *DC* (Domestic GDP growth, Govt effectiveness index, policy stability index, the Chinn-Ito Index, domestic credit to GDP) is a vector of domestic variable controls. These control variables follow Engel and Park (2018). Since many of these variables are only available at annual frequency, we restrict our regression to an annual frequency (end of year data).

The regression estimates are displayed in table 2. The first column shows estimates without controls, and with local controls alone, both global and domestic controls and global controls alone in column (2) to column (4). These estimates largely confirm the existing literature that finds a relationship between $LCratio_{i,t}$ and inflation targeting $IT_{i,t}$. All four specifications of the IT coefficient are significantly positive. Inflation targeting countries tend to have a significantly higher LC ratio. In the sample, countries that adopt inflation targets never return to non inflation targeting. The positive relationship therefore captures a low frequency difference induced by changing from a non-inflation targeter to an inflation targeter within a country.[6]

More interestingly, in addition to whether a country is an inflation targeter, we see that the coefficient estimates for $(IT_{i,t} = 1) \times ln(\frac{reserves}{GDP})$ are significantly positive in all specifications. This indicates that an inflation targeter with a higher reserves to GDP ratio tends to have a higher local currency share for their sovereign debt. This differs for non inflation targeters, who appear to have insignificant estimates across different specifications. The empirical results here suggests that on top of being an inflation targeter, which is a long-term institution change for some EMs, the management of foreign reserves seems to have an interesting correlation with the Sovereign currency portfolio at a higher frequency.

**Foreign reserves and exchange rate sensitivity.**

International investors who hold sovereign debt in local currency must bear exchange rate risk. But exchange rate risk is important only insofar as it is correlated with the investor's own stochastic discount factor. This suggests that well diversified international investors would be indifferent to exchange rate movements attributable solely to local factors within an individual emerging economy. We next provide some empirical evidence on the relationship between foreign reserves and

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[5]These are de jure inflation targeters, where a Central Bank explicitly states inflation control as their objective and has an explicit target rate or range for inflation.

[6]A time invariant difference between inflation targeters and non-inflation targeters is captured by a country fixed effect.

Table 2: Local currency debt ratio and foreign reserves

$$LCratio_{i,t} = \alpha_i + \beta_1 IT_{i,t} + \beta_2(IT=0) \times ln\frac{reserves}{GDP}_{i,t} + \beta_3(IT=1) \times ln\frac{reserves}{GDP}_{i,t} + controls_{i,t} + \varepsilon_{i,t}$$

| | no control | global controls | all controls | local controls |
|---|---|---|---|---|
| *IT dummy* | 0.492*** | 0.220** | 0.114** | 0.340*** |
| | (0.136) | (0.091) | (0.041) | (0.065) |
| $(IT=0) \times ln\frac{reserves}{GDP}$ | -0.017 | 0.018 | -0.003 | -0.033** |
| | (0.020) | (0.020) | (0.008) | (0.015) |
| $(IT=1) \times ln\frac{reserves}{GDP}$ | **0.179*** | **0.094*** | **0.042** | **0.108*** |
| | **(0.046)** | **(0.026)** | **(0.015)** | **(0.018)** |
| VIX | | -0.000 | -0.002** | |
| | | (0.001) | (0.001) | |
| UST_5y | | -0.030*** | -0.027*** | |
| | | (0.007) | (0.002) | |
| US GDP growth | | 1.124 | -0.691 | |
| | | (1.083) | (0.832) | |
| Domestic GDP growth | | | 0.283* | 0.205 |
| | | | (0.149) | (0.141) |
| Chinn-Ito Index | | | -0.015 | -0.050 |
| | | | (0.046) | (0.037) |
| Govt Effectiveness | | | 0.129*** | 0.148*** |
| | | | (0.027) | (0.023) |
| Political Stability | | | 0.015 | 0.009 |
| | | | (0.015) | (0.010) |
| $\frac{Domestic\ credit}{GDP}$ | | | 0.002*** | 0.003*** |
| | | | (0.000) | (0.000) |
| N | 311 | 311 | 270 | 270 |
| Within $R^2$ | 0.14 | 0.28 | 0.47 | 0.39 |

Discroll Kraay (1998) Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

exchange rate movements in EMs that is attributable to global factors.

We first look at some cross-sectional evidence. To capture the exchange rate movements that are relevant for investors, we regress the log change of EM exchange rates on the log change of the VIX index country by country at quarterly frequency:

$$\Delta s_t = \alpha + \beta^{VIX} \Delta ln VIX_t + \varepsilon_t \tag{2}$$

where $s_t$ is the log of exchange rate of EM currency per USD.

In this quarterly regression, we obtain a $\beta^{VIX}$ for each emerging country. This can be interpreted as a measure of how sensitive the EM currency is to changes global shocks as measured by the VIX index, an often-used measure of foreign investor risk appetite/stochastic discount factor/state.

Figure 2 below plots separately the time series mean of reserves to GDP v.s. the beta obtained from eq (2) for each country for inflation targeters and non inflation targeters.[7] On the left hand panel, we see a negative slope that indicates a country with a higher mean reserves to GDP tends to have a lower $\beta^{VIX}$ for inflation targeting countries. The regression line of this scatter plot has a p-value of 0.04. In contrast, we see an insignificantly positive relationship on the right hand side for non-inflation targeters. Taken together, this empirical evidence indicates that inflation targeting countries with high reserves have their exchange rate less sensitive to global movements.

Now we turn our focus to some time series evidence. In this exercise, we first conduct a panel regression at quarterly frequency:
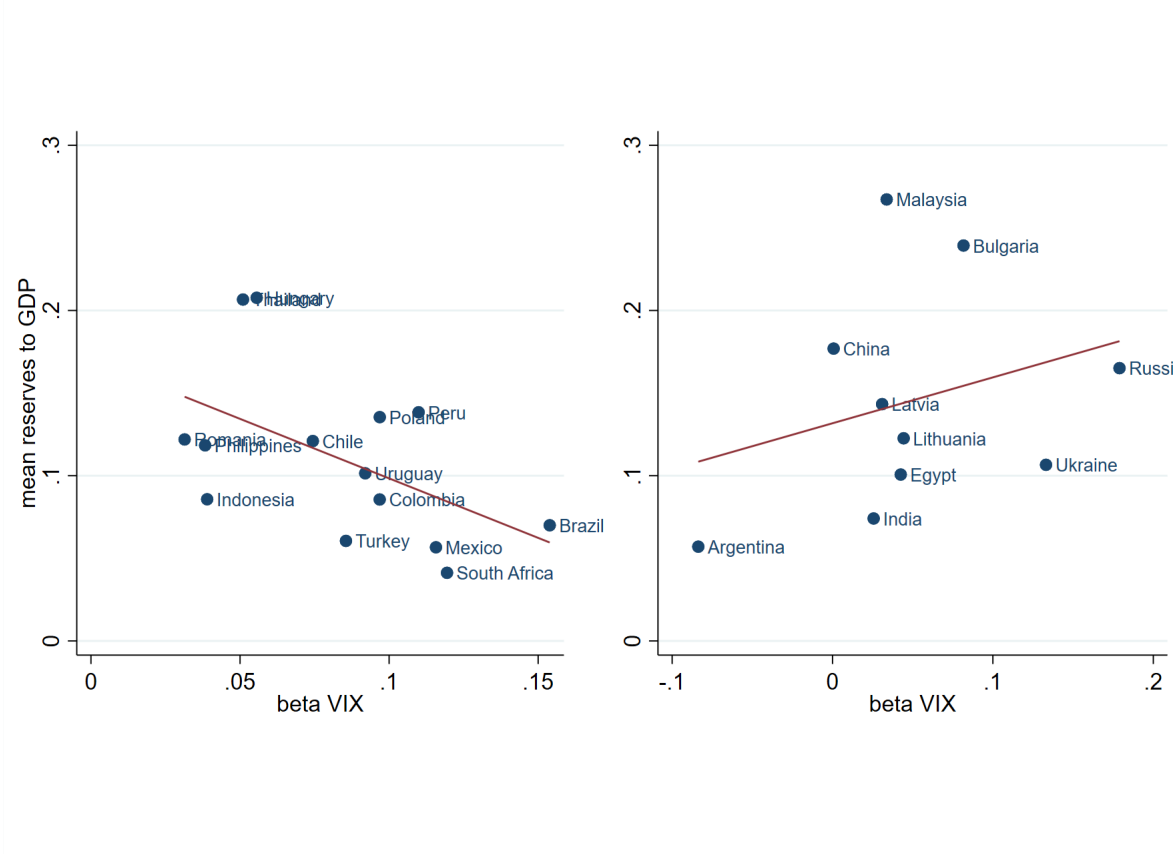
$$\Delta s_{i,t} = \alpha_i + \beta_i^{VIX} \Delta ln VIX_t + \sum_{t=0}^{T} \delta_t T_t + \varepsilon_{i,t} \tag{3}$$

where $T_t$ is a time dummy.

Note that we allow for country-dependent coefficients on the VIX as well as time fixed effects, so the regression can capture the most aggregate related exchange rate movements. We produce fitted values for the dependent variable $\Delta \hat{s}_{i,t}$ and fitted values for the residual $\hat{\varepsilon}_{i,t}$. $\Delta \hat{s}_{i,t}$ can be interpreted as the exchange rate movements that are related to global factors and the residual will be country specific exchange rate movements.

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[7]Strictly speaking, inflation targeters in this cross-sectional picture are countries that become inflation targeter during the sample period, and non inflation targeters are those who never counted as inflation targeters throughout the sample.

Figure 2: mean reserves to GDP and sensitivity of exchange rate to VIX index ($\beta^{VIX}$)

After decomposing the exchange rate into two components, we investigate how a country's foreign reserves co-move with different components with the follow panel regression at quarterly frequency:

$$\Delta ln(\frac{reserves}{GDP})_{i,t} = \alpha_i + \beta_1 IT_{i,t} + \beta_2(IT_{i,t}=0)\Delta\hat{s}_{i,t} + \beta_3(IT_{i,t}=1)\Delta\hat{s}_{i,t} + \beta_4(IT_{i,t}=0)\hat{\varepsilon}_{i,t} + \beta_5(IT_{i,t}=1)\hat{\varepsilon}_{i,t} + \varepsilon_{i,t}$$

(4)

In table 2, we report the regression estimates of eq (4). The coefficient on $IT_{i,t}$ is negative, meaning that inflation targeting countries decumulate reserves more than non-inflation targeter on average. The more interesting result is that only the coefficient on $(IT_{i,t}=1)\Delta\hat{s}_{i,t}$ is significant and the estimate is negative. This means that for inflation targeting countries, there is decumulation of reserves when the currency depreciates due to its global component. This is not the case for currency depreciation that is due to country specific factors, as the coefficient estimate of $(IT_{i,t}=1)\hat{\varepsilon}_{i,t}$ is small and insignificant. This foreign reserves management behaviour is consistently with a "lean against the wind" story for inflation targeters, in which Central Banks use their existing

Table 3: Global factor exchange rates and change of reserves

| | $\Delta ln(\frac{reserves}{GDP})_{i,t}$ |
|---|---|
| *IT dummy* | -0.030*** |
| | (0.009) |
| $(IT = 0) \times$ global exchange rate change | -0.129 |
| | (0.190) |
| $(IT = 1) \times$ global exchange rate change | **-0.241\*\*** |
| | **(0.105)** |
| $(IT = 0) \times$ local exchange rate change | 0.103 |
| | (0.088) |
| $(IT = 1) \times$ local exchange rate change | -0.042 |
| | (0.108) |
| *N* | 2647 |
| Within $R^2$ | 0.01 |

Discroll Kraay Standard errors with 5 lags in parentheses.

* p<0.1, ** p<0.05, *** p<0.01

reserves to intervene in face of pressure on the currency coming from global shocks.[8]

**Foreign reserves and local currency sovereign spreads.**

We now provide some evidence from the pricing data. For a global investor who invests in EM local currency sovereign bonds, the sovereign spread is defined as the yield over the US treasury of the same maturity:

$$\text{local currency sovereign spread} \equiv \text{local currency sovereign yield} - \text{US Treasury yield} \quad (5)$$

Since the local currency sovereign yield is in a different currency denomination than the US Treasury, investing in local currency sovereign bonds involve two risks: pure credit risk and exchange rate risk. As in Du and Schreger (2017), one can use the cross currency swap to eliminate the currency risk. This could be thought of as swapping the US Treasury yield to the EM currency and treat it a risk-free benchmark in EM currency. So we add and subtract the cross currency swap (CCS) to equation (5) and result in:

$$\text{local currency sovereign spread} \equiv \underbrace{\text{local currency sovereign yield} - \text{US Treasury yield} - \text{CCS}}_{\text{local currency sovereign risk (LCCS, pure credit risk)}}$$
$$+ \underbrace{\text{CCS}}_{\text{exchange rate risk}}$$
$$(6)$$

We directly obtain these sovereign spreads and the decomposition from the Du and Schreger

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[8]The effect is not mechanical. Mechanically, a depreciation of the local currency (an increase $\Delta s_t$) will increase $\frac{reserves}{GDP}$ because the reserves are in foreign currency and GDP is measured in local currency. Therefore, the mechanical relationship should result in a positive coefficient.

database. Note that because of a non-nesting sample with Arslanalp and Tsuda (2014), we are left with just 16 countries in this exercise. With a quarterly panel fixed effect setting, we regress the local currency sovereign spread and each of its component with the reserves to GDP:

$$y_{i,t} = \alpha_i + \beta_1 ln(\frac{reserves}{GDP})_{i,t} + \beta_2 ln(\frac{Govtdebt}{GDP})_{i,t} + \beta_3 ln(\frac{Privdebt}{GDP})_{i,t} + \beta_4 GDPgrowth_{i,t} + \beta_5 lnVIX_t + \varepsilon_{i,t}$$
$$(7)$$

where $y_{i,t} = \{$local currency sovereign spread, exchange rate risk, pure credit risk$\}$ defined in eq (6) and we control for govt external debt to GDP, private external debt to GDP, domestic GDP growth and the VIX index, all of which are known to be important in explaining sovereign risk.

The regression estimates are reported in table 4 below. The first column reports the regression with the local currency sovereign spread as the dependent variable. The coefficient estimate of $ln(\frac{reserves}{GDP})$ is negative and significant, indicating an increase in reserves is associated with a reduction in the sovereign spread. When we look at the component that is associated with this reduction, we find that the reduction in the sovereign spread is primarily associated with a change in the exchange rate risk component in column (2), while there is no significant change attributable to the credit risk component in column (3). [9]

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[9]We control for survey forecast-based expected exchange rate appreciation as a proxy of controlling for first-moment exchange rate movement. The survey forecast is obtained from Bloomberg. It is the forecast value of the end of quarter exchange rate 1 year from the current quarter. We construct the value such that a positive value is an expected appreciation.

Table 4: Sovereign Spreads and Foreign Reserves

| | LC spreads | Exchange rate premium | Local currency credit risk |
|---|---|---|---|
| $ln\frac{reserves}{GDP}$ | -6.327** | -8.250*** | 1.923 |
| | (2.358) | (1.440) | (1.671) |
| $ln\frac{govt\ \text{LC}\ debt}{GDP}$ | -0.063 | 0.038 | -0.102** |
| | (0.052) | (0.047) | (0.043) |
| $ln\frac{govt\ \text{FC}\ debt}{GDP}$ | 0.165*** | 0.116*** | 0.049* |
| | (0.035) | (0.035) | (0.028) |
| $ln\frac{private\ debt}{GDP}$ | 0.111*** | 0.085*** | 0.026*** |
| | (0.022) | (0.016) | (0.009) |
| Survey expected | -1.345 | -7.977*** | 6.632*** |
| appreciation | (1.849) | (1.395) | (1.148) |
| GDP growth | -0.395 | 0.034 | -0.429 |
| | (0.722) | (0.781) | (0.398) |
| Dom_credit_GDP | 0.008 | 0.022** | -0.014** |
| | (0.010) | (0.009) | (0.006) |
| ln VIX | 2.376*** | 1.604*** | 0.772*** |
| | (0.278) | (0.163) | (0.189) |
| $N$ | 628 | 628 | 628 |
| $R^2$ | 0.31 | 0.31 | 0.25 |

Discroll Kraay standard errors with 5 lags in parentheses

* p<0.1, ** p<0.05, *** p<0.01

## 3 A model of currency composition and foreign reserves

So far, we have shown that the share of local currency sovereign debt is significantly positively related to foreign exchange reserve holdings and moreso for inflation targeting countries, while at the same time, for inflation targeting countries, reserves tend to be reduced in response to a depreciation in nominal exchange rates that is driven by global risk factors. Finally, we showed that reserve holdings are significantly associated with lower sovereign spreads, and in particular the component of sovereign spreads that is driven by currency risk. How can we account for these different phenomena with a single model, and furthermore, show how each all these findings are interrelated?

In this section, we lay out a small open economy tradable-nontradable model to reconcile the empirical findings and study the interaction between sovereign debt currency composition and foreign reserves. The key mechanism is that local currency debt for the sovereign is a good hedge for domestic shocks, since the local currency depreciates in domestic bad times. However, risk averse international investors charge a risk premium on local currency bonds because of exchange rate risk, since bad times in the domestic economy are positively correlated with bad times for international investors. Reserves management dampens the sensitivity of the exchange rate to global shocks. This acts so as to reduce the local currency risk premium. The presence of financial frictions in the domestic economy, as described more fully below, leads the Central Bank to accumulate reserves

in placid periods, and deploy reserves in response to downturns in the global economy. This acts so as to stabilize the domestic real exchange rate. On top of this, inflation targeting can further reduce the affect of global shocks on the nominal exchange rate. Intuitively, by providing insurance to global investors via a more stable exchange rate during global bad times, the Sovereign enjoys a lower insurance premium (risk premium) in the issuance of instruments (local currency debt) that insure against domestic shocks.

The small open economy has five types of agents:

1. Representative households who borrow subject to a collateral constraint as in Bianchi (2011); Davis et al. (2020). This constraint gives rise to a pecuniary externality and a role for reserves accumulation.[10].

2. Domestic financial intermediaries who purchase bonds from private households, financed by borrowing in international financial markets. This supports the effectiveness of foreign exchange rate intervention, as in Gabaix and Maggiori (2015).

3. A benevolent Central Bank that chooses an optimal reserve management policy.

4. A Sovereign that issues debt in foreign and local currency to smooth and front-load government spending.

5. Risk averse international investors who provide funding to the small open economy.

## 3.1 The Sovereign problem

We begin by describing the problem of the Sovereign. The Sovereign must provide public goods to citizens, where we assume for simplicity that public goods are produced by government spending on the internationally traded goods. The Sovereign desires to front-load government spending and as a result must borrow from international investors. [11] In addition to total borrowing, the Sovereign can choose in which currency to issue debt. In addition, we assume that in the absence of monetary policy commitment, the Sovereign can choose the overall inflation rate of the economy.

This gives us has four choice variables for the Sovereign: government spending ($G_t^T$), debt in foreign currency ($B_t^{FC}$), debt in local currency ($B_t^{LC}$) and, in the absence of monetary policy commitment, the choice of the aggregate price index ($P_t$).

The Sovereign receives tax revenue from the private households which is proportional to the country's tradable endowment. We assume the sovereign receives a constant share $\tau$ of the tradable endowment.[12] If we let the total tradable endowment be $(1 + \tau)y_t^T$, we assume that the Sovereign

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[10]Foreign reserves accumulation as a tool to correct for pecuniary externality is studied in Arce et al. (2019) and Kim and Zhang (2020)

[11]We assume that the Sovereign does not borrow directly from private households. This allows us to separate the problem of currency denomination of Sovereign debt cleanly from that of optimal FX intervention. Relaxing this assumption would not materially affect the benefits of intervention for the Sovereign portfolio choice, but would make the analysis more complicated.

[12]Tax revenue could be proportional to GDP without changing any of the qualitative results, so long as real GDP and the tradable endowment are positively correlated. Assuming revenue as proportional to the tradable endowment simplifies the analysis.

receives $\tau y_t^T$ per period, and and the private economy gets $y_t^T$. To simplify notation, denote $\tau y_t^T \equiv y_t^G$.

Given this discussion, the Sovereign budget constraint is:

$$\text{Budget constraint in LC: } p_t^T G_t^T + \frac{S_t}{R_t^{FC}} B_t^{FC} + \frac{1}{R_t^{LC}} B_t^{LC} \leq p_t^T y_t^G + S_t B_{t-1}^{FC} + B_{t-1}^{LC}$$

where $p_t^T$ is the nominal price of the traded good, $S_t$ is the nominal exchange rate, $R_t^{FC}$ and $R_t^{LC}$ are respectively the rate of return on foreign and local currency debt. We assume that the law of one price is satisfied for traded goods, so that $p_t^T = S_t$. Now write this equation in real terms as

$$\text{Budget constraint in LC: } G_t^T + \frac{1}{R_t^{FC}} B_t^{FC} + \frac{1}{R_t^{LC} p_t^T} B_t^{LC} \leq y_t^G + B_{t-1}^{FC} + \frac{B_{t-1}^{LC}}{p_t^T} \tag{8}$$

As we will show later, the price index can be expressed in the following form.

$$\begin{aligned} P_t &= [\alpha(p_t^T)^{1-\xi} + (1-\alpha)(p_t^N)^{1-\xi}]^{\frac{1}{1-\xi}} \\ &= p_t^T [\alpha + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}]^{\frac{1}{1-\xi}} \end{aligned} \tag{9}$$

Because prices are fully flexible, changing $P_t$ has no consequences for the the relative price $\frac{p_t^N}{p_t^T}$. The inflation setter can pick a $P_t$ which translates to the $p_t^T$ and changes the real value of local currency debt.

## 3.2 Sovereign with full inflation targeting

We first discuss the problem with full inflation targeting. Although our core analysis focuses on the case without full inflation targeting, looking at this case simplifies some analysis. For the case with full inflation targeting, we assume the Sovereign solves the debt portfolio problem, taking the monetary policy as given (as in Ottonello and Perez (2018)).[13] Therefore, the choice variables are $G_t^T, B_t^{FC}, B_t^{LC}$ but not $P_t$. In this case, we also assume the monetary policy enforces full price stability, therefore $P_t = P$ for all $t$. Setting a constant nominal price level has no consequences for the relative price $\frac{p_t^N}{p_t^T}$, which is determined by (18) in the private economy.

The Sovereign maximizes the total discounted sum of the utility from government spending $(G_t)$ by choosing the local currency $(B_t^{LC})$ and foreign currency debt $(B_t^{FC})$:

$$\max_{G_t^T, B_t^{FC}, B_t^{LC}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t [\frac{G_t^{1-\sigma}}{1-\sigma} - l(\frac{P_t}{P_{t-1}})] \right] \tag{10}$$

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[13] An equivalent way of modeling inflation targeting is to allow the Sovereign to choose price index but makes inflation (infinitely) costly. i.e. the inflation cost $l()$ below is so high such that $l'(.) \to \infty$

subject to the budget constraint (8). $l$ is a convex loss function of inflation, since the inflation is stable in this particular setting, it is irrelevant for now.

First order conditions with respect to government spending and foreign currency debt are:

$$\text{FOC } G_t^T : \ \lambda_t = u_{T,t} = \frac{1}{G_t^{T\sigma}} \tag{11}$$

$$\text{FOC } B_t^{FC}: \ \underbrace{u_{T,t}}_{MBFC} = \underbrace{\beta R_t^W [E_t u_{T,t+1}]}_{MCFC} \tag{12}$$

*MBFC* and *MCFC* stand for marginal benefit and cost of FC debt.

The choice of local currency debt is determined by the first order condition:

$$
\begin{aligned}
\text{FOC } B_t^{LC}: \ \underbrace{u_{T,t}}_{MBLC} \ &= \ \underbrace{\beta R_t^{LC} [E_t u_{T,t+1} \frac{p_t^T}{p_{t+1}^T}]}_{MCLC} \\
&= \ \beta R_t^{LC} [E_t(u_{T,t+1})E_t(\frac{p_t^T}{p_{t+1}^T}) + cov_t(u_{T,t+1}, \frac{p_t^T}{p_{t+1}^T})] \\
&= \ \beta R_t^{LC} [E_t(u_{T,t+1})E_t(\frac{S_t}{S_{t+1}}) + cov_t(u_{T,t+1}, \frac{S_t}{S_{t+1}})]
\end{aligned} \tag{13}
$$

where *MBLC* and *MCLC* represent the marginal benefit and cost of LC debt. This condition illustrates that the Sovereign has a possible risk hedging benefit of issuing debt in local currency due to the possibility that the nominal exchange rate depreciates in bad times for the Sovereign - i.e. when the marginal utility of government spending is high.

### 3.3 International investors.

Unlike the conventional analysis of sovereign borrowing we assume that international investors are risk averse. From the investors point of view, they price the assets according to their asset pricing equation.

$$E_t[\Gamma_{t+1}^* R_t^W] = 1 \text{ and } E_t[\Gamma_{t+1}^* R_t^{LC} \frac{S_t}{S_{t+1}}] = 1$$

where $\Gamma_{t+1}^*$ is the stochastic discount used by the international investor.

The investor's no arbitrage condition implies:

$$
\begin{aligned}
E_t[\Gamma_{t+1}^* R_t^W] \ &= \ E_t[\Gamma_{t+1}^* R_t^{LC} \frac{S_t}{S_{t+1}}] \\
&= \ R_t^{LC}[E_t(\Gamma_{t+1}^*)E_t(\frac{S_t}{S_{t+1}}) + cov_t(\Gamma_{t+1}^*, \frac{S_t}{S_{t+1}})]
\end{aligned} \tag{14}
$$

Equation (14) implies that investors will require a premium on local currency debt in the case where $cov_t(\Gamma_{t+1}^*, \frac{S_t}{S_{t+1}}) < 0$, since in this case, the local currency will depreciate when investors have high marginal utility of funds - i.e. in bad times for investors.

**With risk averse investor - Local currency premium**

We plug in (14) into the Sovereign's first order condition for local currency debt eq(13). This yields:

$$u_{T,t} = \beta \frac{E_t(\Gamma^*_{t+1} R^W_t)}{[E_t(\Gamma^*_{t+1}) E_t(\frac{S_t}{S_{t+1}}) + cov_t(\Gamma^*_{t+1}, \frac{S_t}{S_{t+1}})]} [E_t(u_{T,t+1}) E_t(\frac{S_t}{S_{t+1}}) + cov_t(u_{T,t+1}, \frac{S_t}{S_{t+1}})]$$

$$u_{T,t}[E_t(\Gamma^*_{t+1}) E_t(\frac{S_t}{S_{t+1}}) + cov_t(\Gamma^*_{t+1}, \frac{S_t}{S_{t+1}})] = \beta R^W_t E_t(\Gamma^*_{t+1}) [E_t(u_{T,t+1}) E_t(\frac{S_t}{S_{t+1}}) + cov_t(u_{T,t+1}, \frac{S_t}{S_{t+1}})]$$

Divide the whole equation by $E_t(\Gamma^*_{t+1}) E_t(\frac{S_t}{S_{t+1}})$, to get:

$$\underbrace{u_{T,t}[1 + \underbrace{\frac{cov_t(\Gamma^*_{t+1}, \frac{S_t}{S_{t+1}})}{E_t(\Gamma^*_{t+1}) E_t(\frac{S_t}{S_{t+1}})}}_{\text{local currency risk premium}}]}_{MBFC} = \underbrace{\beta R^W_t [E_t u_{T,t+1}]}_{MCFC} + \underbrace{\beta R^W_t \frac{cov_t(u_{T,t+1}, (\frac{S_t}{S_{t+1}}))}{E_t(\frac{S_t}{S_{t+1}})})}_{\text{hedging benefit}} \qquad (15)$$

Compared to the first order condition of foreign currency debt, in additional to the hedging benefit, local currency debt is more expensive because of the local currency risk premium. The optimal portfolio choice between foreign currency debt and local currency debt boils down to how much the hedging benefit of local currency debt is worth relative to the higher interest cost compared to the foreign currency debt. But both the hedging benefit and the local currency premium are critically dependent on the stochastic process for the nominal exchange rate. These will depend not just on the monetary policy rule (the behaviour of the price level), but also the process for the real exchange rate. In section (3.7) below, we show how foreign exchange reserves management influences the real exchange rate.

## 3.4 Sovereign without full inflation targeting

For the case without full inflation targeting, we assume the Sovereign solves the debt portfolio problem as above, and in addition, chooses the price index $P_t$ each period. Therefore, the choice variables are $G^T_t, B^{FC}_t, B^{LC}_t, P_t$.

The Sovereign maximizes the total discounted sum of government spending ($G_t$) by choosing the local currency ($B^{LC}_{t+1}$), foreign currency debt ($B^{FC}_{t+t}$) and $P_t$:

$$\max_{G^T_t, B^{FC}_t, B^{LC}_t, P_t} E_0 \left[ \sum_{t=0}^{\infty} \beta^t [\frac{G^{1-\sigma}_t}{1-\sigma} - l(\frac{P_t}{P_{t-1}})] \right]$$

subject to budget constraint (8). $l$ is a convex loss function of inflation and chosing a inflation $\frac{P_t}{P_{t-1}}$ which is different from the target rate induce a disutility for the sovereign (and also households),

The first order conditions to $G^T_t, B^{FC}_t, B^{LC}_t$ are the same as in equation (11, 12, 13) above. How-

ever, the difference between inflation targeting and non inflation targeting can be seen from the first order condition for $P_t$, which is:

$$\text{FOC } P_t : \underbrace{l'(\tfrac{P_t}{P_{t-1}})\tfrac{1}{P_{t-1}}}_{\text{MC of inflation}} = u'(G_t)\frac{\partial (p_t^T)^{-1}}{\partial P_t}B_{t-1}^{LC}$$

$$= \underbrace{-u'(G_t)\frac{B_{t-1}^{LC}}{P_t^T}\frac{1}{P_t}}_{\text{MB of inflation}} \quad (16)$$

Since $B_{t-1}^{LC} < 0$ for borrowing, the marginal benefit of inflation is positive when there is some local currency borrowing. The marginal benefit of inflation is also higher when the marginal utility of $G_t$ is high. In the extreme case where the Sovereign has no disutility of inflation at all, the Sovereign would always inflate away any existing debt with an arbitrarily high inflation rate. i.e. arbitrary high $P_t, P_t^T$ and $S_t$. Then, ex-ante, according to the investors pricing equation, $(E_t[\Gamma_{t+1}^* \tfrac{S_t}{S_{t+1}}] = \tfrac{1}{R_t^{LC}})$, $R_t^{LC}$ would be arbitrarily high. Then, with local currency debt determined according to first order condition of $B_t^{LC}$ (13), the Sovereign would issue no local currency debt, regardless of the hedging benefit and the covariance term $cov_t(\Gamma_{t+1}^*, \tfrac{S_t}{S_{t+1}})$.

It is apparent from this discussion that the critical determinant of local currency borrowing is the distribution of the nominal exchange rate. The behaviour of the nominal exchange rate will depend partly on the degree of commitment to the inflation target, as described above. But unless the exchange rate is explicitly pegged, it will also depend on the process for the real exchange rate. As we now show, the real exchange rate will in turn depend on the foreign reserves management policy of the central bank. In order to explain this, we turn to the household problem, then the role of intermediary financiers, and finally the optimal reserve management policy of the central bank.

## 3.5 Household problem

The household sector is same as Bianchi (2011), but using the calibration of Schmitt-Grohe and Uribe (2020) to allow for an exacerbated role for pecuniary externalities.[14]

A continuum of identical households receive tradable ($y_t^T$) and non-tradable endowments ($y^N$, a constant). They choose the consumption of tradable goods ($c_t^T$) and non tradable goods ($c_t^N$) within each period. Households can sell one period maturity foreign currency denominated bonds to financial intermediaries at a gross rate $R_t$. By assumption, households and financial intermediaries do not trade in local currency bonds, perhaps due to 'orginal sin' at the private level. This is empirically realistic, since it is widely recognized that most corporate EM borrowing is in foreign currency. [15]

Household preferences are described as:

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[14]This is critical, since as shown in Davis et al. (2020), it gives rise to a role for countercyclical foreign exchange rate intervention.
[15]See Du and Schreger (2017), Wu (2020) for evidence.

$$\max_{c_t^T, c_t^{NT}, b_t^{FC}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right] \tag{17}$$

subject to

$$c_t = [\alpha^{1/\xi}(c_t^T)^{(\xi-1)/\xi} + (1-\alpha)^{1/\xi}(c_t^N)^{(\xi-1)/\xi}]^{\xi/(\xi-1)}$$

where $c_t$ is aggregate consumption. The period budget constraint, written in units of the domestic currency, is given by

$$p_t^T c_t^T + p_t^N c_t^N + S_t \frac{b_t^{FC}}{R_t} \leq p_t^T (1+\tau) y_t^T + p_t^N y^N + S_t b_{t-1}^{FC} - T_t + TR_t^{CB}$$

where $b_t^{FC} < 0$ denotes new foreign currency borrowing at time $t$. $p_t^T$, and $p_t^N$ are the prices for tradable and non-tradable goods and $S_t$ is the nominal exchange rate, (the price of foreign currency). $T_t$ represents a tax paid to the government, and $TR_t^{CB}$ is a transfer from the Central Bank. Finally, $\Pi_t$ represents profits of financial intermediaries, which are described below.

Households face a borrowing constraint that depends on the value of GDP, given by:

$$\text{credit constraint: } -S_t \frac{b_t^{FC}}{R_t} \leq \kappa(p_t^T y_t^T + p_t^N y^N)$$

where $\kappa$ is a parameter that determines the tightness of the borrowing constraint.

**Household equilibrium conditions**

The market clearing condition for tradable and non-tradable goods are:

$$y^N = c_t^N$$

$$p_t^T c_t^T + S_t \frac{b_t^{FC}}{R_t} \leq p_t^T y_t^T + S_t b_{t-1}^{FC} - T_t + TR_t^{CB}$$

Households first order conditions lead to the equilibrium relative price of non-tradables given by:

$$\frac{p_t^N}{p_t^T} = \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{y^N}\right)^{\frac{1}{\xi}} \tag{18}$$

The household's Euler equation is:

$$u_{T,t} - \mu_t = \beta R_t [E_t u_{T,t+1}] \tag{19}$$

where $u_{T,t}$ is the derivative w.r.t to $c_t^T$ and $\mu_t$ is the multiplier on the borrowing constraint.

The complementary slackness condition of the borrowing constraint is:

$$S_t \frac{b_t^{FC}}{R_t} + \kappa(p_t^T y_t^T + p_t^N y^N) \geq 0 \text{ and with equality if } \mu_t > 0$$

## 3.6 Financial sector

As in [Gabaix and Maggiori](2015), households cannot directly access international financial markets. In order to borrow, they sell bonds denominated in foreign currency to a set of competitive one-period lived financial intermediaries. Financial intermediaries borrow from international investors, in foreign currency, and lend to households. Financial intermediaries begin each period with zero net worth, and satisfy a balance sheet condition;

$$\left( \frac{b_t^{fs}}{R_t} + \frac{F_t^{fs}}{R_t^W} \right) = 0$$

where $b_t^{fs}$ represents bonds purchased from domestic households at price $\frac{1}{R_t}$, and $F_t^{fs}$ represent bonds purchased from international investors at price $\frac{1}{R_t^W}$. Intermediary profits are then $(b_t^{sf} + F_t^{fs})$. To prevent intermediaries absconding with receipts from foreign lenders (which equal $-\frac{F_t^{fs}}{R_t^W}$ in real terms, intermediaries are limited by the incentive constraint that discounted profits must be at least equal to $\Gamma \mid \frac{F_t^{fs}}{R_t^W} \mid$ times receipts at the beginning of period $t+1$, which are $-\frac{F_t^{fs}}{R_t^W}$. This leads to a wedge between domestic and world returns on foreign currency bonds, given by

$$R_t = R_t^W - \frac{\Gamma}{\beta} \frac{F_t^{fs}}{R_t^W} \tag{20}$$

where $F_t^{fs} < 0$ is the financial sector borrowing from abroad, which, in equilibrium will equal the household's borrowing from financial intermediaries. Higher household borrowing will increase the trading activity of financiers, increasing the wedge between domestic and world rates of return.

## 3.7 The Central Bank and reserves management

Although households can only borrow from financial intermediaries, they can also hold bonds issued by the Central Bank. Given the Central Bank's constraint on reserves, as described below, households must have a non-negative position in Central Bank bonds.

**Notation.** By bond market clearing we have:

$$b_t^{FC} + b_t^{fs} + b_t^{cb} = 0 \, , \, F_t = F_t^{fs} + F_t^{cb}$$

Here, the first equality is the domestic bond market clearing condition, which says that bonds traded within the domestic market must sum to zero, where $b_t^{fs}$ and $b_t^{cb}$ respectively represent bonds issues by the financial intermediaries and the Central Bank. The second equation defines net foreign assets, which is the sum of the claims of the financial intermediaries $F^{fs}$ and the Central

Bank $F^{cb}$ on the rest of the world.[16]

In addition, the balance sheets of the financial intermediaries and the Central Bank must be satisfied, as follows:

$$\frac{b_t^{fs}}{R_t} + \frac{F_t^{fs}}{R_t^W} = 0 \,, \, \frac{b_t^{cb}}{R_t} + \frac{F_t^{cb}}{R_t^W} = 0$$

Noting that taxes paid to the Sovereign are given by $T_t = \tau y_t^T$, it follows that the households balance of payments condition may be written as

$$p_t^T c_t^T - S_t \frac{F_t}{R_t^W} \leq p_t^T y_t^T - S_t F_{t-1} \tag{21}$$

and the condition (20) becomes

$$R_t = R_t^W + \frac{\Gamma}{\beta} \left( \frac{-F_t + F_t^{cb}}{R_t^W} \right) \tag{22}$$

Equation (21) indicates that despite the presence of financial intermediaries, households effectively borrow at the world interest rate, since intermediary profits are rebated to households in a lump-sum transfer. But despite that, (22) implies that Central Bank sterilized intervention will have non-neutral effects on the economy. When the central bank increases its reserve holdings, it must borrow from the private sector. The private sector in turn will attempt to maintain its total borrowing $-F_t$ by selling more bonds to the intermediaries. But this will tend to push up domestic interest rates above foreign interest rates, and as a result, leads to a fall in total private sector borrowing.

So far however we have not explained the motivation between Central Bank foreign reserves management. We now turn to this.

### 3.7.1 The Central Bank optimization

The Central Bank has two functions. First, it manages reserves through sterilized intervention, buying reserves with bonds issued to the private sector, while also depleting reserves by retiring bonds. As discussed above, sterilized intervention has real effects in this model due to the frictions in financial markets associated with financial intermediaries.[17] Since the Central Bank can buy or sell reserves on the world market without going through the financial intermediaries, its intervention policy can affect the domestic rate of return faced by households and thereby affecting the total external position of households. The Central Bank may also pursue a monetary policy rule of strict price stability (stabilizing the CPI). This is the case of full inflation commitment, as described above in the Sovereign's problem. Alternatively, without full commitment, we assume that the Sovereign (the fiscal authority) chooses the inflation rate as part of its fiscal decision making. The

---

[16]We note that the net foreign asset definition here represents the net claims that the private sector holds indirectly through the asset holdings of the Central Bank and the financial intermediaries. It excludes the debt of the Sovereign, as described above.

[17]See Davis et al. (2020) for a detailed discussion of the mechanics of intervention in a similar model.

Central Bank maximizes the welfare of the domestic household taking into account the first order conditions and collateral constraints on the household. In addition, we assume that optimal reserves are chosen under discretion, so the Central Bank chooses reserves in each period taking the actions of the future policy maker as given. Finally, as in Davis et al. (2020), we assume that reserves are constrained to be non-negative. Therefore, the Central Bank problem is:

$$\max_{F_t^{cb}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{\{[\alpha^{1/\xi}(c_t^T)^{(\xi-1)/\xi} + (1-\alpha)^{1/\xi}(c_t^N)^{(\xi-1)/\xi}]^{\xi/(\xi-1)}\}^{1-\sigma}}{1-\sigma} \right] \tag{23}$$

subject to

$$\text{Budget constraint: } p_t^T c_t^T + S_t \frac{F_t}{R_t^W} \leq p_t^T y_t^T + S_t F_{t-1}$$

$$\text{Household Euler equation: } u_{T,t} - \mu_t = \beta [E_t u_{T,t+1}(R_t^W - \frac{\Gamma}{\beta}(F_t - F_t^{cb}))]$$

$$\text{Borrowing constraint: } -\frac{F_t}{R_t^W} \leq \kappa(y_t^T + \frac{1-\alpha}{\alpha}(\frac{c_t^T}{y^N})^{\frac{1}{\xi}} y^N)$$

$$F_t^{cb} \geq 0$$

The optimal Central Bank reserve policy in this model is based on the analysis of Davis et al. (2020). The critical feature of the model is that the collateral constraint depends on the real exchange rate, which in turn depends on the consumption of traded goods and total private sector borrowing. Since the presence of the real exchange rate in the collateral constraint represents a pecuniary externality, private agents do not explicitly take account of the effect of their borrowing on the real exchange rate and the probability of a sudden stop. The key difference of Central Bank optimization and the household optimization hinges on the Central Bank internalizes the pecuniary externality and is able to achieve via reserves managment ($F_t^{cb}$). By changing the reserves level, Central Bank alters the interest rate the households are facing (equation 22). It therefore change the consumption path of households and the real exchange rate.

As shown by Schmitt-Grohe and Uribe (2020), if there is sufficient complementarity in consumption between traded and non-traded goods, a fall in private sector borrowing may give rise to a self-fulfilling deleveraging driven sudden stop. For high levels of initial debt, there may be multiple equilibrium levels of private consumption and borrowing - in one equilibrium the collateral constraint is slack and borrowing and traded goods consumption is high with a high real exchange rate, while another equilibrium involves low borrowing, consumption and a much lower real exchange rate. In our paper we follow the analysis of Davis et al. (2020) by assuming that agents select the unconstrained equilibrium when it exists. But this still leaves the possibility of a sudden collapse in capital inflows and a discrete real exchange rate depreciation if a rise in the world interest rate leads households to delever beyond a critical threshold. As a result, there is a role for the Central Bank in using foreign reserves intervention to avoid a sudden stop. In particular,

as is shown below, the Central Bank will wish to accumulate reserves in normal times in order to reduce the probability of sudden stops, and run down reserves in response to a rise in global interest rates, to limit the deleveraging of the private sector. The detailed mechanics of foreign exchange intervention are described more fully in Appendix A and in Davis et al. (2020).

## 3.8 Price index and exchange rate

We have already noted that the link between the Sovereign's problem and foreign reserve management of the Central Bank is the behaviour of the nominal exchange rate. To be more precise, we can define the consumer price index for this economy as $P_t$, which given the form of consumer preferences is:

$$
\begin{aligned}
P_t &= [\alpha(p_t^T)^{1-\xi} + (1-\alpha)(p_t^N)^{1-\xi}]^{\frac{1}{1-\xi}} \\
&= [\alpha(p_t^T)^{1-\xi} + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}(p_t^T)^{1-\xi}]^{\frac{1}{1-\xi}} \\
&= \{[(p_t^T)^{1-\xi}][\alpha + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}]\}^{\frac{1}{1-\xi}} \\
&= p_t^T[\alpha + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}]^{\frac{1}{1-\xi}} \\
&= S_t^T[\alpha + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}]^{\frac{1}{1-\xi}}
\end{aligned}
\tag{24}
$$

where the last equality follows from the law of one price in traded goods and the normalization of the foreign price level.

Rearranging the CPI definition (24) gives the conditions for the nominal exchange rate:

$$
S_t = \underbrace{P_t}_{\text{price index factor}} \times \underbrace{[\alpha + (1-\alpha)(\frac{p_t^N}{p_t^T})^{1-\xi}]^{-\frac{1}{1-\xi}}}_{\text{real exchange rate factor}}
\tag{25}
$$

Two forces drive the nominal exchange rate in this model, a nominal price index factor that depends on monetary policy and a domestic factor that is related to the relative price of tradable and non-tradable goods. Foreign reserves management can stabilize the second factor, while inflation policy can influence the nominal price index. In both cases, the reduction in the sensitivity of the nominal exchange rate to global shocks can reduce the currency risk premium imposed by international investors, thereby fostering the issue of local currency debt on the part of the Sovereign.

## 3.9 An analytical example

Section 4 below constructs a full quantitative analysis of the model. Here we provide some further intuition with the help of analytical approximation to the optimal local currency bond portfolio. We make use of the portfolio approximation techniques in Devereux and Sutherland (2011). Using this, we may obtain an analytical expression for the optimal share of local currency bonds issued by the Sovereign. To keep things simple, we focus here only on the case with full inflation targeting,

which highlights the link between foreign reserves management and local currency sovereign debt. To begin, we rewrite the Sovereign's budget constraint as:

$$G_t + D_{t-1}R^W_{t-1} + D^L_{t-1}R^x_{t-1} = y^G_t + D_t \tag{26}$$

where $D_t \equiv -\left(\frac{B^{FC}_t}{R^W_t} + \frac{B^{LC}_t}{R^{LC}_t p^T_t}\right)$ represents the value of new debt issued by the Sovereign in time $t$, $D^L_t \equiv -\frac{B^{LC}_t}{R^{LC}_t p^T_t}$ is the value of local currency debt, and $R^x_{t-1} \equiv R^W_{t-1} - R^{LC}_{t-1}\frac{p^T_{t-1}}{p^T_t}$ is the ex-post (time $t$) excess return on foreign currency debt over local currency debt. For simplicity in what follows, we further define $\tilde{R}^{LC}_{t-1} = R^{LC}_{t-1}\frac{p^T_{t-1}}{p^T_t}$.

We may then rewrite the portfolio optimization conditions for the Sovereign and the international investors as

$$E_{t-1}R^x_{t-1}G^{-\sigma}_t = 0 \tag{27}$$

$$E_{t-1}R^x_{t-1}\Gamma^*_t = 0 \tag{28}$$

We then combine (27) and (28) with the Euler equation for optimal provision of government spending.

$$G^{-\sigma}_t = \tilde{\beta}E_t R^W_t G^{-\sigma}_{t+1} \tag{29}$$

Following Devereux and Sutherland (2011), we take a 2nd order approximation of (27) and (28) around a non-stochastic steady state , combined with a first order approximation of (26), and ( 29). Here we make a slight change in the model so as to ensure the existence of a non-stochastic steady state by re-defining the time discount factor to be endogenous to the size of government consumption, so that $\tilde{\beta} = \omega G^{-\eta}_t$. It is assumed moreover that in steady state, the Sovereign is a net debtor. [18]

For this example we make the additional assumptions about the shocks to $Y^G_t$ and $R^W_t$;

$$\log(Y^G_t) = \bar{Y}^G_t + \varepsilon_y$$

$$\log(R^W_t) = \bar{R}^W_t + \varepsilon_R$$

where $\varepsilon_y$ and $\varepsilon_R$ are mean zero i.i.d. random variables.

For a variable $z$, we define $\hat{z}$ as the log deviation from steady state, except for $\hat{D}$ and $\hat{R}^x_t$, which is defined below. Then the 2nd order approximation of (27) can be written as

---

[18]In a steady state, we must have $\omega \bar{G}^{-\eta} R^W = 1$. We could alternatively introduce portfolio adjustment costs, which would serve the same purpose as an endogenous time discount factor, following the arguments of Schmitt Grohe and Uribe (2003). Note to simplify the exposition, we assume that the Sovereign does not take into account the effect of spending on the time discount factor. This doesn't affect the qualitative results of this section. Moreover, the value of $\eta$ can be very small while still ensuring the existence of a steady state.

$$E_t\left(\hat{R}_t^x + \frac{1}{2}(\hat{R}_t^{2W} - \hat{\tilde{R}}_t^{2LC}) - \sigma\hat{G}_t\hat{R}_t^x\right) = \mathscr{O}(\varepsilon^3) \tag{30}$$

$$E_t\left(\hat{R}_t^x + \frac{1}{2}(\hat{R}_t^{2W} - \hat{\tilde{R}}_t^{2LC}) - \hat{\Gamma}_t^*\hat{R}_t^x\right) = \mathscr{O}(\varepsilon^3) \tag{31}$$

where $\hat{R}_t^x \equiv \hat{R}_t^W - \hat{\tilde{R}}_t^{LC}$, and $\mathscr{O}(\varepsilon^3)$ denotes that the approximation is up to the second order.

We may approximate (26), and ( 29) up to the first order. [19] This gives:

$$\hat{G}_t + \frac{1}{\beta}\hat{D}_{t-1} + \frac{\bar{D}}{\beta\bar{G}}\hat{R}_{t-1}^W - \frac{\bar{D}}{\beta\bar{G}}^L\hat{R}_{t-1}^x = y_g\hat{y}_t^G + \hat{D}_t + \mathscr{O}(\varepsilon^2) \tag{32}$$

$$E_t\sigma\hat{G}_{t+1} = (\sigma - \eta)\hat{G}_t + \hat{R}_t^W + \mathscr{O}(\varepsilon^2) \tag{33}$$

where $\hat{D}_{t-1} = \frac{D_{t-1} - \bar{D}_t}{\bar{G}}$, $\bar{\beta} = \frac{1}{\bar{R}^W}$ is the reciprocal of the steady state world interest rate, and $y_g = \frac{\bar{Y}^G}{\bar{G}}$.

Equation (32) reflects the fact that up to a first order, the steady state value of $R_t^x$ is zero, so the first order response of $D_t^L$ does not enter (32). Moreover, from the definition of $R_{t-1}^x$, we may write

$$\hat{R}_{t-1}^x = \hat{R}_{t-1}^W - \hat{R}_{t-1}^{LC} + \hat{S}_t - \hat{S}_{t-1} \tag{34}$$

where we have used the fact that $p_t^T = S_t$ from above.

Finally, we make the assumption that the stochastic discount factor of international investors is a function of the global interest rate $R_t^w$, and moreover, up to a first order approximation, we have

$$\hat{\Gamma}_t^* = -\gamma\hat{R}_t^W + \mathscr{O}(\varepsilon^2) \tag{35}$$

We wish to obtain the optimal response of government spending $\hat{G}_t$ in order to obtain the equilibrium local currency portfolio from (30) and (31). Using (32) and iterating forward, we obtain the approximate inter-temporal budget constraint condition as:

$$E_t\sum_{i=0}^{\infty}\beta^i\left(y_gY_{t+i}^G - G_{t+i} - \frac{\bar{D}}{\beta\bar{G}}\hat{R}_{t+i-1}^W\right) + \frac{\bar{D}}{\beta\bar{G}}^L\hat{R}_{t-1}^x + \frac{1}{\beta}\hat{D}_{t-1} = \mathscr{O}(\varepsilon^2) \tag{36}$$

where we have used the fact that $E_t R_t^x = 0$ up to the first order.

Now substituting in (33) and summing, using the assumptions on $\hat{Y}_t^G$ and $\hat{R}_t^W$ we obtain (ignoring the order notation hereafter)

$$\hat{G}_t = (1 - \beta\vartheta)\left(y_gY_t^G - \frac{\bar{D}}{\beta\bar{G}}\hat{R}_{t-1}^W - \beta(\frac{\bar{D}}{\beta\bar{G}} + \frac{1}{\sigma})\hat{R}_t^W + \frac{\bar{D}}{\beta\bar{G}}^L\hat{R}_{t-1}^x + \frac{1}{\beta}\hat{D}_{t-1}\right) \tag{37}$$

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[19]Note that because (30) and (31) are accurate only up to second order, in order to determine the optimal portfolio, the other equations can be approximated up to first order.

where $\vartheta = \frac{\sigma - \eta}{\sigma} < 1$.

Now, equating (30) with (31), and using (35) and (37), dropping the time notation since this describes a constant portfolio, we may derive the optimal local currency portfolio

$$\frac{\bar{D}}{\beta \bar{G}}^L = \frac{1}{Var(\hat{S})} \left( \underbrace{\underbrace{-y_g Cov(\hat{Y}^g, \hat{S})}_{\text{hedging domestic shock}} + \underbrace{\beta(\frac{\bar{D}}{\beta} + \frac{1}{\sigma})Cov(\hat{R}^W, \hat{S})}_{\text{hedging global shock}}}_{\text{hedging benefit}} - \underbrace{\frac{\gamma}{1 - \beta \vartheta}Cov(\hat{R}^W \hat{S})}_{\text{local currency risk premium}} \right) \quad (38)$$

Expression (38) captures the intuition discussed in the more general model above. The sovereign would like to have a positive local currency debt in order to hedge both income (tax revenue) risk and world interest rate risk, so long as the covariance between revenue and the nominal exchange rate is negative, and so long as the nominal exchange rate depreciates in response to positive world interest rate shocks, given that the sovereign is a debtor, and interest rate shocks tend to depress current fiscal spending. On the other hand, the final term in (38) indicates that a positive covariance between world interest rate shocks and the nominal exchange rate tends to increase the cost of local currency debt, given risk averse lenders, and ceteris paribus, would reduce the sovereign's optimal local currency portfolio.

This expression takes the process for the nominal exchange rate as given. But given condition (24) above, we know that nominal exchange rate process is driven partly by foreign exchange reserve management and the real exchange rate. Foreign reserve intervention can reduce the covariance between the world interest rate and the nominal exchange rate.[20] In principle, this could reduce or increases the sovereign's issue of local currency debt since the lower direct hedging benefit of a reduced covariance between world interest rates and the nominal exchange rate goes against the fall in costs of issuing local currency debt when this covariance falls. In the quantitative analysis below we find that the second channel clearly dominates, and foreign reserves management unambiguously leads to an increase in local currency debt issue. [21]

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[20]The endogenous stochastic process for the nominal exchange rate implied by foreign reserve management cannot be described by first order approximation, since it is discontinuous at the point where the collateral constraint binds. This is described in Appendix A. Despite this, optimal foreign reserve management always reduces $(\hat{R}_t^W, \hat{S}_t)$

[21]This example can be extended to allow for lack of inflation commitment on the part of the Sovereign. In that case, we must add (16) to (30) and (31) in the approximation. The approximation becomes more cumbersome however, due to the fact that the first order approximation of (16) involves a first order portfolio term $\hat{D}_{t-1}^L$, and requires a higher order (3rd order) approximation of the optimal portfolio equations, as in Devereux and Sutherland (2010).

# 4 Calibration and quantitative evaluation

## 4.1 Calibration strategy

We calibrate our model to Brazil, a typical and widely studied emerging economy. We separate our parameters into three blocks. The first block contains parameters that are standard and are directly taken from the literature. The second block is estimated from the data and the last block is calibrated to match data moments.

For the first block, we take the CRRA coefficient ($\sigma$) of 5 for both households and the government. We take the elasticity of substitution between tradable and non-tradable goods ($\xi$) of 0.45 from Schmitt-Grohe and Uribe (2020), and Akinci (2017). As discussed at length in their paper, this is the empirically relevant value.[22] We take the weight on tradable goods ($\alpha$) of 0.30, which is commonly used in the literature.

The second block is comprised of parameters directly estimated from the Brazilian data. We use the one year CDS rate to measure the $r^W$ ($r^W \equiv R^W - 1$). To get the exogenous component of the country's funding cost, we first run a regression of $CDS_t^{1y} = \alpha_0 + \beta_0 VIX_t + \varepsilon_t$. We use the fitted value from this regression $\hat{CDS}_t^{1y}$ as $r^W$. We then estimate the AR(1) coefficient of $r^W$. The estimated persistent coefficient is 0.4 and the standard deviation of the innovation is 0.038. The mean value of $r^W$ is 0.02. For the endowment process, we estimate the AR(1) process after a quadratic detrending. The estimated persistent coefficient is 0.65 and the standard deviation of innovation is 0.03.

For the last block, we use a set of parameters to match the data moments in the Brazilian data. Although the parameters are calibrated jointly, we can give a heuristic description of how the empirical moments inform specific parameters. The discount factor ($\beta$) and the parameter on the collateral constraint ($\kappa$) are calibrated jointly to match the total private debt to GDP and implied crisis probability of 5-8% in the literature. The parameter on the financial friction ($\Gamma$) is useful for matching reserves sensitive to exchange rate movements ($\beta_3$ in table 2). The calibrated value is $\beta = 0.895$, $\kappa = 0.21$ and $\Gamma = 0.06$. We then parameterize the investor stochastic discount factor as $\Gamma_{t+1}^* = e^{(-r_t^W + \gamma \varepsilon_{t+1}^r - 0.5\gamma^2 \sigma_r^2)}$, where $\varepsilon_{t+1}^r$ is the innovation of $r^W$ and $\sigma_r^2$ is the unconditional variance of $r_t^W$. We calibrate the parameter $\gamma$ to 44 to match the average forward exchange premium of 6.9% in the data. We parameterize the inflation disutility cost using a quadratic cost function: $\delta(\pi_t - \pi^{target})^2$ where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\pi^{target}$ is the inflation target by the Central Bank of Brazil (4.5%). The parameter $\delta$ is set to 8170 to match to realized inflation in Brazil (6.3%).

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[22] A low elasticity of substitution is critical for the link between world real interest rates, the real exchange rate, and the need for FX interventions as discussed above (see also the Appendix).

[23] Quadratic detrend.

| Symbols | Description | Values | Notes |
|---|---|---|---|
| **Parameters from the literature** | | | |
| $\sigma$ | CRRA coefficient | 5 | Standard literature value |
| $\xi$ | Elasticity of substitution between T and NT goods | 0.45 | Schmitt-Grohe and Uribe (2020),Akinci (2017) |
| $\alpha$ | weight on tradable goods | 0.3 | Standard literature value |
| **Parameters from the data** | | | |
| $\rho^{r^W}$ | Persistence of $r^W$ | 0.4 | Estimated AR(1) of 1yr Brazilian CDS |
| $\sigma^{r^W}$ | sd of innovation of $r^W$ | 0.038 | Same |
| $Er^W$ | Mean of $r^W$ | 0.02 | Same |
| $\sigma$ | Persistence of $y^T$ | 0.65 | Estimated AR(1) on Brazil GDP[23] |
| $\beta$ | sd of innovation of $y^T$ | 0.03 | Same |
| $\pi^{target}$ | Target inflation rate | 4.5% | Central Bank of Brazil target |
| **Parameters from moment matching** | | | |
| $\beta$ | Discount factor | 0.895 | External private debt to GDP (17-18%) |
| $\Gamma$ | Financial friction parameter | 0.06 | Matching regression $\beta$ in table 2 |
| $\kappa$ | Credit constraint parameter | 0.21 | Crisis probability (6-7%) |
| $\gamma$ | Investor SDF $m = \exp^{-r+\gamma\varepsilon_\tau - 0.5\gamma^2\sigma_\tau^2}$ | $\gamma = 44$ | Foreign exchange premium (6.9%) |
| $k$ | Sovereign inflation disutility cost and functional form | $\delta=8170$ | Inflation rate of 6.3%  Quadratric cost $\delta(\pi - \pi^{target})^2$ |
| $\tau$ | Share of endowment to the government | 0.097 | Total government debt to GDP (9.8%) |

## 4.2  Results

**Policy functions**

Figure 3 shows the optimal reserves to GDP (y-axis) as a function of existing private debt to GDP (x-axis). The three panels correspond to the case when the world interest rate is at a low, middle and high level. Each of the figures demonstrates four phases of reserves management. First, when the private debt to GDP is low, there is no need to accumulate reserves. This is because the economy is very far from a sudden stop crisis. Second, as the private debt to GDP rises, there is an accumulation of reserves. This is because the economy enters a region where there is a positive probability that the collateral constraint is binding in the next period. Third, when the debt to GDP is sufficiently high, the constraint binds and the central bank finds it optimal to decumulate reserves to support private consumption. In this sense, the severity of the crisis is dampened. Finally, as private debt to GDP goes even higher, the central bank hits the non-negative reserves constraint. The reserves to GDP ratio then goes to zero.

This figure illustrates the lean against the global wind property of the optimal reserves policy. As the interest rate goes up (from the upper to the lower panel), the central bank decumulates reserves earlier as the economy hits the binding region quicker. The peak of reserves to GDP ratio is located at more towards to left-hand-side of the figure as interest rate rises. The central bank also uses up the reserves faster, as shown by reaching the zero reserves region earlier at the right-hand side of the figure.

Figure 4 shows the equilibrium price schedule offered by for the local currency debt price as a function of local currency debt derived from the simulated model. As local currency debt increases, risk averse investors increase the interest rate required on debt, so the price sinks. The red line illustrates the debt price in the absence of FXI, while the blue line shows the same policy function in the case of optimal FXI. It is apparent that FXI leads to an increase in the price at which the sovereign can issue local currency debt across the full range of debt issue.
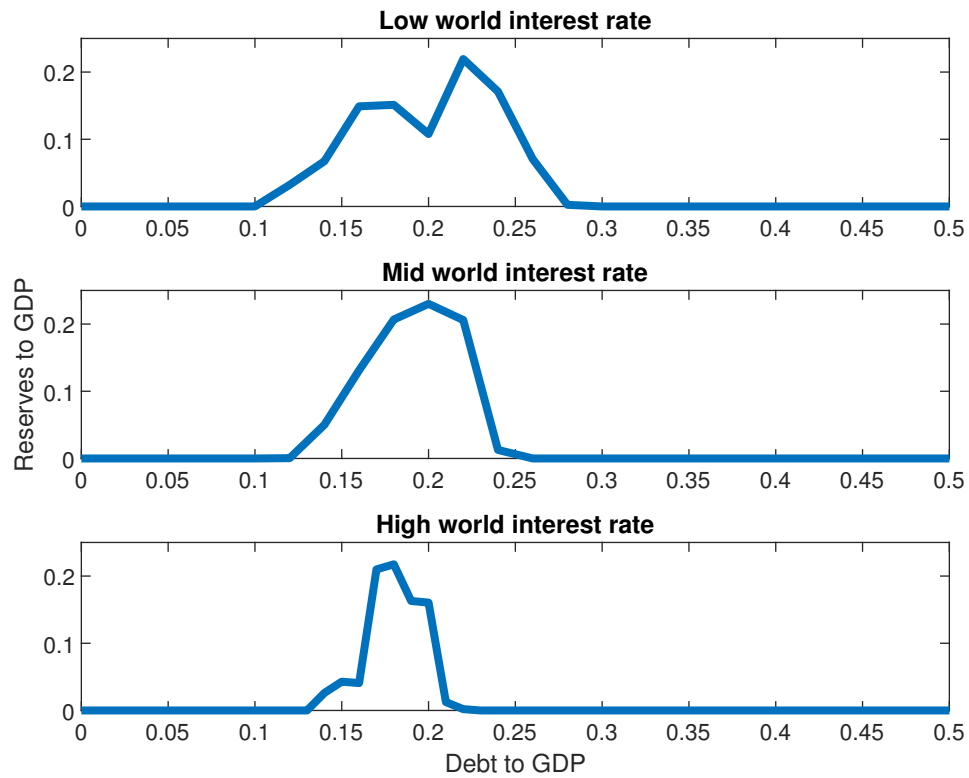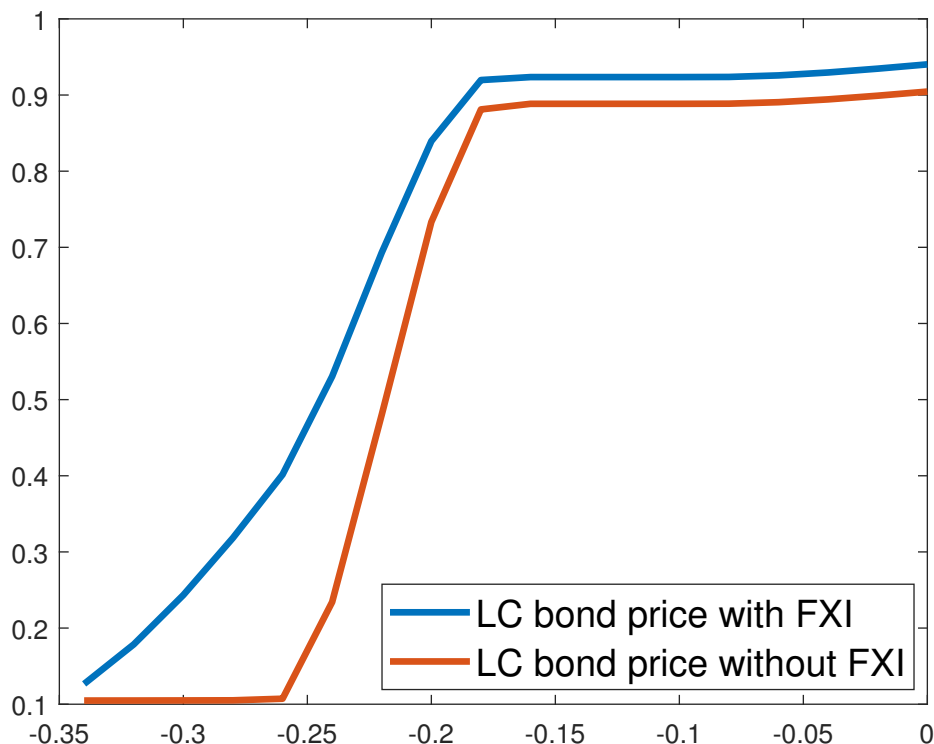
Figure 3: Policy function of optimal FXI

Figure 4: Price schedule of local currency debt



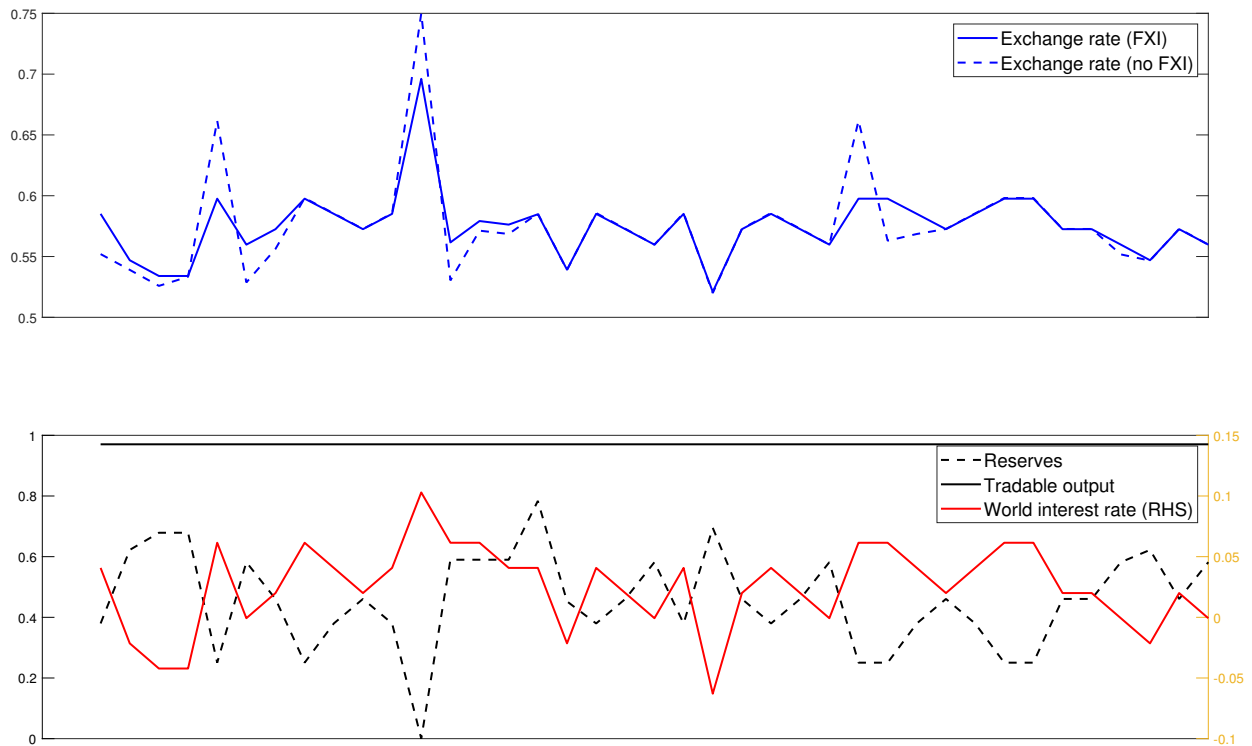**The dynamics of the exchange rate and foreign exchange reserves**

Figure 5 below illustrates the workings of the part of the model that governs the dynamics of foreign reserves and the exchange rate. For a better illustration, this figure is generated using the parameterization discussed above, but assumes a constant endowment of traded goods, but allows for a stochastic world interest rate. The two panels show a simulated path of the exchange rate, reserves and world interest rate. The top panel of the figure shows the behaviour of the nominal exchange rate with and without optimal foreign exchange intervention. The figure also assumes full inflation targeting, so the nominal exchange rate is driven only by movements in the relative price of non-traded to traded goods. The bottom panel of the figure shows an exogenous stochastic process for the world interest rate and the optimal foreign exchange reserve policy followed by the Central Bank, subject to a non-negativity constraint on reserves.

It is clear from the figure that the optimal foreign exchange management policy dampens the volatility of the nominal exchange rate. During normal times (in the middle of the time series), the exchange rate is less volatile with FXI, reflected by the blue solid line (FXI case) and is always further from the mean without FXI - the dashed blue line. There are three crises in the simulated path, which are all caused by a spike in foreign interest rate (endowment is assumed to be constant at this simulation). At the left and right of the time series, a sudden stop crisis is prevented by FXI, leading to a a substantially less depreciated exchange rate. In the middle of the figure a sudden

stop crisis still occurs regardless of the FXI. But even so, the case with FXI has a less depreciated exchange rate due to a decumulation of reserves (the bottom panel). In this manner, the FX policy makes local currency debt safer from the perspective of international investors.

The bottom panel of the figure shows that foreign exchange reserves are strongly negatively correlated with world interest rates - reserves increase in times of low interest rates and are then depleted in face of interest rate spikes. For some large and persistent increases in interest rates, reserves hit their lower bound.

Figure 5: Simulations for Reserves and Exchange Rates under Optimal FXI



### Quantitative results

Table 5 shows the results of the simulated model moments. The model targeted moments are all close to those of Brazilian data. The local currency risk premium for Brazil is measured at 6.9%. The simulations with optimal FXI get closest to this. [24] The inflation rate is set close to the historical inflation rate of 6.3 percent over the sample. Government debt is estimated at roughly 10 percent of GDP.
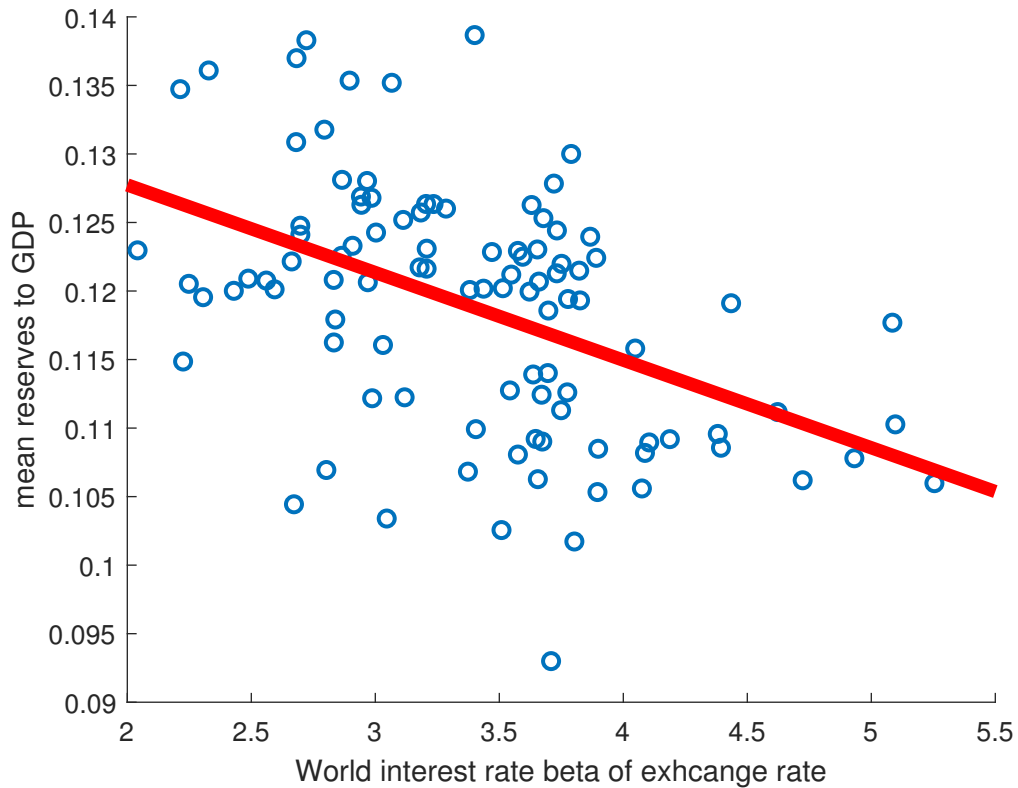
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[24]The simulated policy functions in Figure 4 confirm that the local currency spread is reduced by FXI for the same states of FC and LC holdings along each point of the interest rate grid.

The untargeted moments in the simulated model concern the level of FX reserves and the ratio of debt in local currency. In the model with endogenous FX policy (column (2)), the mean reserves to GDP ratio is 21 percent, somewhat higher than 15 percent observed in the data sample. But the model does a good job in matching the share of government debt in local currency - 50 percent in the data, 53 percent in the model with FXI and endogenous monetary policy, and 68 percent with both FXI and inflation targeting. We note that in the absence of endogenous FX intervention, local currency debt represents a smaller fraction of the total at 40%. This accords well with the empirical evidence presented in section 2 above. Despite the fact that the FXI case in column (2) has a higher LC share and a higher overall debt, we can see a lower equilbrium local currency spreads of 7.4%, constrast to 8.5% in the no FXI case (column (3)).

Figure 6 shows a scatter plot from multiple simulations of the model We simulate the economy for 100 times and record the mean reserves to GDP. We also obtain an exchange rate sensitivity to world interest rates by regressing the change of nominal exchange rate on change of world interest rate in the model. The scatter plot illustrates the negative relationship between the beta coefficient of the exchange rate regressed on the world interest rate (on the horizonal) and the level of FX reserves to GDP (on the vertical). Comparing this figure to to 2 above, we see that the simulated model accurately represents the impact of FX reserves on the sensitivity of the exchange rate to global shocks.

| Table 5 **Moments from the model simulation** | | | | | |
|---|---|---|---|---|---|
| Variable | Brazil Data | Model with FXI | Model no FXI | Model with full IT and FXI | Model with full IT and no FXI |
| | (1) | (2) | (3) | (4) | (5) |
| **Targeted** | | | | | |
| Private Debt to GDP | 17-18% | 17.9% | 18.2% | 24.8% | 25.3% |
| Crisis Probability | NA | 4.17% | 6.25% | 3.4% | 7.3% |
| regression $\beta_3$ in table 2 of $\Delta reserves$ on $\Delta \hat{s}_{i,t}$ | -0.34 for Brazil, -0.24 for Panel | -0.27 | NA | -0.27 | NA |
| Government LC spread $r_{LC} - r_{FC}$ | Data exchange rate premium measured by cross currency swap is 6.9% | 7.4% | 8.5% | 4.5% | 4.8% |
| Realized Inflation Rate | 6.3% | 6.6% | 7.5% | 4.5% | 4.4% |
| Total Gov Debt to GDP | 5+4.8%=9.8% (FC+LC) | 10.8 | 9.9% | 16.4% | 11.9% |
| **Untargeted** | | | | | |
| Reserves to GDP | 15% | 21% | 0% | 21% | 0% |
| FC gov. debt to GDP | 5% | 5.1% | 5.8% | 5.2% | 6.44% |
| LC gov. debt to GDP | 4.9% | 5.7% | 4.1% | 11.1% | 4.75% |
| LC gov debt share | 50% | 53.2% | 40.0% | 68% | 64% |

Figure 6: Scatter plot of model implied reserves to GDP and exchange rate sensitivity to global shock



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**Appendix**

# 5 Sudden stops and the collateral constraint

Following Schmitt-Grohe and Uribe (2020), we assume that the collateral constraint on household borrowing depends on the current value of GDP, and that the elasticity of substitution in preferences between traded and non-traded goods is sufficiently low that there can arise an underborrowing equilibrium driven by self-fulfilling expectations. Figure 5, taken from Schmitt-Grohe and Uribe (2020), illustrates the relationship between net private sector borrowing and the collateral constraint. The figure illustrates the left and right hand side of the borrowing constraint, written first as a steady state condition

$$-F = \kappa(y_t^T + \frac{(1-\alpha)}{\alpha}(y^T + F(1 - \frac{1}{R^W}))^{\frac{1}{\xi}}) \tag{39}$$

and secondly as a 'short-run' condition

$$-F_t = \kappa(y_t^T + \frac{(1-\alpha)}{\alpha}(y^T - \frac{F_t}{R_t^W} + F_{t-1})^{\frac{1}{\xi}}) \tag{40}$$

where for simplicity we have assumed that traded good output is constant at $y^T$, and non-traded good output is normalized to unity.
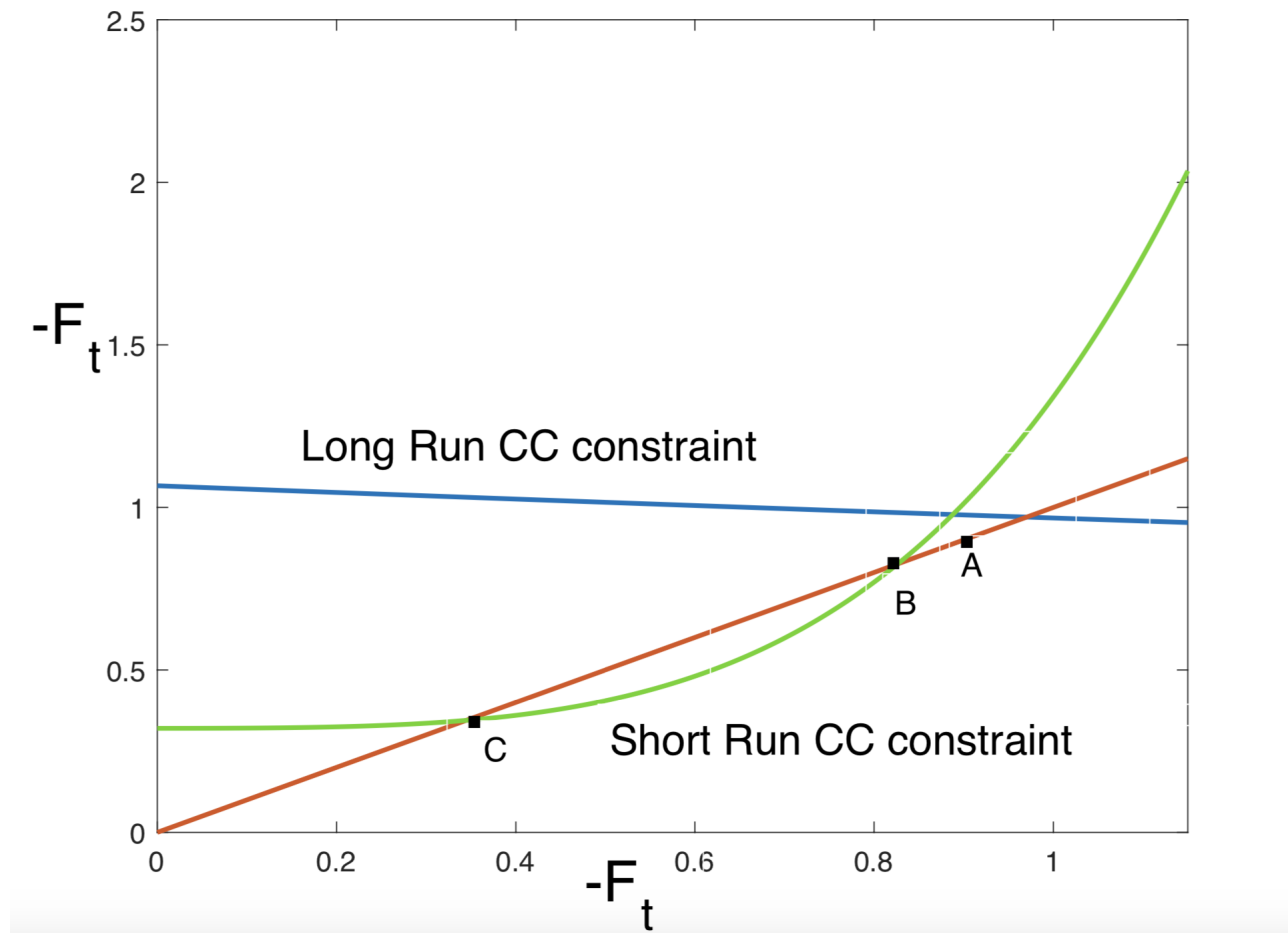
Equation (39) describes a downward sloping relationship on the left hand side, as a higher net foreign debt $-F$ tightens the borrowing constraint by reducing traded good consumption, depreciating the real exchange rate, and reducing the value of collateral. The intersection of this with the $45^0$ line indicates the maximum possible long run level of net foreign debt. Point A in the graph could be a steady state debt level in which the collateral constraint is not binding.

However, note that, conditional on $-F_{t-1}$, the right hand side of (40) is increasing in $-F_t$. Then, in some cases, coinciding with point A, there may be other short run equilibria, where the collateral constraint binds if agents reduce their debt sufficiently, causing a fall in the right hand side of (40) more than the fall in the left hand side. This can occur if the elasticity of substitution between traded and non-traded goods is low, so a fall in current consumption of traded goods leads to a large fall in the relative price of non-traded goods, and the short run borrowing constraint intersects the $45^0$ line with a slope greater than unity. Points B and C in the figure are both potential equilibria.

In the quantitative solution of the model, in the case of multiple equilibria, it is necessary to adopt an equilibrium selection mechanism. Schmitt-Grohe and Uribe (2020) discuss a number of alternative strategies for selecting an equilibrium in a quantitative evaluation of their model. We follow Davis et al. (2020) in assuming that if equilibrium $A$ exists, agents coordinate on that equilibrium, but if not, then they coordinate on equilibrium $C$. The argument is that equilibrium $B$ is unstable in a traditional sense. The implication of this equilibrium selection assumption is that small increases in the world interest rate can lead to precipitous declines in consumption of traded

goods, depreciating real exchange rates, and reversals in the current account. This implies that a rising world interest rate can cause large 'sudden stops'. This link between world interest rates and sudden stops is present only due to the calibration of the model with low intra-temporal elasticity of demand and the potential for multiple equilibria as in figure 5.

Figure 7: Long run and short run borrowing constraint



document

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Table 5: Data source

Sample period: 2004Q1-2019Q1

| Variable | Data source |
|---|---|
| Reserves | IMF IFS |
| Local currency share | Arslanalp and Tsuda (2014) |
| Local currency sovereign spreads | Du and Schreger (2016) |
| VIX | FRED |
| US Treasury 5Y | FRED |
| US GDP | World Bank WDI, IMF IFS |
| Domestic GDP | World Bank WDI, IMF IFS |
| Chinn-Ito Index | Chinn Ito (2006) |
| Govt Effectiveness | World Bank WDI |
| Political Stability | World Bank WDI |
| Domestic credit to GDP | World Bank WDI |
| External public debt to GDP | BIS International Debt Statistics |
| External private debt to GDP | BIS International Debt Statistics, BIS locational bank statistics |
| Nominal exchange rates | IMF IFS |
| Inflation targeting | Ogrokhina and Rodriguez (2018) |

Table 6: Country specific data information

| Inflation Targeter | Inflation target year | Non Inflation Targeter |
|---|---|---|
| Brazil | 1999 | Argentina |
| Chile | 1999 | Bulgaria |
| Colombia | 1999 | China |
| Hungary | 2001 | Egypt |
| India | 2016 | Lithuania |
| Indonesia | 2005 | Lativa |
| Korea | 2001 | |
| Mexico | 2001 | |
| Peru | 2002 | |
| Philippines | 2002 | |
| Poland | 2002 | |
| Romania | 2005 | |
| Russia | 2013 | |
| South Africa | 2000 | |
| Thailand | 2000 | |
| Turkey | 2006 | |
| Uruguay | 2002 | |
| Ukraine | 2015 | |