The permanent and temporary effects of stock splits on liquidity in a dynamic semiparametric model

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Abstract

We develop a dynamic framework to detect the occurrence of permanent and transitory breaks in the illiquidity process. We propose various tests that can be applied separately to individual events and can be aggregated across different events over time for a given firm or across different firms. We use this methodology to study the impact of stock splits on the illiquidity dynamics of the Dow Jones index constituents and the effects of reverse splits using stocks from the S&P 500, S&P 400 and S&P 600 indices. Our empirical results show that stock splits have a positive and significant effect on the permanent component of the illiquidity process while a majority of the stocks engaging in reverse splits experience an improvement in liquidity conditions.

Keywords: Amihud illiquidity, Difference in Difference, Event Study, Nonparametric Estimation, Reverse Split, Structural Change

JEL Classification: C12, C14, G14, G32

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1 Introduction

One commonly held market explanation for why companies split their stock is the theory that this creates "wider" markets.[1] Reducing the price level should make it easier for a bigger pool of retail (uninformed) investors to buy into the stock and allow existing investors to sell part of their holdings more easily to other investors thereby increasing the investor base and the volume of transactions. This in turn should lead to improved liquidity conditions and reduce the cost of capital to companies. However, there are other theoretical arguments presented in Copeland (1979) that may point to a decrease in liquidity following a stock split, such as increases in real transaction costs.[2] In his empirical study, Copeland found that liquidity worsened following stock splits on average. However, his dataset terminated over fifty years ago and at that time stocks were traded in 1/8ths of a dollar and the trading system was pit based rather than electronic. More recent work we review below has found mixed results. We will evaluate Copeland's findings with updated data and modern statistical methods.

Many of the empirical studies about stock splits distinguish between short-term and long-term effects but do so in an informal way statistically.[3] We use a recently developed time series model, Hafner et al. (2023) henceforth HLW, to capture this distinction formally and to provide new methodology to test the effects on liquidity of stock splits. We base our analysis on the popular Amihud (il)liquidity measure proposed by Amihud (2002), which is considered an empirical proxy of Kyle's price impact parameter. But rather than

[1] Announcing their 4 for 1 split in June of 2020, the Apple Board of Directors gave the reason "to make the stock more accessible to a broader base of investors."

[2] As Weld et al. (2009) say "commissions paid by investors on trading ten \$35 shares are about ten times those paid on a single \$350 share". Nowadays, trading costs for retail traders would not scale in this way but nevertheless the total costs would be higher.

[3] Copeland (1979) for example worked with a dynamic regression model for trading volume estimated separately for before and after.

averaging over daily measures as in the usual approach, we feed the daily measures directly into our model. Our model is a member of the class of multiplicative error models (MEM) that have been applied to many different positive-valued financial time series including volatility, duration between trades, and transaction volume, see e.g. Engle (2002). We allow for a nonparametric time trend to capture the secular improvement in liquidity that has happened for most stocks from the 1960s to the present day. The MEM model and its applications and developments over the last 20 years are reviewed in Cipollini and Gallo (2022).[4] Within our econometric model, we test the following hypotheses

> $\mathbf{H}_0^1$: *Stock splits have no permanent effect on the level of liquidity against the alternative that they do have a permanent effect.*
>
> $\mathbf{H}_0^2$: *Stock splits have no additional temporary effect on the level of liquidity against the alternative that they do have a temporary effect.*

Our test of $\mathbf{H}_0^1$ is similar to that used in the regression discontinuity literature and the structural break literature - we look for discrepancy between forward- and backward-looking trend estimates based on local linear kernel smoothing, Fan and Gijbels (1996). Our test involves the estimated permanent component of liquidity based on the model prewhitened liquidity series, see Hafner et al. (2023). We establish the large sample properties of our statistic under the null and alternative hypotheses. Since the tests are based on smoothing methods, there is the issue of bias and we use several different methods to account for that in our inference procedures including the recent work of Armstrong and Kolesár (2020). We consider individual event-specific tests, tests that pool across different splits for the same firm, and tests that pool across firms as well. One issue with these tests is whether

[4]An alternative approach can be based on fitting linear models to the logarithm of liquidity, but we avoid this approach here primarily because taking logarithms emphasizes values close to zero, the so-called inlier problem, and deemphasizes large positive values of illiquidity, which are of much more interest and concern to policymakers.

they account for major events like the tech bubble and bust of 1999-2001 and the Global Financial Crisis of 2008 that occur at the same time as some forward splits and reverse splits. To control for this we develop a difference in difference (DID) test statistic that estimates the difference in jumps at the specified time relative to the jumps observed in a control group of stocks that did not experience a split at the given time.

Our test of $\mathbf{H}_0^2$ are designed in the spirit of event studies as e.g. in Fama et al. (1969), i.e., we look at abnormal liquidity and cumulative abnormal liquidity and determine whether these quantities are consistent with the null of no change. Our tests are robust to the presence of a permanent break and are looking additionally for short-term adjustments to the new level of liquidity. We propose a nonparametric method for estimating the critical values for these test statistics under the assumption that the shock process is stationary and mixing and show that it is asymptotically valid. Since we focus on liquidity rather than firm valuation, we concentrate on the post split effects rather than the post announcement effects, although our test statistics are computed in some cases over periods including the announcement as well as the split itself.

We next discuss our empirical results. Our results broadly support findings in Copeland (1979) using a more recent sample of Dow Jones index component stocks. Specifically, we document that stock splits cause significant shifts in the long-term Amihud illiquidity trend while no additional significant effects on short-run Amihud liquidity dynamics are detected. This seems to suggest that the market quickly adjusts to the new pricing regime. The change in the long-term trend of illiquidity is predominantly positive, implying that liquidity conditions deteriorate in the long run after stock splits. We also investigate whether the effects of reverse stock splits on the illiquidity process are symmetric to the ones documented for stock splits. Our empirical study uses low-price stocks from the constituents of the S&P 500, S&P 400 and S&P 600 indices with a reverse split during the past 30 years. The results suggest that a majority of the low-price stocks engaging in reverse splits experience an improvement in liquidity conditions. However, we find limited effects on the short-run illiquidity dynamics. The fact that reverse splits result in a significant decrease in the illiquidity trend for the majority of those stocks is in line with the results in Han (1995). The pronounced improvement in liquidity conditions for our sample of stocks with low pre-event price levels is consistent with the evidence in Blau et al. (2023). This could be linked to the fact that short-selling activity increases after reverse splits in part because reverse splits ease the constraints on short selling for low-priced stocks (Kwan et al. (2015)).

Li and Ye (2023) (henceforth LY) develop an innovative model capturing the effects of discrete prices and discrete quantities (lot sizes) on liquidity. Their model implies that there should be a U-shaped relationship between a stock's nominal price and its liquidity. Their empirical analysis confirms some of their theoretical predictions, in particular, they find that most splits improve liquidity. They are working with a different measure of liquidity, the quoted bid-ask spread (which they obtain at the millisecond frequency and then average over [-180,-60] days and [60,180] days. They make cross sectional predictions the centerpiece of their empirical work, whereas we are working with lower frequency daily data and fit a time series model. Nevertheless, we try to build on some features of their model. Specifically, we also compute a measure of the permanent effect of a stock split based on our proxy for one of their model implied liquidity measures. In this case, we find different empirical results, which we discuss below.

Literature Review. There is a substantial literature on the effect of stock splits on firm valuation starting with Dolley (1933) who studied stock splits between 1921-1931 and found (split-adjusted) price increases around the time of the split. Fama et al. (1969) introduced a new methodology based on the market model for stock returns that we now call event study and applied it to 940 splits between 1927-1959. They argued that Dolley and other studies used windows that were too short and consequently did not control for the price appreciation trend established prior to the split decision and they find that most of the effect occurs before the split itself consistent with this sample selection interpretation: firms that take the decision to split their stock tend to have had a period of high price appreciation prior to their decision. Other studies following Fama et al. (1969) did find significant short-term/long-term effects on firm valuation. Weld et al. (2009) summarize a lot of this evidence. They discuss the proposed reasons why companies split their stock and how this evidence stacks up with those theories. These include: signaling management's confidence about the future, optimal trading ranges for retail investors, and optimal real tick size for market making. However, they find that none of the existing theories are able to explain the observed constant nominal stock prices over a large part of the 20th century (in contrast with the CPI, which went up manyfold over the same period).[5]

Although the early focus of the empirical work was on firm valuation, there has been a lot of subsequent work looking at other outcome variables. Ohlson and Penman (1985) examine stock return volatilities before and after the execution dates (ex-dates) of stock splits. They find an increase of approximately 30% in the return standard deviations following the ex-date. This holds for daily and weekly returns and persists for a long while. Lamoureux and Poon (1987) find that of 215 splits, eighty-seven showed a statistically significant drop in split-adjusted, market-adjusted average daily volume, while twenty-seven exhibited a significant increase. Of forty-nine reverse splits, fifteen exhibited a statistically significant increase and two exhibited a significant decrease in split-adjusted, market-adjusted volume. Lakonishok and Lev (1987) report that trading volume temporarily increased on announce-

[5]Since the time period considered by that article, it appears that some of the facts have changed regarding stock splits. Specifically, the frequency of stock splits has fallen considerably up to a few years ago. Reflecting this reduction in the frequency of splits, the average price of large caps (S&P500) has increased from around $50 in 2000 to more than $200 in 2022, Mackintosh (2022). However, recently there has been an upsurge in stock splits, for example: Walmart (3:1 on February 22nd 2024), Nvidia (10:1 on June 7th 2024), Chipotle (50:1 on June 25th 2024), and Broadcom (10:1 on July 12th, 2024).

ment day and decreased after the split announcement. Consistent with this, Huang et al. (2015) find that there is a liquidity improvement on announcement day, as well as in the period between announcement day and execution date. However, the liquidity declined after the ex-date to the level before the announcement. The authors concluded that liquidity improvement is a short-lived effect of stock splits. Copeland (1979) provided an empirical study that worked with weekly trading volume mostly for stocks from 1963-1974. He developed a dynamic regression model for trading volume, his main liquidity measure. He estimated the model for 25 firms before their split and then separately for after their split. He tested the differences using classical F tests. He found that volume increases less than proportionately after a split. He also found that brokerage revenues increase, and that percentage bid-ask spreads increased following stock splits, i.e., liquidity worsened. On the other hand, other researchers (Chern et al. (2008); Guo et al. (2008); Yu and Webb (2009)) found that stock splits reduce bid-ask spreads, and increase the number of small traders who are attracted to the lower price on ex-date, indicating liquidity improvement. Mohanty and Moon (2007) also found a significant improvement in the average trading volume, comparing the 12-month post-split announcement period with the one prior to the announcement. Han (1995) studied the effect of reverse splits on liquidity using bid-ask spread, trading volume, and the number of non-trading days as liquidity proxies. He finds that liquidity significantly improves after reverse splits relative to a control group matched on industry, size, and share price. Lin et al. (2009) find contrarily that following forward stock splits a nontrading type of liquidity proxy improves. The upshot of this literature is that the evidence of the impact of stock splits on company-specific liquidity is, to this date, inconclusive.

The remainder of the paper is organized as follows. The following section recalls the Dynamic Autoregressive Liquidity (DArLiq) model introduced in HLW and defines a measure for the permanent effect. Section 3 presents the econometric methodology including estimation of the dynamic model and various tests of permanent and temporary effects. Sections 4 and 5 present an empirical application where we use our framework to analyze the effect of stock splits and reverse splits on the long-run trend and short-run illiquidity dynamics. Section 6 concludes. Some additional figures for the empirical analysis are collected in Appendix D of the supplementary material.

2 The dynamic model for illiquidity

2.1 Liquidity measures

We are concerned with stock market liquidity, which is essentially the ability to buy and sell shares quickly at a good price. Kyle (1985), O'Hara (1995), Hasbrouck (2006) identify three aspects of liquidity: trading costs and price impact; depth; and resiliency. Common ways of measuring liquidity using high frequency trade and quote data on limit order book include: quoted bid-ask spreads, effective spreads, realized spreads, quoted depth and weighted depth, and transaction volume. High frequency intraday data is now very complex and requires a lot of computation and in any case is not available for all markets of interest for a long period of time.[6] For these reasons, low-frequency measures are very popular. There are several methods widely used to measure liquidity using lower-frequency data. The usual approach is to work with daily data and then average to a monthly or longer

[6] Quoted bid-ask spreads are not such a good measure of liquidity or transaction costs because trades often occur outside of the quoted prices. The effective spread, which is the difference between the trade price and the mid-quote price, is a better measure of execution cost, as reflected in SEC rule 605, Foresight (2012). Nevertheless, there are many complex issues in calculating effective spreads with high frequency trade and quote data in a legally integrated market such as the US, where separate venues exist without synchronized timestamps so that for example establishing the time priority of messages at say the millisecond frequency across different venues is difficult. For example, it takes at least 4 milliseconds for messages to travel between New York and Chicago, according to the laws of physics.

horizon. Some of these measures are based on the autocovariance function of daily returns such as the Roll measure, see Li and Linton (2022) for a recent contribution. Some are based on the frequency of zero returns in say a month, the so-called LOT model of Lesmond et al. (1999). These approaches are less easy to adapt to create a meaningful daily measure of liquidity in the absence of intradaily data. We focus on the Amihud illiquidity, Amihud (2002), which can be interpreted as a price impact measure of liquidity, and which has a natural daily counterpart. This has been used in countless studies (this paper is cited more than 13500 times) as: a dependent variable measuring market quality; and an independent variable as a priceable factor in asset pricing cross-sectional regressions, Amihud (2002). Goyenko et al. (2009) and Fong et al. (2017) compare a large number of low-frequency liquidity measures, and generally speaking the Amihud measure does pretty well. There are several variations on the Amihud measure, and we present some of them in Table 1.

In the empirical study, our main results are obtained with the measure $\ell_t = \sigma_t / V_t$, where σ_t is the high-low measure of daily volatility. One advantage of this measure is that it avoids the zero problem sometimes encountered with daily returns, see HLW. Furthermore, it has been argued that this version leads to a more efficient low-frequency liquidity measure than the classic daily Amihud proxy, see Lacava et al. (2023). Our main empirical results are qualitatively robust to the use of alternative liquidity measures except the LY measure, which gives quite different results. This seems to arise because of the additional presence of the nominal price level.

Table 1: Alternative Daily Liquidity or Illiquidity measures.

Paper	**Daily Measures**
Amihud (2002)	$\frac{\left\|P_t^C - P_{t-1}^C\right\|}{V_t \times P_{t-1}^C}$
Amivest	$\frac{V_t \times P_{t-1}^C}{\left\|P_t^C - P_{t-1}^C\right\|}$
Lacava et al. (2023)	$\frac{\sigma_t}{V_t}$
Barardehi et al. (2021)	$\frac{\left\|\ln(P_t^C) - \ln(P_t^O)\right\|}{V_t}$
Fong et al. (2018)	$\frac{a\sigma_t}{V_t^{1/2}}$
Li and Ye (2023)	$\frac{2\sigma_t L_t P_t}{V_t}$
Copeland (1979)	V_t
Kyle and Obizhaeva (2016)	$c\frac{V_t^{1/3}}{\sigma_t^{2/3}}$
Hui and Heubel (1984)	$\frac{\left(P_t^H - P_t^L\right)}{P_t^L}\frac{mcap_t}{V_t}$
Danyliv et al. (2014)	$\log_{10}\left(\frac{V_t}{P_t^H - P_t^L}\right)$

Note: σ_t is a daily measure of volatility such as the high-low measure $\frac{P_t^H - P_t^L}{P_t^C}$. V_t is trading volume, L_t is round lot size, and $mcap_t$ is the market capitalization. The high, low, close and open prices are denoted P_t^H, P_t^L, P_t^C, and P_t^O.

2.2 Time series model

Suppose that the non-negative time series ℓ_t follows a multiplicative dynamic stochastic process

$$\ell_t = g(t/T)\lambda_t\zeta_t \tag{1}$$

$$B(L)\lambda_t = \omega + C(L)\ell^*_{t-1}, \tag{2}$$

where $g(.)$ is a positive but unknown function of rescaled time while $\ell^*_t = \ell_t/g(t/T) = \lambda_t\zeta_t$ is the rescaled liquidity, and ζ_t is a sequence of non-negative random variables with conditional mean one and finite unconditional variance denoted by σ^2_ζ. We here consider the special case where the lag polynomials satisfy $B(L) = 1 - \beta L$ and $C(L) = \gamma$, where $\beta, \gamma > 0$ and $\beta + \gamma < 1$ in which case the process λ_t is mean stationary. The component $g(.)$ captures the long-term slowly evolving trend in the process, which is necessary to include for many stocks and indices due to the longer term improvements in liquidity that we have observed. The dynamic process λ_t represents short-term stationary predictable variation around this trend, which is also needed for liquidity time series that have this moderately persistent deviations from the trend as documented in HLW. The last component ζ_t represents the new information, the shock driving the process. In HLW we investigated the tail thickness issue and found Pareto type tails but tail thickness parameters in the range five to ten. When it comes to estimation, there is an identification issue because we can multiply and divide the two components g, λ by constants and obtain the same value of liquidity. To resolve this we suppose that $E(\lambda_t) = 1$, which is achieved by setting $\omega = 1 - \beta - \gamma$. The properties of this model and the overall estimation strategy are discussed in HLW.[7]

[7] A more general approach could be based on Inoue et al. (2021) where all the parameters are allowed to change over time in a smooth fashion. This would be a purely nonparametric model that includes ours as a special case (at least when the shocks are assumed to be i.i.d.). Our model leans on the old idea of trend+cycle decomposition and imposes more structure with the benefit of faster convergence rates for some quantities.

The Amihud (2002) liquidity measure is formally defined as $A_t = \sum_{s\in I_t} \ell_s/n_t$. Usually, A is measured monthly, quarterly, or annually by averaging the daily values of ℓ_s within period I_t of length n_t days.[8] Note that our trend function $g(t/T)$ can be interpreted as the model-specific counterpart of the low-frequency measure A_t. The key feature of our model is that we allow the detrended liquidity $\ell_t^* = \ell_t/g(t/T)$ to have nontrivial short-term predictability through the process λ_t so that shocks to liquidity have persistent effects on the level of liquidity relative to trend. In practice, both short-term and long-term predictability is present in this series, and it is important to take account of both features when carrying out inferential procedures and in predicting future liquidity.

Our model implies that $E(\ell_t \mid \mathcal{F}_{t-1}) = g(t/T)\lambda_t$ and $E(\ell_t) = g(t/T)$, which is the basis of our estimation strategy. It also implies that $\mathrm{var}(\ell_t \mid \mathcal{F}_{t-1}) = g(t/T)^2\lambda_t^2\sigma_{\zeta t}^2$, where $\sigma_{\zeta t}^2 = \mathrm{var}(\zeta_t \mid \mathcal{F}_{t-1})$. If we additionally assume that ζ_t is i.i.d., then $\mathrm{var}(\ell_t \mid \mathcal{F}_{t-1})$ is, apart from a constant, the square of $E(\ell_t \mid \mathcal{F}_{t-1})$, which may be rather restrictive. Therefore, since we do not need this for estimation, we shall not require this, preferring the weaker assumption that $\zeta_t - 1$ is a martingale difference sequence.

We suppose that the liquidity trend function g is almost everywhere a smooth function of (rescaled) time, specifically, it has two continuous derivatives. The function g captures the long-run variation of liquidity, which is generally smoothly varying (and for most assets shows improvements over time). However, we allow the possibility of permanent shifts in liquidity (structural change) by allowing the function g to be discontinuous at a finite set of known points $u_1 = t_1/T, \ldots, u_m = t_m/T \in (0, 1)$. For a given point $u \in (0, 1)$ define the left and right limits of the function and its first two derivatives

$$\lim_{u\uparrow u} g^{(r)}(u) = g_-^{(r)}(u), \quad \lim_{u\downarrow u} g^{(r)}(u) = g_+^{(r)}(u), \quad r = 0, 1, 2,$$

[8] Acharya and Pedersen (2005) modify A_t by multiplying it by lagged market price to take care of nonstationarity and then winsorizing to reduce the effect of outliers. We instead explicitly model the trend to account for nonstationarity. In HLW we discussed the issue of heavy tails and how to deal with them.

which we assume are well defined and finite. We allow that $g_{-}^{(r)}(u_i) \neq g_{+}^{(r)}(u_i)$ for $i = 1, \ldots, m$ and $r = 0, 1, 2$, but for all $u \notin \{0, u_1, \ldots, u_m, 1\}$ we maintain that $g_{-}^{(r)}(u) = g_{+}^{(r)}(u) = g(u)$, for $r = 0, 1, 2$.[9] We adopt the convention that $g^{(r)}(.)$ are CADLAG (continuous from the right with limits from the left), that is, $g^{(r)}(u_i) = g_{+}^{(r)}(u_i)$, and so we may write for $r = 0, 1, 2$ and $u \in [0, 1]$

$$g^{(r)}(u) = g_0^{(r)}(u) + \mathcal{J}_i^{(r)} \mathbb{1}(u \in [u_i, u_{i+1}))$$

for some baseline function $g_0(.)$ that is twice continuously differentiable, where $\mathbb{1}(.)$ is the indicator function, and $\left|\mathcal{J}_i^{(r)}\right| \leq C < \infty$. One question of interest is whether a break has occurred, and, if one has occurred, how big is the effect in terms of the level of g. The size of the jump at point u_i is measured in level terms and percentage terms, respectively, by:

$$\mathcal{J}_i = \mathcal{J}_i^{(0)} = g_{+}(u_i) - g_{-}(u_i), \quad \mathcal{J}_{\%i} = 2\frac{g_{+}(u_i) - g_{-}(u_i)}{g_{+}(u_i) + g_{-}(u_i)}. \tag{3}$$

This is the magnitude of the permanent effect of the split on the level of Amihud illiquidity of a given firm at time u_i (the effect that remains in the absence of further changes).

LY develop an interesting framework for thinking about discreteness in markets. In their Corollary 1, they present a formula for their model implied nominal bid ask spread (in the case of continuous pricing but discrete quantities)

$$S_t^L = \frac{2\sigma\lambda_j L}{DVol_t} P_t^2, \tag{4}$$

where $\sigma\lambda_j$ can be interpreted as a volatility measure (they assume that value evolves in continuous time like a Poisson jump process with constant volatility) and $DVol_t$ is dollar trading volume, L is the round lot size, and P_t is the stock price level; they call this the square rule. We interpret $\sigma\lambda_j/DVol_t$ as our Amihud illiquidity measure, in which case we find that the percentage bid-ask spread, $s_t^L = S_t^L/P_t$, can be written in our notation as

$$s_t^L = 2L \times \ell_t \times P_t.$$

[9] Note that $g_{+}^{(r)}(0)$ and $g_{-}^{(r)}(1)$ are also assumed to be well defined finite quantities.

Let P^+ (ℓ^+) be the price (illiquidity) right after a split and P^- (ℓ^-) be the price (illiquidity) just before a split so that $P^+ = \varsigma P^-$, where ς is (one over) the split factor. The percentage effect of a stock split at u_i on the implied percentage bid-ask spread, can be measured in this framework by $2\left(L\ell^+P^+ - L\ell^-P^-\right)/\left(L\ell^+P^+ + L\ell^-P^-\right)$, where we drop the time subscripts for convenience. Assuming that the lot size does not change over the split period we can cancel L and P^- from the numerator and denominator, and replacing $\ell^\pm$ by the corresponding permanent component $g^\pm$ yields our LY measure of the permanent effect of a split at u_i on liquidity:

$$\mathcal{J}_{\%i}^{LY} = 2\frac{\varsigma_i g^+(u_i) - g^-(u_i)}{\varsigma_i g^+(u_i) + g^-(u_i)}, \tag{5}$$

where ς_i is (one over) the split factor.[10] For forward splits, $\varsigma_i < 1$ and so $\mathcal{J}_{\%i}^{LY}$ could be negative when $\mathcal{J}_{\%i}$ is positive - a split could make Amihud illiquidity worse but the LY implied bid ask spread better in percentage terms. From this it is quite clear why the LY measure gives different results.

3 Econometric methodology

3.1 Estimation of the model

We observe a sample of daily illiquidities $\{\ell_t,\ t = 1, \ldots, T\}$ for a given firm. We first estimate the trend process by a local linear kernel smoother designed to be robust to potential breaks at known distinct points $u_0 = 0 < u_1 < u_2 < \cdots < u_m < 1 = u_{m+1}$. Specifically, we define $\widehat{g}(u) = F(\widehat{\alpha}_0(u))$ and $\widehat{g}'(u) = F'(\widehat{\alpha}_0(u))\widehat{\alpha}_1(u)$, where for $u \in [u_i, u_{i+1})$,

$$(\widehat{\alpha}_0(u), \widehat{\alpha}_1(u)) = \arg\min_{\alpha_0,\alpha_1} \sum_{t=1}^{T} K_h(t/T-u)\left\{\ell_t - F(\alpha_0 + \alpha_1(t/T-u))\right\}^2 1\left(u_i \le t/T < u_{i+1}\right). \tag{6}$$

[10]On the other hand, if the round lot size changes in the same proportion, the effect cancels out and $\mathcal{J}_{\%i}^{LY} = \mathcal{J}_{\%i}$.

Here, $K_h(.) = K(./h)/h$, where K is a continuous kernel supported on $[-1, 1]$ and h is a bandwidth, while F is a known function. The choice of $F : \mathbb{R} \to \mathbb{R}_+$ imposes non-negativity on $\widehat{g}(u)$, albeit at the cost of nonlinear optimization. For most series this step may be unnecessary and the choice of $F(y) = y$ yields the local linear regression, which has an explicit formula

$$\begin{pmatrix} \widehat{\alpha}_0(u) \\ \widehat{\alpha}_1(u) \end{pmatrix} = S_T(u)^{-1}\Delta_T^{-1}Q_T(u)$$

$$S_T(u) = \frac{1}{T}\sum_{t=1}^{T} 1\left(u_i \leq t/T < u_{i+1}\right) K_h(t/T-u) \begin{pmatrix} 1 & \frac{t/T-u}{h} \\ \frac{t/T-u}{h} & \left(\frac{t/T-u}{h}\right)^2 \end{pmatrix}$$

$$Q_T(u) = \frac{1}{T}\sum_{t=1}^{T} 1\left(u_i \leq t/T < u_{i+1}\right) K_h(t/T-u) \begin{pmatrix} 1 \\ \frac{t/T-u}{h} \end{pmatrix} \ell_t,$$

where $\Delta_T = \text{diag}(1, h)$. We continue the presentation with the case $F(x) = x$ for simplicity. HLW used a local constant estimator with an additional explicit boundary adjustment. Our method provides an automatic boundary adjustment that preserves the rate of convergence of the estimator at all points $u \in [0, 1]$. At interior points of the interval (u_i, u_{i+1}) the estimator is the standard local linear with two-sided smoothing, but for points $u_i + ch$ with $c \in [0, 1]$ only time points to the right of u_i are included and at points $u_{i+1} - ch$ with $c \in [0, 1]$ only points to the left of u_{i+1} are included. This is exactly what happens with the local linear estimator at a boundary point of the covariate support, see Fan and Gijbels (1996). The estimator $\widehat{g}(u)$ is CADLAG everywhere on $[0, 1]$ and is continuous everywhere except at the points $\{u_0, u_1, \ldots, u_{m+1}\}$, consistent with the posited behaviour of $g(u)$. Robinson (1989) established large sample properties of local constant kernel estimators in a similar time series setting without structural breaks, and Francisco-Fernández et al. (2003) established uniform consistency. Kristensen (2012) and others have extended these results to settings with abrupt changes like ours. We carry out the estimation of the dynamic parameters of λ_t using this jump robust estimator. In the supplementary material we discuss how we select bandwidth h.

Define the detrended liquidity $\widehat{\ell}_t^* = \ell_t/\widehat{g}(t/T)$, $t = 1, \ldots, T$. We use GMM to estimate the dynamic parameters $\theta = (\beta, \gamma)^\intercal$ from the conditional moment restriction $E(\ell_t^*|\mathcal{F}_{t-1}) = \lambda_t$, where $\ell_t^* = \ell_t/g(t/T)$, $t = 1, \ldots, T$. We work with the residual $\ell_t^*/\lambda_t(\theta) - 1$, which is a martingale difference sequence at the true parameter values $\beta = \beta_0, \gamma = \gamma_0$. In practice, we use $\widehat{\ell}_t^*/\widehat{\lambda}_t(\theta) - 1$, where $\widehat{\lambda}_t(\theta) = 1 - \beta - \gamma + \beta\lambda_{t-1} + \gamma\widehat{\ell}_{t-1}^*$. Then define $\rho_t(\theta, \widehat{g}) = z_{t-1}(\widehat{\ell}_t^*/\widehat{\lambda}_t(\theta) - 1)$ and

$$\widehat{\theta}_{GMM} = \arg\min_{\theta\in\Theta} \left\| M_T(\theta, \widehat{g}) \right\|_W, \quad M_T(\theta, \widehat{g}) = \frac{1}{T}\sum_{t=1}^{T} \rho_t(\theta, \widehat{g}), \tag{7}$$

where W is a weighting matrix, while $z_t \in \mathcal{F}_t$ are instruments, for example lagged values, while for a vector a and matrix W, $\|a\|_W^2 = a^\intercal W a$. This provides initial root-T consistent estimators of θ under the conditions of HLW, which are based on Chen et al. (2003). These conditions include undersmoothing (relative to what would be optimal for estimation of $g(u)$), that is, the first round bandwidth sequence h satisfies $Th^4 \to 0$.

Given consistent estimates of $\theta, g(.)$ one can improve $\widehat{\theta}_{GMM}$ and $\widehat{g}(.)$ for efficiency gains or simplicity of standard errors by working with the "pre-whitened" liquidity, $\ell_t^\dagger = \ell_t/\lambda_t$. We have $E\left(\ell_t^\dagger\right) = g(t/T)$, which provides an alternative local moment condition for estimation of g, and one that is purged of the short-run variation induced by the dynamic process λ_t. We let $\widetilde{g}(u) = \widetilde{\alpha}_0(u)$, where $(\widetilde{\alpha}_0(u), \widetilde{\alpha}_1(u))$ are defined as the minimizers of (6) with ℓ_t replaced by $\widehat{\ell}_t^\dagger = \ell_t/\widehat{\lambda}_t$, where $\widehat{\lambda}_t = \widehat{\lambda}_t(\widehat{\theta}_{GMM}, \widehat{g})$, and $\widehat{\theta}_{GMM}, \widehat{g}(.)$ were estimated in the previous procedure. HLW show that the preliminary estimation of λ_t by $\widehat{\lambda}_t$ has no effect on the large sample distributions of the "pre-whitened" estimator $\widetilde{g}(u)$. As discussed in HLW, the benefit of working with $\widetilde{g}(u)$ is that its large sample variance is much simpler to estimate than the large sample variance of $\widehat{g}(u)$ (which requires long-run variance estimation), which tends to benefit inference in finite and large samples.

To estimate the jump size at points u_i we compute two different estimates of $g(u_i)$, a left sider and a right sider. Specifically, we let $\widehat{g}_+(u_i) = \widehat{\alpha}_{0,+}(u_i)$ and $\widehat{g}_-(u_i) = \widehat{\alpha}_{0,-,}(u_i)$, $i = 1, \ldots, m$, where: $(\widehat{\alpha}_{0,+}(u_i), \widehat{\alpha}_{1,+}(u_i))$ minimize (6), while $(\widehat{\alpha}_{0,-}(u_i), \widehat{\alpha}_{1,-}(u_i))$ minimize (6) with $1\left(u_i \leq t/T < u_{i+1}\right)$ replaced by $1\left(u_{i-1} \leq t/T < u_i\right)$. We further let $\widetilde{g}_+(u_i) = \widetilde{\alpha}_{0,+}(u_i)$ and $\widetilde{g}_-(u_i) = \widetilde{\alpha}_{0,-,}(u_i)$, $i = 1, \ldots, m$, where: $(\widetilde{\alpha}_{0,+}(u_i), \widetilde{\alpha}_{1,+}(u_i))$ minimize (6) with ℓ_t replaced by $\ell_t/\widehat{\lambda}_t$, while $(\widetilde{\alpha}_{0,-}(u_i), \widetilde{\alpha}_{1,-}(u_i))$ minimize (6) with ℓ_t replaced by $\ell_t/\widehat{\lambda}_t$ and $1\left(u_i \leq t/T < u_{i+1}\right)$ replaced by $1\left(u_{i-1} \leq t/T < u_i\right)$. Note that $\widehat{g}_-(u_i), \widetilde{g}_-(u_i)$ consistently estimate $g_-(u_i)$.

The raw and percentage sizes of the jump can be estimated by the first round or second round estimates, and for preserving space we just present here the second round versions

$$\widetilde{\mathcal{J}}_i = \widetilde{g}_+(u_i) - \widetilde{g}_-(u_i), \quad \widetilde{\mathcal{J}}_{\%i} = \frac{2\left(\widetilde{g}_+(u_i) - \widetilde{g}_-(u_i)\right)}{\widetilde{g}_+(u_i) + \widetilde{g}_-(u_i)}. \tag{8}$$

3.2 Test of permanent effect

3.2.1 Single Split

We first consider the null hypothesis that $g_-(u_i) = g_+(u_i)$ versus the alternative that $g_-(u_i) \neq g_+(u_i)$ for a given u_i. One may also consider the "kinked" case where g is continuous but its first or higher order derivatives are not continuous, which is a more subtle change in the liquidity trend, but our focus is on jumps (or breaks) in the level of liquidity. There is a large literature on testing for structural change (at an unknown point) in parametric models, Perron (1989), and in nonparametric regression, Muller (1992), Delgado and Hidalgo (2000), and Vogt (2015). In our case the change point(s) are known in advance so the methodology is more similar to that of the regression discontinuity literature, see Imbens and Lemieux (2008), Cattaneo and Titiunik (2022). The main difference here is that we have a fully specified dynamic model in the background that provides a framework for the construction of standard errors appropriate for the type of dependence found in this type of data, and the model also suggests alternative estimates of the jumps based on the prewhitened liquidity. We test for the presence of a discontinuity at u_i by computing an adjusted t-statistic based on the final stage one-sided local linear estimators. We employ different ways of conducting inference in this nonparametric setting that take account of the smoothing bias, including some recently proposed methods.

Define the standard error and bias and the infeasible test statistic where for simplicity here we suppose that a common bandwidth h (in practice our estimate of $h_{\Delta,opt}(u_i)$ defined above) is used:

$$SE(u_i) = \sqrt{||K^+||^2 \left(\sigma_+^2(u_i) + \sigma_-^2(u_i)\right)/Th}, \quad b(u_i) = \frac{1}{2}h^2\mu_2(K^+)\left(g_+^{(2)}(u_i) - g_-^{(2)}(u_i)\right),$$

$$\tau(u_i) = \frac{\widetilde{\mathcal{J}}_i - \mathcal{J}_i}{SE(u_i)}. \tag{9}$$

Here, K^+ is the right boundary kernel derived from K, which is defined in the supplementary material. Note that the estimators subscripted $+$ are asymptotically independent of the estimators subscripted $-$, because $K^+ \times K^- = 0$, no matter what the correlation structure of the errors (this explains why there is no covariance term in the standard errors). In practice, we replace $\sigma_\pm^2(u_i)$ by estimates $\widetilde{\sigma}_\pm^2(u_i)$, where

$$\widetilde{\sigma}_\pm^2(u_i) = \widetilde{g}_\pm(u_i)^2 \times \widehat{\sigma}_\zeta^2, \quad \widehat{\sigma}_\zeta^2 = \frac{1}{T}\sum_{t=1}^{T}\left(\widehat{\zeta}_t - \overline{\widehat{\zeta}}\right)^2.$$

Let $\widetilde{\tau}(u_i)$ denote the feasible statistic with $SE(u_i)$ replaced by the estimated version $\widetilde{SE}(u_i)$.

Theorem 1. *Suppose that the conditions A1-A3 given in the supplementary material hold and that conditions A4-A6 of HLW hold. Furthermore, suppose that $Th^5 \to \gamma$, where $\gamma \in [0, \infty)$. Then, we have as $T \to \infty$*

$$\widetilde{\tau}(u_i) \Longrightarrow N(\rho_i, 1), \quad \rho_i = \lim_{T\to\infty}\frac{b(u_i)}{SE(u_i)}. \tag{10}$$

We test the hypothesis that $\mathcal{J}_i = 0$ by comparing the value of $\widetilde{\tau}(u_i)$ (with $\mathcal{J}_i = 0$) with some critical value that we discuss below.

Stock splits are not the only reason why the liquidity process can experience discontinuities, there may be other firm specific reasons as well as macroeconomic effects and regulatory changes. These other breaks do not impact $\widehat{g}(u_i)$ or $\widehat{g}_{\pm}(u_i)$ when they occur at sufficiently different timepoints, since our kernel estimators are local, but they may affect estimation of θ (although since the distortion is only affecting a small fraction h of the observations, this may not be catastrophic) and hence $\widetilde{g}(u_i)$ and $\widetilde{g}_{\pm}(u_i)$. One solution to this is to identify the other break points using the methodology of Delgado and Hidalgo (2000) and then to implement $\widehat{g}(u_i), \widetilde{g}(u_i)$ using all the identified breakpoints in (6). Since the break points are estimated at a faster rate than the break magnitudes, their estimation does not affect the limiting behavior of the test statistic $\widetilde{\tau}(u_i)$. The alternative solution is to work with $\widehat{g}_{\pm}(u_i)$ throughout, although the test statistics associated with this require long run variance estimation.

In some cases, the event of interest takes place at the same time as other structural changes affecting all stocks, such as during decimilization or the Global Financial Crisis. In this case, we propose to include a control group to eliminate common trends at the change time. This amounts to a diff in diff test, Angrist and Pischke (2009). Specifically, suppose that we have a "treatment" stock labelled with an S subscript and a "control" stock labelled with an C subscript.[11] We suppose that the univariate dynamic model holds for both stocks separately but that the corresponding shocks ζ_{St} and ζ_{Ct} may be correlated. We define $\widetilde{g}_{S,\pm}(u_i)$, $\widetilde{g}_{C,\pm}(u_i)$, $\widetilde{\mathcal{J}}_{C,i}$, $\mathcal{J}_{C,i}$, $\widetilde{\mathcal{J}}_{S,i}$, and $\mathcal{J}_{S,i}$ for the given period u_i, and then define the diff-in-diff statistic as

$$\tau_{did}(u_i) = \frac{\left(\widetilde{\mathcal{J}}_{S,i} - \widetilde{\mathcal{J}}_{C,i}\right) - \left(\mathcal{J}_{S,i} - \mathcal{J}_{C,i}\right)}{SE_{did}(u_i)} \tag{11}$$

$$SE_{did}(u_i) = ||K^+|| \sqrt{\frac{\left(\sigma^2_{S,+}(u_i) + \sigma^2_{C,+}(u_i) - 2\sigma_{S,C,+}(u_i)\right) + \left(\sigma^2_{S,-}(u_i) + \sigma^2_{C,-}(u_i) - 2\sigma_{S,C,-}(u_i)\right)}{Th}}$$

[11] In the empirical section we do a synthetic control method explained there.

$$b_{did}(u_i) = \frac{h^2}{2} \times \mu_2(K^+) \times \left(\left(g^{(2)}_{S,+}(u_i) - g^{(2)}_{S,-}(u_i) \right) - \left(g^{(2)}_{C,+}(u_i) - g^{(2)}_{C,-}(u_i) \right) \right)$$

$$\widetilde{\sigma}_{S,C,\pm}(u) = \widetilde{g}_{S,\pm}(u_i) \times \widetilde{g}_{C,\pm}(u_i) \times \widehat{\sigma}_{\zeta_S,\zeta_C}, \quad \widehat{\sigma}_{\zeta_S,\zeta_C} = \frac{1}{T}\sum_{t=1}^{T}(\widehat{\zeta}_{St} - \overline{\widehat{\zeta}}_S)(\widehat{\zeta}_{Ct} - \overline{\widehat{\zeta}}_C).$$

Define $\widetilde{\tau}_{did}(u_i)$ like $\tau_{did}(u_i)$ but with $\widetilde{SE}_{did}(u_i)$ in place of $SE_{did}(u_i)$.

THEOREM 2. *Suppose that the conditions A1-A4 given in the supplement hold and that condition A4-A6 of HLW hold. Furthermore, suppose that $Th^5 \to \gamma$, where $\gamma \in [0, \infty)$. Then, we have as $T \to \infty$*

$$\widetilde{\tau}_{did}(u_i) \Longrightarrow N(\rho_{did}, 1), \quad \rho_{did} = \lim_{T\to\infty} \frac{b_{did}(u_i)}{SE_{did}(u_i)}.$$

We test the hypothesis that $\mathcal{J}_{S,i} = \mathcal{J}_{C,i}$ by comparing the value of $\widetilde{\tau}_{did}(u_i)$ (with $\mathcal{J}_{S,i} - \mathcal{J}_{C,i} = 0$) with some critical value that we discuss below. We comment that the control group approach heavily relies on being able to find stock(s) that are not themselves influenced by the effect on the treatment group, i.e., where spillover effects from S to C are not anticipated. One may use a single stock as control or construct a synthetic control, see Abadie (2021).

Inference. We consider several approaches for inference; we focus on the statistic $\widetilde{\tau}(u_i)$, but the same arguments apply to $\widetilde{\tau}_{did}(u_i)$. The central approach we adopt is to assume that the bias terms e.g., $b(u_i)$, are of smaller order such that one does not need to account for it in the CLT and one does not need to estimate $g^{(2)}_{\pm}(u_i)$. This holds under the case that $g^{(2)}_{+}(u_i) = g^{(2)}_{-}(u_i)$, i.e., the curve has a level shift but the curvature from the left and the right are equal. It also holds when this condition is violated provided that $\gamma = \lim_{T\to\infty} Th^5 = 0$, which is referred to as the undersmoothing case. In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\widetilde{\tau}(u_i)$ with the normal critical values $\pm z_{\alpha/2}$ for a size α test.

An alternative approach is to consistently estimate the bias and subtract it from the estimator, which can be done in a number of ways either explicitly or implicitly but for consistent estimation of the bias another bandwidth has to be used. A popular method called jackknife is due to Schucany and Sommers (1977) in which we replace the estimator $\widetilde{g}_{\pm}(u_i;h)$ computed with whatever bandwidth h by $2\widetilde{g}_{\pm}(u_i;h/2)-\widetilde{g}_{\pm}(u_i;h)$, which removes the bias effect from the limiting distribution but raises the variance by making the implicit kernel of higher order and hence raising its L_2 norm, see Härdle and Linton (1994). Calonico et al. (2014) advocate an explicit bias correction with the same bandwidth used in the estimation of g. This bias correction eliminates the bias asymptotically but leads to an additional contribution to the variance, which needs to be accounted for. In the application we consider a parametrically guided bias correction based on the pilot parametric model that has been used for bandwidth selection as outlined in the supplementary material. We suppose that $g_+(u), g_-(u)$ are globally polynomial with parameters $\sum_{j=0}^{p} a_j^i u^j$ on the interval $[u_i, u_{i+1})$, in which case the bias is $b(u_i) = h^2\mu_2(K)r(u_i)/2$, where $r(u_i) = \sum_{j=2}^{p} j(j-1)(a_j^{i+1}-a_j^i)u_i^{j-2}$, and we estimate the parameters a_j^i by segmented global polynomial regression on the separate regimes. One does not need to adjust the standard error in our case because the estimates of a_j^i are root-T consistent.[12] In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\widetilde{\tau}(u_i) - \widetilde{b}(u_i)/\widetilde{SE}(u_i)$ with $\pm z_{\alpha/2}$ for a size α test.

Armstrong and Kolesár (2020) suggest an alternative approach that provides "honest" confidence intervals. In this case one estimates an upper bound on the bias terms. Specifically, let $\rho_{\max} = \max_{1\leq i\leq J}\sup_{\mathcal{G}}\lim_{T\to\infty}(\mid b(u_i)\mid /SE(u_i))$ be the upper bound over the class of functions $\mathcal{G}$. We let $\widetilde{\rho}_{\max} = \max_{1\leq i\leq J}(\mid \widetilde{b}(u_i)\mid /\widetilde{SE}(u_i))$ be the estimated upper bound described below. In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\widetilde{\tau}(u_i)$ with $cv_{1-\alpha}(\widetilde{\rho}_{\max})$, where $cv_{1-\alpha}(\widetilde{\rho}_{\max})$ is the $1-\alpha$ critical value of the folded normal distribution $\mid N(\widetilde{\rho}_{\max},1)\mid$ for a size α test. In one implementation Armstrong and

[12]Note that in our case there are also bias terms from $\widehat{\theta}_{GMM}$, which are of smaller order since we required that $Th^4 \to 0$ for that theory.

Kolesár (2020) use a global polynomial model for $g_+(u), g_-(u)$ on the interval $[u_i, u_{i+1})$ to estimate the bias $b(u_i)$, as we have done above for bandwidth selection and bias correction.

Our tests are valid against all fixed departures for which $\mathcal{J}_i \neq 0$, since $\widetilde{\tau}(u_i) \xrightarrow{P} \infty$ (with $\mathcal{J}_i = 0$) in this case. They also have power against some small departures, specifically, power lies between zero and one for alternatives such that $\mathcal{J}_i = \delta_i \Delta_T$ for some sequence $\Delta_T \to 0$ such that $\sqrt{Th}\Delta_T \to \Delta \neq 0$. Our tests also have power against local alternatives regarding the timing of the break (when there may be some small anticipation or delay in the effects), such as the break point occurring at $u_i \pm ch$ for some $c \in [0, 1]$, see Hidalgo and Seo (2013) for some discussion. This is relevant since a number of authors include quarantine periods in their before and after analysis, see for example Copeland (1979) and Li and Ye (2023). One can also construct confidence intervals for $\mathcal{J}_i$ and $\widetilde{\mathcal{J}}_{\%i}$ using any of the above approaches without imposing $\mathcal{J}_i = 0$, since the distribution theory is stated in this general case.

To save space, we do not report the simulation results here, but keep them available upon request. We note that both the undersmoothing method and the honest confidence interval approach work well overall, especially when the sample size is large. In smaller samples (e.g. n=1000), the honest confidence interval approach performs slightly better than the undersmoothing approach. For the bias correction method, we observe that there is an over-rejection issue when the true trend does not have a break and this does not improve as the sample size increases. Therefore, a more sophisticated bias correction method should be considered but we do not pursue this direction in this paper.

3.2.2 Multiple Splits

We provide a joint test of the null hypothesis of no breaks at any of the u_i versus the alternative of one or more breaks. We may also average across a sample of firms $j = 1, \ldots, n$ with breaks at times u_{ji}, $i = 1, \ldots, m_j$, so we suppose that N is the total number of events being considered, where $N \leq \sum_{j=1}^{n} m_j$. We consider either of the statistics

$$W = \sum_{i=1}^{N} \widetilde{\tau}(u_i)^2, \quad M = \max_{1 \leq i \leq N} \left| \widetilde{\tau}(u_i) \right|, \tag{12}$$

Provided the splits occur at different times the above arguments regarding the asymptotic variances follow. Under the null hypothesis of no breaks anywhere, W is asymptotically distributed as $\sum_{i=1}^{N} (Z_i + \rho_i)^2$ and M is asymptotically distributed as $\max_{1 \leq i \leq N} |Z_i + \rho_i|$, where Z_i are i.i.d. standard normal random variables (the individual t-statistics are mutually independent in large samples given the physical separation between u_i and u_j).

An alternative approach is to work with a directional test. Suppose that we pool the jumps across the splits (either for a given firm or across firms) as follows

$$\widetilde{\tau}_w = \frac{\sum_{i=1}^{N} w_i \widetilde{\tau}(u_i)}{\sqrt{\sum_{i=1}^{N} w_i^2}}, \tag{13}$$

where w_i is a (possibly stochastic) weighting scheme such as market cap or equal weighting. Then we may show that under the null hypothesis, we have (as $T \to \infty$ for N fixed) $\widetilde{\tau}_w \implies N(\rho_w, 1)$, where $\rho_w = \sum_{i=1}^{N} w_i \rho_i / \sqrt{\sum_{i=1}^{N} w_i^2}$. The individual statistics are uncorrelated across distinct points u_i. We may test the null hypothesis by comparing $\widetilde{\tau}_w$ with the normal critical values in the undersmoothing case or by comparing $\widetilde{\tau}_w$ with critical values $cv_{1-\alpha}(\widetilde{\rho}_{w,\max})$ in the "honest" confidence interval case, where $cv_{1-\alpha}(\widetilde{\rho}_{w,\max})$ is the $1-\alpha$ critical value of the folded normal distribution $\mid N(\widetilde{\rho}_{w,\max}, 1) \mid$ for a size α test. Here, the worst case ratio is $(\sum_{i=1}^{N} w_i / \sqrt{\sum_{i=1}^{N} w_i^2}) \times \max_{1 \leq i \leq N} \sup_{\mathcal{G}} \lim_{T \to \infty}(\mid b(u_i) \mid / SE(u_i))$. This test is more directional in its intent, and will not reject all null hypotheses, only those for which the discontinuities tend to go in the same direction, i.e., for which $\sum_{i=1}^{N} w_i \mathcal{J}_i \neq 0$. This is similar to the principle underlying the variance ratio tests and the usual way in which event studies are conducted through cumulative abnormal returns.

3.3 Test of temporary effects

We next consider how to allow for temporary effects or rather short-term adjustments that eventually die out. One approach might be to include dummy variables in the dynamic equation, that is, let

$$\lambda_t = 1 - \beta - \gamma + \beta\lambda_{t-1} + \sum_{j=1}^{J} \delta_j D_{jt} + \gamma\ell^*_{t-1}, \tag{14}$$

where D_{jt} is a dummy variable that is one in period t_j and zero otherwise. To allow for anticipation effects and slow transmission one could focus on times around the known intervention point, that is, if t_j is a stock split day, include dummy variables for $t_j - E, \ldots, t_j + E$ for some event window $\mathcal{E}$ of length $J = 2E + 1$. With multiple splits one could include dummy variables around all the key dates. Under this modelling assumption the level of the process λ_t is affected for all $t \geq t_1$, with a flexible effect between t_1 and t_J, but after t_J the effect decreases rapidly as $t - t_J \to \infty$ and the long run effect of the intervention is zero. One can estimate the parameters δ by GMM jointly with β, γ, but the estimates of δ_j so-formed are not consistent.

Instead of estimating the model (14), we propose a test of the null hypothesis that $\delta_1 = \ldots = \delta_J = 0$ against the general alternative under the assumption of i.i.d. shocks ζ_t. In fact our test is also valid when $\zeta_t - 1$ is only a stationary mixing martingale difference sequence. Our test is based on the residuals from the null estimation, which is the common practice in stock market event studies. Here, we just present the single event setting and take a simple approach. We suppose that the event window is given by $\{t_1 - E, \ldots, t_1 + E\}$. Define the residuals $\widehat{\zeta}_t = \ell_t / \widetilde{g}(t/T)\widehat{\lambda}_t$, $t = 1, \ldots, T$, where the estimation of $\widehat{\theta}$ and $\widetilde{g}(\cdot)$ are described above. Under our conditions, these residuals are asymptotically equivalent (as $T \to \infty$) to the true unobserved ζ_t. We define abnormal illiquidity and cumulative abnormal illiquidity at times $\tau = 0, \ldots, 2E$ as follows:

$$\widehat{AIL}_\tau = \widehat{\zeta}_{t_1 - E + \tau} - 1, \quad \widehat{CAIL}(\tau) = \sum_{s=0}^{\tau} \widehat{AIL}_s. \tag{15}$$

We do not use the usual normal critical values here because this is not likely to be a good assumption in view of the fact that $\zeta_t \geq 0$ and that the event window is typically short, i.e., E is finite so that a CLT does not apply. We use instead nonparametrically estimated critical values. Suppose that $\zeta_t - 1$ is a stationary mixing process with marginal distribution F that is unknown and let F_{e_τ} denote the marginal distribution of the stationary mixing series $\{e_{r,\tau}\}$, where $e_{r,\tau} = \sum_{s=0}^{\tau}(\zeta_{r+s} - 1)$. We estimate the distributions F and F_{e_τ} based on the data not including the event window, $S = \{1, \ldots, T\} \setminus \{t_1 - E, \ldots, t_1 + E\}$. Specifically, letting $\widehat{e}_{r,\tau} = \sum_{s=0}^{\tau}(\widehat{\zeta}_{r+s} - 1)$, we define

$$\widehat{F}_{\widehat{e}_\tau}(x) = \frac{1}{T_S} \sum_{t \in S} 1\left(\widehat{e}_{t,\tau} \leq x\right), \quad x \in \mathbb{R},$$

where T_S is the cardinality of the set S, and $\widehat{F}(x) = \widehat{F}_{\widehat{e}_0}(x)$. Then define the critical values $\widehat{F}_{\widehat{e}_\tau}^{-1}(\alpha/2), \widehat{F}_{\widehat{e}_\tau}^{-1}(1-\alpha/2)$. We reject the null hypothesis (for a given $\tau = 0, \ldots, 2E$) if $\widehat{CAIL}(\tau)$ is outside the interval $[\widehat{F}_{\widehat{e}_\tau}^{-1}(\alpha/2), \widehat{F}_{\widehat{e}_\tau}^{-1}(1-\alpha/2)]$. There is a large literature about estimation of distribution functions of residuals in time series settings, see for example Koul and Ling (2006) and Escanciano (2010).

Theorem 3. *Suppose that conditions A1-A4 in the supplementary material hold. Then, as $T \to \infty$, $\widehat{F}_{\widehat{e}_\tau}^{-1}(\alpha/2) \xrightarrow{P} F_{e_\tau}^{-1}(\alpha/2)$ and $\widehat{F}_{\widehat{e}_\tau}^{-1}(1-\alpha/2) \xrightarrow{P} F_{e_\tau}^{-1}(1-\alpha/2)$, and therefore the rejection frequency of our test converges to α under the null hypothesis.*

We may average across events (same firm different events or across firms) and obtain a CLT when the number of events being averaged across is large. Provided the timing of the stock splits across firms does not coincide very much, the standard errors can be based on the "as if independence" assumption. Specifically, let

$$\widehat{AAIL}(\tau) = \sum_{i=1}^{N} w_i \widehat{AIL}_{i\tau}, \quad \widehat{ACAIL}(\tau) = \sum_{i=1}^{N} w_i \widehat{CAIL}_i(\tau) \tag{16}$$

denote the averaged abnormal and cumulative abnormal illiquidity across N events. For large N and T, these statistics are asymptotically normal and can be compared with the critical values $\pm z_{\alpha/2}\sqrt{\sum_{i=1}^{N} w_i^2 \widehat{\sigma}_{\zeta i}^2}$ and $\pm z_{\alpha/2}\sqrt{(\tau+1)\sum_{i=1}^{N} w_i^2 \widehat{\sigma}_{\zeta i}^2}$, respectively, where $\widehat{\sigma}_{\zeta i}^2$ is the estimated variance of the corresponding ζ for the particular firm event.

Our tests are applied using the jump consistent estimate of the trend and so the null hypothesis here includes the possibility of a permanent change to liquidity at the specified points. We should expect to see that adjustments, if any, should be relatively quick if markets are efficient and so there should not be much of a role for slow dynamic responses.

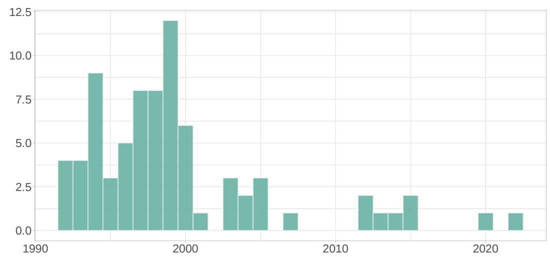
4 Empirical study: stock splits

4.1 Data description

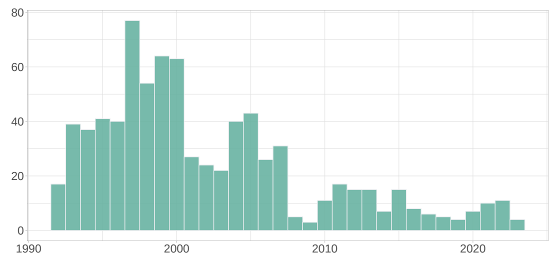
In our application, we use the proposed framework to analyze whether stock splits have permanent and temporary effects on the illiquidity process. We consider historical daily price and volume data for component stocks of the Dow Jones, the S&P 500 (Large caps), S&P 400 (mid caps) and S&P 600 (small caps) indices.[13] The sample period starts from each asset's first available data point (after June 15, 1992) until December 31, 2023. We plot in Figure 1 the number of splits by year. We observe that stock splits happen more often during periods of strong market performance. For example, numerous splits took place during the build-up phase of the dot-com bubble between 1992 and 2000, and a pronounced drop in the occurrence of splits can be observed across most stock indices in the aftermath of its collapse. Likewise, there were hardly any splits during the Global Financial Crisis. This empirical evidence suggests that periods of high price appreciation could be one of the factors motivating firms' decision to split their stock, in line with Fama et al. (1969).

We summarize in Table 2 the frequency of different split sizes. The vast majority of the splits in the sample are two-for-one or lower, and larger stock splits occur less frequently.

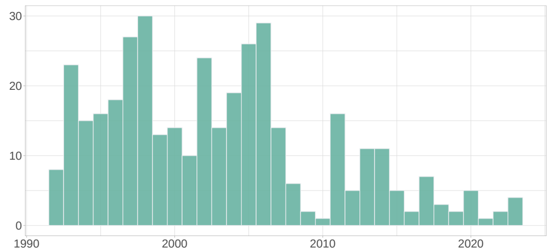
[13]The price and volume data as well as the stock split information are retrieved from the CRSP database.



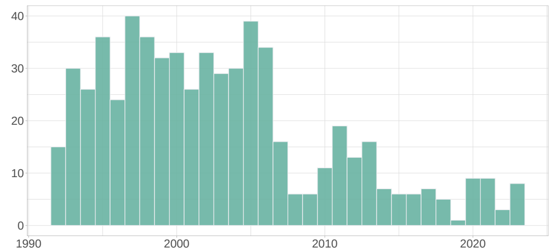
(a) Dow Jones index.



(b) S&P 500 index.



(c) S&P 400 index.



(d) S&P 600 index.

Figure 1: Number of splits per year.

For the Dow Jones index, large stock splits could be motivated by index inclusion reasons as the Dow Jones weights its constituents based on their stock price.[14] We focus on the most common split factor in our empirical study, i.e. the two-to-one stock splits, leaving us with 519 distinct index constituents and 943 splits for the analysis below.

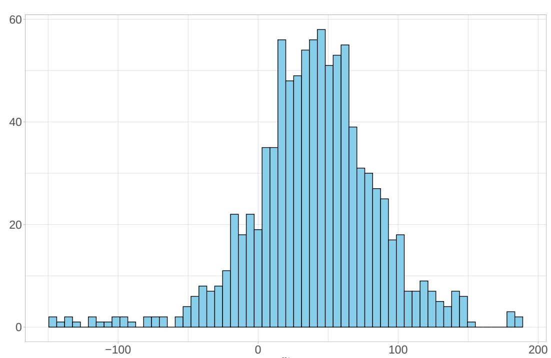
Table 2: Distribution of different split sizes 1992-2023.

Split Size	<1	$1-1.25$	$1.25-1.5$	1.5	2	3	4	>4	Total
Dow Jones	0	0	1	7	61	3	3	2	77
S&P 500	17	2	14	167	518	42	11	13	788
S&P 400	18	2	47	94	201	14	6	1	383
S&P 600	61	8	47	204	263	17	4	1	611

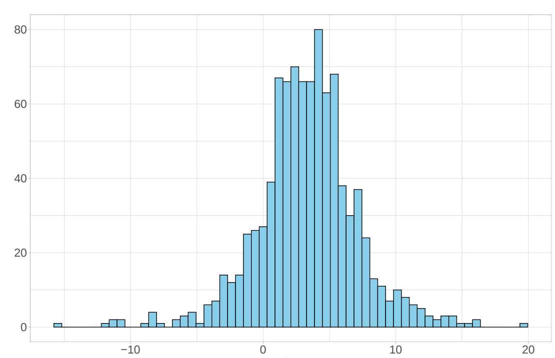
[14] For instance, Apple joined the Dow Jones index in 2015 after undergoing a seven-to-one stock split in June 2014. The split brought Apple's stock price into a more comparable range to the other constituents.

4.2 Test for permanent effects

We assess whether a permanent shift in illiquidity occurs at the time of the i^{th} split for stock j, denoted as u_i^j. This is achieved by testing for potential discontinuities in the long-run trend function g at u_i^j. More specifically, we estimate $g^{\pm}(u_i^j)$ using the local linear approach and construct the test statistics $\tau(u_i^j)$ as described in Section 3.2. To facilitate the computation of the variance of the two estimates $\hat{g}^{\pm}(u_i^j)$, we work with the improved estimator obtained by smoothing out ℓ_t, i.e. $\ell_t/\hat{\lambda}_t$. We first look at the size of the jump in percentage terms for each split as defined in (3), i.e. $\widetilde{\mathcal{J}}(u_i^j)$. Panel (a) of Figure 2 presents the distribution of $\widetilde{\mathcal{J}}(u_i^j)$ obtained using the undersmoothing method. The results show that the majority (85.47%) of the distribution lies in the positive domain, indicating significant increases in the long-run illiquidity trend level and a corresponding negative effect of stock splits on long-run liquidity conditions. Additionally, we find that 81.65% of the splits yield a positive $\widetilde{\mathcal{J}}(u_i^j)$ statistic when we consider the bias correction method.



(a) $\widetilde{\mathcal{J}}$.



(b) $\widetilde{\tau}$.

Figure 2: Cross-split distribution of $\widetilde{\mathcal{J}}$ and $\widetilde{\tau}$.

We compute the test statistic $\tau(u_i^j)$ using the undersmoothing (τ^{US}), bias correction (τ^{BC}) and honest confidence interval (τ^{HCI}) approaches developed in Section 3.2.[15] In panel

[15]In the undersmoothing and bias correction cases, the τ_w statistics should be compared to standard normal critical values (± 1.96 for $\alpha = 5\%$). For the honest confidence interval approach, we should compare

(b) of Figure 2, we present the distribution of $\tau(u_i^j)$ obtained with the undersmoothing method. We observe that the majority (67.13%) of the test statistics are positive and exceed 1.96, suggesting a significant deterioration in liquidity conditions following stock splits. This result is robust across different inference approaches, with respectively 65.43% and 58.64% of the test statistics being significant when one uses the bias correction and honest confidence interval approaches.

The timing of stock splits might sometimes coincide with other market events affecting all stocks simultaneously. Examples of such instances include the introduction of decimalization in US stock markets in the early 2000s and the Global Financial Crisis. To control for these significant events, we use the stock market index ETF (SPY) as a control group to eliminate common trends at the event date and isolate the effects of stock splits from other market-wide events. We report in Table 3 the difference-in-difference (DID) test statistics computed as in (11). We observe that 82% of the test statistics are positive and around 50% are so. This confirms that stock splits negatively impact long-run liquidity conditions even when accounting for potential confounding effects from concurrent market-wide events.

Table 3: Summary of τ^{DID} statistics.

	Min	25%	Median	75%	Max	$\tilde{\mathcal{J}}_{\%w}/\tau_w > 0(\%)$	significant (%)
τ_w^{DID}	-11.47	0.24	1.94	4.73	15.37	81.79%	49.57%

4.2.1 Aggregating multiple splits by firm

We first look at the average jump size in percentage terms aggregated across all splits for each stock. The values of the statistics are summarized in the second and third rows of

the τ_w statistics with the critical values from the corresponding folded normal distribution obtained based on the worst-case-scenario bias.

Table 4. The first row provides the number of splits.[16] We observe that stock splits increase the illiquidity trend level by 41% in the median case. The interquartile range suggests a high degree of variability around this central tendency.

Table 4: Summary of the average $\tilde{\mathcal{J}}_{\%w}$ statistics, p-values of the joint test W and the aggregated statistics for directional tests.

	Min	25%	Median	75%	Max	$\tilde{\mathcal{J}}_{\%w}/\tau_w > 0(\%)$	significant (%)
# of splits	1	1	2	2	6		
$\tilde{\mathcal{J}}^{US}_{\%w}$	-146.13%	18.91%	40.76%	62.74%	178.60%	88.05%	
$\tilde{\mathcal{J}}^{BC}_{\%w}$	-181.52%	14.90%	37.07%	62.58%	208.03%	84.01%	
p^{US}_W	0.00	0.00	0.00	0.01	0.93		83.43%
p^{BC}_W	0.00	0.00	0.00	0.05	1.00		74.95%
τ^{US}_w	-15.76	1.80	4.45	7.13	20.95	87.86%	73.99%
τ^{BC}_w	-16.94	1.43	4.70	7.82	18.53	83.62%	71.68%
τ^{HCI}_w	-17.34	1.59	5.02	7.84	19.11	84.78%	58.00%

Note: The statistics are aggregated by firm. $\tilde{\mathcal{J}}_{\%w}$ is the average jump in percentage as defined in (3). p_W is the p-value of the aggregated statistic $W = \sum_{i=1}^m \tau(u_i)^2$. τ_w is the statistic computed as in (13) and is asymptotically $N(0,1)$ under the null hypothesis. US stands for undersmoothing and BC stands for bias correction. HCI stands for honest confidence interval where we compare the τ_w statistics with $cv^{HCI}_{1-\alpha}$.

We aggregate the individual test statistics $\tau(u^j_i)$ for each company j to provide a joint test of the null hypothesis of no breaks at any point of the illiquidity series for this stock. The fourth and fifth rows of Table 4 report the p-values for the statistic $W^j = \sum_{i=1}^m \tau\left(u^j_i\right)^2$ for the undersmoothing and bias correction approaches. In the undersmoothing case, the p-values associated with the W^j statistics are below 5% for 83% of the stocks considered in our analysis. When considering the bias correction approach, the p-values suggest that the effects are significant for 75% of the stocks. These results indicate strong evidence against the null hypothesis of no long-term effect on liquidity from the stock split events.

The last three rows of Table 4 provide the directional average statistics (τ_w) defined as in (13) and computed using an equal weighting scheme. Each row reports the summary results using the undersmoothing, bias correction and honest confidence interval inference

[16] Our analysis includes a total of 519 stocks: 257 stocks with one two-to-one split, 154 stocks with two, 71 stocks with three, and 37 stocks with more than three splits within the sample period.

approaches. Most τ_w statistics are positive – 88% for undersmoothing, 84% and 85% for bias correction and honest confidence interval. In addition, 74% of the statistics are positive and significant, with a median value of 4.45 for τ_w. This indicates a significant difference in pre- and post-split long-run illiquidity trends on average. For the honest confidence interval method, we observe a lower percentage of significant statistics but a majority of the companies still experience a significant increase in the illiquidity trend level (58%).

4.2.2 Alternative illiquidity measures

This section evaluates the robustness of our main results to the use of the alternative illiquidity measures introduced in Section 2.1. The results are summarized in Table 5 and confirm that our main empirical findings remain robust across the different measures considered. The only exception is the LY implied bid-ask spread measure in the last row, which yields notably different results. This diverging result may stem from the additional presence of the nominal price level in the measure, which changes by the size of the split factor between the pre- and post-split date. This feature of the illiquidity measure may introduce a mechanical downward effect in the tests computed based on it.

Table 5: Summary of the aggregated statistics for directional tests.

ℓ_t	Min	25%	Median	75%	Max	$\tau_w > 0(\%)$	significant (%)
$\frac{\sigma_t}{V_t}$	-15.76	1.80	4.45	7.13	20.95	87.86%	73.99%
$\frac{\lvert R_t^{CC}\rvert}{V_t}$	-9.59	3.41	5.30	7.26	13.78	94.61%	87.67%
$\frac{\lvert R_t^{OC}\rvert}{V_t}$	-12.27	0.85	2.70	4.40	12.98	83.82%	59.73%
$\frac{\sigma_t}{\sqrt{V_t}}$	-9.13	2.44	5.40	8.05	18.81	88.82%	76.88%
$(\frac{\sigma_t^2}{V_t})^{\frac{1}{3}}$	-12.63	2.66	5.57	8.52	19.11%	88.05%	79.19%
$\frac{\sigma_t}{V_t}P_{t-1}^M$	-16.71	1.45	4.62	7.27	20.27	84.59%	70.91%
$\frac{\sigma_t}{V_t}\text{mcap}_t$	-10.92	2.28	5.10	7.77	17.85%	90.56%	77.84%
$\frac{\sigma_t}{V_t}P_t$	-15.84	-6.80	-3.59	-1.31	7.40	13.68%	68.59%

Note: R^{CC} is close-to-close return while R^{OC} is the open-to-close intra-day return. P^M is the price of the market index. The significant (%) column shows the percentage of positive and significant τ_w statistics for the first seven measures and the percentage of negative and significant τ_w statistics for the last measure.

4.3 Test for temporary effects

We plot in Figure 3 the aggregated test statistics for the temporary effects across all splits for all considered firms as defined in (16). The blue line represents the aggregated Cumulative Abnormal Illiquidity (ACAIL) test statistics at horizons ranging from 45 days before to 45 days after the event, i.e. $\widehat{ACAIL}(\tau), \quad \tau = 0, \ldots, 90$. The red lines are the 2.5% and 97.5% quantiles, i.e. $z_{\alpha/2}\sqrt{\tau \sum_{i=1}^{N} w_i^2 \widehat{\sigma}_{\zeta i}^2}$. We find very limited evidence supporting a statistically significant effect of stock splits on short-term illiquidity. In addition, there is no clear direction for the temporary effects.

We plot in Figures 12 and 13 (Appendix D.3) the aggregated test statistics across events of a given firm as defined in (16). The directions of the temporary effects for different firms are mixed, with most stocks exhibiting insignificant increases and decreases in liquidity over the event window. These unclear patterns are consistent with the mixed evidence in previous literature on the effect of stock splits on market liquidity, with some authors reporting liquidity improvements following a split (Lamoureux and Poon (1987)) and others documenting a reduction in liquidity or muted effect in the post-split period (Lakonishok and Lev (1987)). A notable exception is the Apple stock, which experiences a significant increase in cumulative abnormal illiquidity before the split execution date persisting in the post-split period. This is in line with the prediction from the signaling theory of Brennan and Copeland (1988) who model a firm's decision to split its stock as a costly signal about its prospects, which is associated with a (at least) temporary decrease in the stock liquidity.

To summarize, our empirical evidence suggests that stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but very limited effects on the short-run illiquidity dynamics. The documented increase in stocks' long-run illiquidity following a split challenges somewhat the predictions from the optimal price range and optimal tick size (Angel (1997)) theories that splitting firms should experience an increase

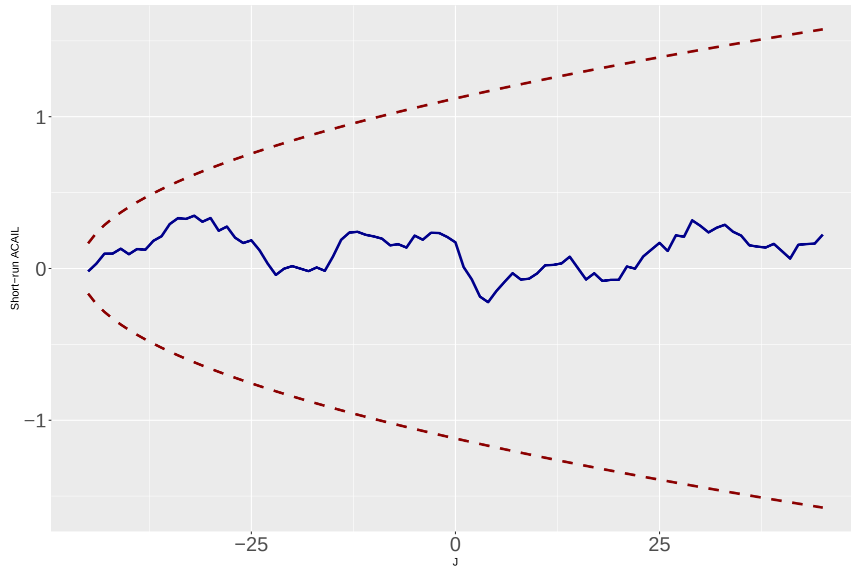


Figure 3: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the corresponding 2.5% and 97.5% quantiles.

in the liquidity of their stock in the long run. See also Goyenko et al. (2006) for an in-depth discussion of the short- and long-run liquidity effects of stock splits.

5 Reverse stock splits

We investigate whether the effects of reverse stock splits on illiquidity are symmetric to the ones documented for stock splits in Sections 4.2 and 4.3. The analysis in the literature shows that the reverse splits have a more pronounced improvement in liquidity conditions for low-price stocks. This could be because reverse splits ease the constraints on short selling for low-priced stocks, see e.g. Kwan et al. (2015). Therefore, in this part of the analysis, we focus our attention on the constituents of the S&P 500, S&P 400 and S&P 600 indices with a pre-event price level below \$5. In total, we have 53 stocks in our sample and there is only one reverse split for each stock in the sample. The sample period starts from each asset's first available data point (after June 15, 1992) until December 31, 2023.

We estimate the $g^{\pm}(u_0)$ functions using the local linear approach and construct the test

Table 6: τ_w statistics for directional tests..

	AAON	ACLS	AIG	ARWR	ASRT	BANR	BCEI	BCOR	BKNG	C	CAR
Split size	1-4	1-4	1-20	1-10	1-4	1-7	1-111.6	1-10	1-6	1-10	1-10
τ_w	-0.09	-2.71	-21.37	-10.53	4.42	15.40	-8.41	-7.87	-11.77	8.64	-17.14
$\tau_{w,SC}$	-2.63	-1.02	-6.75	-9.52	3.98	5.02	-14.07	-3.48	-8.38	0.15	-23.24

	CBB	CCOI	CIEN	CIVI	COO	CPE	CPF	CSII	CYTK	EPAC	EXPR
Split size	1-5	1-20	1-7	1-111.6	1-3	1-10	1-20	1-10	1-6	1-5	1-20
τ_w	-0.01	-2.20	3.83	-8.41	0.49	-2.71	-9.85	-6.97	-24.56	-14.86	-0.46
$\tau_{w,SC}$	3.77	-3.49	-6.72	-14.53	0.44	-1.51	-16.14	-9.19	-17.34	-6.90	-0.63

	FBP	FTR	HAFC	HPR	HSKA	IART	KEM	KLXE	LCI	LPI	MSTR
Split size	1-15	1-15	1-8	1-50	1-10	1-2	1-3	1-5	1-4	1-20	1-10
τ_w	-4.26	-1.90	-7.65	-4.10	-7.48	4.55	-3.93	1.02	-2.19	-7.99	-19.93
$\tau_{w,SC}$	-4.11	-0.86	-10.26	-1.59	-8.29	5.17	-1.94	2.64	-2.28	-7.75	-9.44

	MTH	NEU	ODP	OPCH	PFBC	PPBI	RRC	SANM	SBCF	SNV	SPPI
Split size	1-3	1-5	1-10	1-4	1-5	1-5	1-15	1-6	1-5	1-7	1-25
τ_w	-0.63	-4.48	-4.76	1.27	-4.15	0.47	-1.33	-13.75	-8.22	-1.69	6.36
$\tau_{w,SC}$	0.19	-7.12	1.09	1.06	-4.49	0.61	-2.30	-9.02	-7.76	-9.33	8.23

	SSP	THRM	TISI	UCBI	UFI	UIS	VIAV	XPO	ZD		
Split size	1-3	1-5	1-10	1-5	1-3	1-10	1-8	1-4	1-4		
τ_w	12.78	1.05	-2.22	0.66	-6.77	-18.41	-2.24	-0.09	-3.33		
$\tau_{w,SC}$	19.07	0.58	-1.53	-3.78	-12.32	-10.75	-0.20	0.25	-3.66		

Note: SC stands for synthetic control.

statistics $\tau(u_0)$ for the permanent effect introduced in Section 3.2. The test statistics $\tau(u^j)$, using an undersmoothing approach, are reported in the second row of Table 6. We observe for 40 out of 53 stocks that there is a negative effect of the reverse splits on the illiquidity trend – corresponding to an improvement in liquidity conditions. Among those, there are 32 stocks for which the decrease in the illiquidity trend after a reverse split is statistically significant.[17] The fact that reverse splits result in a significant decrease in the illiquidity

[17] We also consider the robustness of our results to the use of the bias-corrected and the honest confidence interval approaches. Using bias correction, 37 out of 53 stocks experienced improved liquidity conditions after a reverse split, and it is statistically significant for 34 of them. When adopting the honest confidence interval, 36 out of 53 stocks experienced improved liquidity conditions with 32 being statistically significant.

trend for the majority of the stocks considered is in line with the results in Han (1995). The pronounced improvement in liquidity conditions for our sample of stocks with low pre-event price levels is consistent with the evidence in Blau et al. (2023) that short-selling activity increases after reverse splits – in part because reverse splits ease the constraints on short selling for low-priced stocks (Kwan et al. (2015)).

We also consider the robustness of our results to the inclusion of a control group to account for other events that can potentially impact all stocks at the event time (see Equation (11) in Section 3.2). We use the synthetic control approach of Abadie (2021). For each reverse stock split event, we use the constituents from the same index that did not undergo a reverse split as the donor pool for the control group. We use the pre-event trend to construct the weights, i.e. $\min_{w} \sum (g^S - w' g^C)^2$ subject to $w \geq 0$, and $\mathrm{i}' w = 1$. The superscripts "S" and "C" refer to the "treatment" stock and "control" group respectively. The test statistics are reported in the third row of Table 6. Our conclusion that reverse stock splits significantly decrease the illiquidity trend level is robust to controlling for common changes in stock trends around the event time via the synthetic control approach.

We plot in Figure 4 the average test statistics for the temporary effects of the reverse stock splits aggregated across all firms.[18] The results echo our analysis in Section 4.3 for stock splits, with limited evidence supporting a statistically significant effect of reverse splits on short-term illiquidity. However, we do observe a short-lived worsening in liquidity conditions around the execution date although it is not significant. The test statistics for the temporary effects of individual reverse splits are plotted in Figures 14 to 19 (Appendix D.4). We can see that only a handful of stocks exhibit significant changes in cumulative abnormal illiquidity during the event window and, as in the stock split case, no clear pattern emerges for the impact of reverse splits on short-term illiquidity. We note however that the worsening in liquidity conditions observed for some stocks before the execution date is

[18] See Sections 3.3 and 4.3 for additional detail.

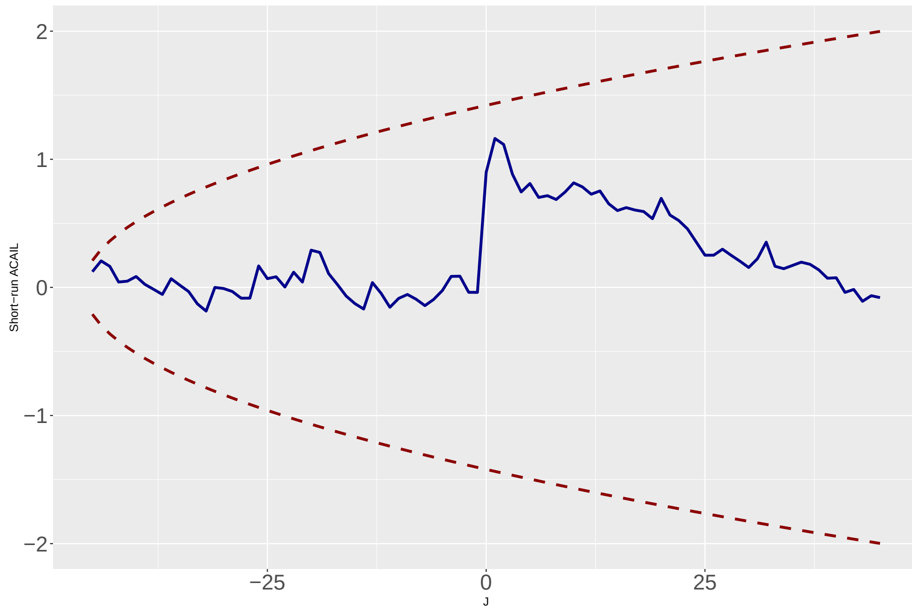


Figure 4: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the corresponding 2.5% and 97.5% quantiles.

consistent with reverse splits signaling a lack of confidence from executives in the prospects of their firm (Han (1995)).[19]

To summarize, our empirical evidence suggests that reverse stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but limited effects on the short-run illiquidity dynamics. The documented decrease in stocks' long-run illiquidity following a reverse split is the symmetric image of our results in Section 4.2 for stock splits and confirms earlier evidence in the literature such as Han (1995).

6 Conclusions

We propose a framework to detect the occurrence of permanent and transitory breaks in the illiquidity process. Our approach builds on the class of dynamic semiparametric models introduced in Hafner et al. (2023), which flexibly capture long-term trends with a non-

[19] See e.g. NewMarket Corporation and Avis Budget Group Inc.

parametric component and short-run variations with an autoregressive component. We develop various tests that can either be applied separately to individual events or can be aggregated across different events – over time for a given firm and/or across different firms. The test for permanent breaks in the long-run component of the illiquidity process is built on differences between forward- and backward-looking trend estimates, which is similar to the approach used in the regression discontinuity and structural break literature. The test for transitory breaks in the short-term illiquidity dynamics is inspired by the event-study approach pioneered by Fama et al. (1969) and is robust to the presence of permanent breaks in the long-run component of illiquidity.

Equipped with this testing framework, we revisit the long-standing debate surrounding the impact of stock splits on firm liquidity. Using a sample of 24 stocks from the Dow Jones index over the period 1992-2023, we find that stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but very limited effects on the short-run illiquidity dynamics. Our results are in line with previous studies documenting a permanent decrease in stock liquidity after a split (e.g. Copeland (1979)) and challenge the common view that stock splits should increase the potential pool of investors and lead to improved liquidity conditions. Finally, we investigate whether the effects of reverse stock splits on the illiquidity process are symmetric to the ones documented for stock splits. Using a sample of 53 low-price stocks from the constituents of the S&P 500, S&P 400 and S&P 600 indices over the same period, we find that reverse splits result in an overall significant decrease in long-run illiquidity but with limited effects on the short-run illiquidity dynamics. The improvement in liquidity conditions is quite pronounced for our sample of stocks with low pre-event price levels. This is consistent with evidence that short-selling activity increases after reverse splits (Blau et al. (2023)) for low-priced stocks as constraints are eased.

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Supplementary Material for “The permanent and transitory effects of stock splits on liquidity in a dynamic semiparametric model”

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A Proof of main results

We make some assumptions.

Definition. *A triangular array process $\{X_{t,T},\, t = 0, 1, 2, \ldots,\, T = 1, 2, \ldots\}$ is said to be alpha mixing if*

$$\alpha(k) = \sup_{T,n\geq 1} \sup_{A\in\mathcal{F}^{n,T}_{-\infty},\, B\in\mathcal{F}^{\infty}_{n+k,T}} |P(AB) - P(A)P(B)| \to 0, \tag{1}$$

as $k \to \infty$, where $\mathcal{F}^{n,T}_{-\infty}$ and $\mathcal{F}^{\infty}_{n+k,T}$ are two σ–fields generated by $\{X_{t,T},\, t \leq n\}$ and $\{X_{t,T},\, t \geq n + k\}$ respectively. We call $\alpha(\cdot)$ the mixing coefficient.

We suppose that ℓ^*_t is stationary and alpha mixing. This can be shown to hold under the parameter resrictions provided ζ_t is i.i.d. It may also hold when ζ_t itself is only described as a stationary mixing process although this can be difficult to establish.

We first consider the estimators that are based on the jump robust smoothing of the raw liquidity.

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Assumption A1. *We suppose that* $g(.) \in \mathcal{G}$*, where for* $0 < c, C < \infty$

$$\mathcal{G} = \left\{ \begin{array}{c} g : g : [0,1] \to \mathbb{R}_+,\ g_\pm(x) \geq c,\ g_+(x) = g_-(x) \text{ except for } x \in \{u_1, \ldots, u_m\} \subset (0,1), \\ \left| g_\pm^{(j)}(x) \right| \leq C \text{ for all } x \in [0,1],\ \text{for } j = 0,1,2 \end{array} \right\}.$$

Assumption A2. *Suppose that* $\{v_t\}$*, where* $v_t = \lambda_t \zeta_t - 1$*, is a stationary alpha mixing sequence with* $E(v_t) = 0$ *and* $E(|v_t|^{\varkappa}) \leq C < \infty$ *for some* $\varkappa > 5/2$ *for* $t = 1, 2, \ldots$*. such that*

$$\sum_{k=1}^{\infty} \alpha(k)^{\frac{\varkappa - 2}{\varkappa}} < \infty. \tag{2}$$

Assumption A3. *Suppose that* K *is symmetric about zero with compact support* $[-1,1]$ *such that* $K(\pm 1) = 0$ *and* K *is twice differentiable where* $K^{(2)}$ *is continuous on* $[-1,1]$*. Let* $||K||_2^2 = \int_{-1}^{1} K(s)^2 ds$*, and* $\mu_j(K) = \int_{-1}^{1} s^j K(s) ds$*,* $j = 0, 1, 2$*.*

Assumption A4. *Suppose that for each* $\tau = 0, \ldots, E$*,* $\{e_{t,\tau}\}$ *is stationary and mixing and satisfies (2) and has Lebesgue density* $f_{e_{t,\tau}}$ *that satisfies* $\sup_x f_{e_{t,\tau}}(x) < \infty$ *and* $\inf_x f_{e_{t,\tau}}(x) > 0$.

Proof of Theorem 1. The only change from HLW is the function class $\mathcal{G}$ that allows jumps at known points. We divide $[0,1]$ into the intervals $[u_0, u_1)$, $[u_1, u_2), \ldots, [u_{m-1}, u_m)$, $[u_m, u_{m+1}]$, where $u_0 = 0$ and $u_{m+1} = 1$. Since we defined $g_-(u_{i+1}) = \lim_{u \uparrow u_{i+1}} g(u)$ we can consider the compact intervals $[u_i, u_{i+1}]$ on which the function is extended to $g(u_{i+1}) = g_-(u_{i+1})$, whereas $g(u) = g(u)$ for all $u < u_{i+1}$. Likewise for the first two derivatives. So defined, the estimation problem on the sub interval $[u_i, u_{i+1}]$ is just like the classic local linear estimator on a compact support. The local linear estimator is robust to known boundary effects. Specifically, at the point u_i, the modified function g obeys a Taylor expansion separately from the right and from the left meaning that depending on whether $t/T - u_i$ is greater than or less than zero

$$\begin{aligned} g(t/T) &= g_\pm(u_i) + (t/T - u_i) g_\pm^{(1)}(u_i) + \frac{1}{2} \left(t/T - u_i\right)^2 g_\pm^{(2)}(u_i) + R_\pm(u_i, t/T - u_i) \\ &= \begin{pmatrix} 1 & \frac{t/T - u_i}{h} \end{pmatrix} \Delta_T \begin{pmatrix} g_\pm(u_i) \\ g_\pm^{(1)}(u_i) \end{pmatrix} + \frac{1}{2} h^2 \left(\frac{t/T - u_i}{h} \right)^2 g_\pm^{(2)}(u_i) + R_\pm(u_i, t/T - u_i), \end{aligned}$$

where $\lim_{\delta \downarrow 0} \delta^2 R_+(u_i, \delta) = 0$ and $\lim_{\delta \uparrow 0} \delta^2 R_-(u_i, \delta) = 0$. By the usual algebra we cancel out the first two terms and what is left in the bias part is a term of order h^2 and a smaller order remainder term. Therefore, our first round jump robust smoother satisfies

$$
\begin{aligned}
\sup_{u\in[0,1]} \left|\widehat{g}(u) - g(u)\right| &\leq \sum_{i=0}^{m} \sup_{u\in[u_i,u_{i+1}]} \left|\widehat{g}(u) - g(u)\right| \\
&\leq \sum_{i=0}^{m} \sup_{u\in[u_i,u_{i+1}]} \left|\widehat{g}(u) - E(\widehat{g}(u))\right| + \sum_{i=0}^{m} \sup_{u\in[u_i,u_{i+1}]} \left|E(\widehat{g}(u)) - g(u)\right| \\
&= O_P\left(\sqrt{\frac{\log T}{Th}}\right) + O_P(h^2).
\end{aligned}
$$

It follows that provided $Th^4 \to 0$ the GMM estimator $\widehat{\theta}$ is $\sqrt{T}$ consistent and asymptotically normal. Furthermore, we can show that $\widetilde{g}_{\pm}(u)$ computed with $\ell_t/\widehat{\lambda}_t$ is equal to the estimator computed with $\ell_t/\lambda_t = g(t/T)\zeta_t = g(t/T) + g(t/T)(\zeta_t - 1)$. This follows because of the root-T consistency of $\widehat{\theta}$ and the arguments in HLW. We then apply the standard arguments for one-sided local linear smoothers $\widetilde{g}_{\pm}(u_i)$ with ℓ_t/λ_t as dependent variable. In particular, we have for $h = O(T^{-1/5})$

$$
\widetilde{g}_{\pm}(u_i) - g_{\pm}(u_i) = \frac{1}{T}\sum_{t=1}^{T} K_h^{\pm}(u_i - t/T)g_{\pm}(t/T)(\zeta_t - 1) + \frac{h^2}{2}\mu_2(K^{\pm})g_{\pm}^{(2)}(u_i) + o_P(T^{-2/5}),
$$

where the local linear estimator is expressed as a kernel estimator with a certain kernel, which facilitates the presentation of its limiting properties, see Fan and Gijbels (1996) (pp 70-72); in this case, the equivalent (right) boundary kernel for any point $u_i - ch$ with $c \in [0,1]$ is

$$
K_c^{+}(u) = (\kappa_0(c) + \kappa_1(c)u)K(u)1(u \in [-c,1]), \tag{3}
$$

$$
\kappa_0(c) = \frac{\mu_{2,c}}{\mu_{0,c}\mu_{2,c} - \mu_{1,c}^2},\ \ \kappa_1(c) = -\frac{\mu_{1,c}}{\mu_{0,c}\mu_{2,c} - \mu_{1,c}^2},
$$

where $\mu_{j,c} = \mu_{j,c}(K) = \int_{-c}^{1} K(u)u^j du$. Similarly, we can define the left boundary kernel $K_c^{-}(u)$. In our case, we only need explicitly the case $c = 0$, and we denote $K^{\pm} = K_0^{\pm}$ and $\mu_j = \mu_{j,0}$. If the original kernel K is symmetric about zero, $||K^{+}||^2 = ||K^{-}||^2$, where $||K||^2 = \int K(u)^2 du$, which we shall assume from now on. Therefore,

$$
\begin{aligned}
\widetilde{g}_{+}(u_i) - \widetilde{g}_{-}(u_i) - (g_{+}(u_i) - g_{-}(u_i)) &= \frac{1}{T}\sum_{t=1}^{T}\left(K_h^{+}(u_i - t/T)g_{+}(t/T) - K_h^{-}(u_i - t/T)g_{-}(t/T)\right)(\zeta_t - 1) \\
&\quad + \frac{h^2}{2}\mu_2(K^{+})\left(g_{+}^{(2)}(u_i) - g_{-}^{(2)}(u_i)\right) + o_P(T^{-2/5}),
\end{aligned}
$$

and the result follows by standard arguments. Note that

$$
\frac{1}{T}\sum_{t=1}^{T} K_h^{+}(u_i - t/T)K_h^{-}(u_i - t/T)g_{+}(t/T)g_{-}(t/T)(\zeta_t - 1)^2 = o_P(T^{-2/5}).
$$

If u_i is irrational, then there is no t such that $u_i = t/T$ and $K_h^+(u_i - t/T)K_h^-(u_i - t/T) = 0$. If u_i is rational then there is at most one t for which $u_i = t/T$ and $K_h^+(u_i - t/T)K_h^-(u_i - t/T) = K^+(0)K^-(0)/h^2$. ■

Proof of Theorem 2. The same arguments apply here to the two separate liquidity series. We have

$$\begin{aligned}
&\widetilde{g}_{S,\pm}(u_i) - \widetilde{g}_{C,\pm}(u_i) - \big(E(\widetilde{g}_{S,\pm}(u_i)) - E(\widetilde{g}_{C,\pm}(u_i))\big) \\
&= \frac{1}{T}\sum_{t=1}^{T} K_h^{\pm}(u_i - t/T)\left(g_{S,\pm}(t/T)(\zeta_{S,t} - 1) - g_{C,\pm}(t/T)(\zeta_{C,t} - 1)\right) + o_P(T^{-1/2}h^{-1/2}),
\end{aligned}$$

which is mean zero and asymptotically normal with asymptotic variance

$$\begin{aligned}
&\frac{\left\|K^{\pm}\right\|_2^2}{Th}\mathrm{var}\left(\left(g_{S,\pm}(u)(\zeta_{S,t} - 1) - g_{C,\pm}(u)(\zeta_{C,t} - 1)\right)\right) \\
&= \frac{\left\|K^{\pm}\right\|_2^2}{Th}\left(g_{S,\pm}^2(u)\sigma_{S,\pm}^2 + g_{C,\pm}^2(u)\sigma_{C,\pm}^2 - 2g_{S,\pm}g_{C,\pm}(u)\sigma_{S,C,\pm}^2\right).
\end{aligned}$$

This follows because $\zeta_{S,t}$ is correlated with $\zeta_{C,t}$, but since $K^+ \times K^- = 0$ (except at the common point 0), the $+$ estimators are essentially uncorrelated asymptotically with $-$ estimators. The bias follows by standard arguments, Fan and Gijbels (1996). ■

Proof of Theorem 3. We show that

$$\sup_{x\in\mathbb{R}}\left|\widehat{F}_{\widehat{e}_\tau}(x) - F_{e_\tau}(x)\right| \xrightarrow{P} 0.$$

For simplicity of notation we replace T_S by T and just suppose we have a standard contiguous sample of size T. Suppose that

$$\widehat{F}_{e_\tau}(x) = \frac{1}{T}\sum_{t=1}^{T} 1\left(e_{t,\tau} \le x\right), \quad \widehat{F}_{e_\tau}(x) = \frac{1}{T}\sum_{t=1}^{T} 1\left(\widehat{e}_{t,\tau} \le x\right),$$

where $e_{t,\tau} = \sum_{s=0}^{\tau}(\zeta_{t+s} - 1)$ and $\widehat{e}_{t,\tau} = \sum_{s=0}^{\tau}(\widehat{\zeta}_{t+s} - 1)$, where $\widehat{\zeta}_t = \zeta_t/\widehat{g}(t/T)\widehat{\lambda}_t$. Under our conditions

$$\max_{1\le t\le T}\left|\widehat{e}_{t,\tau} - e_{t,\tau}\right| = o_P(1). \tag{4}$$

This follows because $\max_{1\le t\le T}\sum_{s=0}^{\tau}\zeta_{t+s} = O_P(T^{1/\varkappa})$ by standard arguments and we have shown that $\max_{1\le t\le T}\left|\widehat{g}(t/T) - g(t/T)\right| = O_P(\log(T)T^{-2/5})$, and likewise $\max_{1\le t\le T}\left|\widehat{\lambda}_t - \lambda_t\right|$.

Since $\varkappa > 5/2$, the result (4) follows. Then (4) says that there is some $\delta_T \to 0$ such that for all $C > 0$, $\Pr\left(A_T^c\right) \to 0$, where the event A_T is defined

$$A_T = \left\{ \max_{1 \leq t \leq T} \left| \widehat{e}_{t,\tau} - e_{t,\tau} \right| \leq C\delta_T \right\}.$$

We first see that

$$\frac{1}{T}\sum_{t=1}^{T} 1\left(e_{t,\tau} \leq x - \max_{1\leq t\leq T}\left|\widehat{e}_{t,\tau} - e_{t,\tau}\right|\right) \leq \widehat{F}_{\widehat{e}_{t,\tau}}(x) \leq \frac{1}{T}\sum_{t=1}^{T} 1\left(e_{t,\tau} \leq x + \max_{1\leq t\leq T}\left|\widehat{e}_{t,\tau} - e_{t,\tau}\right|\right).$$

Then, conditional on A_T, this is further bounded by

$$\widehat{F}_{e_{t,\tau}}(x - C\delta_T) = \frac{1}{T}\sum_{t=1}^{T} 1\left(e_{t,\tau} \leq x - C\delta_T\right) \leq \widehat{F}_{\widehat{e}_{t,\tau}}(x) \leq \frac{1}{T}\sum_{t=1}^{T} 1\left(e_{t,\tau} \leq x + C\delta_T\right) = \widehat{F}_{e_{t,\tau}}(x + C\delta_T).$$

The result follows because by the mean value theorem

$$E\left(\widehat{F}_{e_{t,\tau}}(x \pm C\delta_T)\right) = F_{e_{t,\tau}}(x \pm C\delta_T) = F_{e_{t,\tau}}(x) \pm f_{e_{t,\tau}}(x \pm \lambda C\delta_T)C\delta_T$$

and then for bounded density f the second term goes to zero uniformly in x. Specifically, by the law of total probability and the Bonferroni inequality

$$\begin{aligned}
&\Pr\left(\sup_{x\in\mathbb{R}}\left|\widehat{F}_{\widehat{e}_{t,\tau}}(x) - F_{e_{t,\tau}}(x)\right| > \epsilon_T\right) \\
&\leq \Pr\left(\left\{\sup_{x\in\mathbb{R}}\left|\widehat{F}_{\widehat{e}_{t,\tau}}(x) - F_{e_{t,\tau}}(x)\right| > \epsilon_T\right\} \cap A_T\right)\Pr(A_T) + \Pr(A_T^c) \\
&\leq \Pr\left(\max\left\{\sup_{x\in\mathbb{R}}\left|\widehat{F}_{e_{t,\tau}}(x - C\delta_T) - F_{e_{t,\tau}}(x)\right|, \sup_{x\in\mathbb{R}}\left|\widehat{F}_{e_{t,\tau}}(x + C\delta_T) - F_{e_{t,\tau}}(x)\right|\right\} > \epsilon_T\right) + o(1) \\
&\leq \Pr\left(\sup_{x\in\mathbb{R}}\left|\widehat{F}_{e_{t,\tau}}(x - C\delta_T) - F_{e_{t,\tau}}(x)\right| > \epsilon_T\right) + \Pr\left(\sup_{x\in\mathbb{R}}\left|\widehat{F}_{e_{t,\tau}}(x + C\delta_T) - F_{e_{t,\tau}}(x)\right| > \epsilon_T\right) + o(1).
\end{aligned}$$

We have by the triangle inequality

$$\begin{aligned}
&\sup_{x\in\mathbb{R}}\left|\widehat{F}_{e_{t,\tau}}(x + C\delta_T) - F_{e_{t,\tau}}(x)\right| \\
&\leq \sup_{x\in\mathbb{R}}\left|\widehat{F}_{e_{t,\tau}}(x + C\delta_T) - F_{e_{t,\tau}}(x + C\delta_T)\right| + \sup_{x\in\mathbb{R}}\left|F_{e_{t,\tau}}(x + C\delta_T) - F_{e_{t,\tau}}(x)\right| \\
&\leq \sup_{x\in\mathbb{R}}\left|\widehat{F}_{e_{t,\tau}}(x) - F_{e_{t,\tau}}(x)\right| + \sup_{x\in\mathbb{R}}\left|f_{e_{t,\tau}}(x)\right| \times C\delta_T \\
&= o_P(1).
\end{aligned}$$

The uniform consistency of the empirical process for stationary mixing series is established in Yu (1994). ■

Suppose that there are breaks at other points. Specifically, suppose there is a discontinuity in g at the point u^*. Then

$$\begin{aligned}
\|\widehat{g}-g\|_\infty &= \sup_{u\in[0,1]} \left|\widehat{g}(u)-g(u)\right| \\
&= \sup_{u\in[0,1]} \left|E\left(\widehat{g}(u)\right)-g(u)\right| + O_P\left(\sqrt{\frac{\log T}{Th}}\right) \\
&= \left|\frac{g_+(u^*)-g_-(u^*)}{2}\right| + o_P(1),
\end{aligned}$$

and the estimator of g is inconsistent in the L_∞ norm. However, it is consistent in the L_2 norm, since

$$\begin{aligned}
\|\widehat{g}-g\|_2 &= \left(\int_0^1 \left(\widehat{g}(u)-g(u)\right)^2 du\right)^{1/2} \\
&= \left(\int_0^1 \left(E\left(\widehat{g}(u)\right)-g(u)\right)^2 du\right)^{1/2} + O_P\left(\sqrt{\frac{1}{Th}}\right) \\
&= \left(\left(\int_{u^*-h}^{u^*+h} + \int_0^{u^*-h} + \int_{u^*+h}^1\right)\left(E\left(\widehat{g}(u)\right)-g(u)\right)^2 du\right)^{1/2} + O_P\left(\sqrt{\frac{1}{Th}}\right) \\
&= \left(\int_{u^*-h}^{u^*+h} \left(E\left(\widehat{g}(u)\right)-g(u)\right)^2 du\right)^{1/2} + O(h^2) + O_P\left(\sqrt{\frac{1}{Th}}\right),
\end{aligned}$$

and

$$\int_{u^*-h}^{u^*+h} \left(E\left(\widehat{g}(u)\right)-g(u)\right)^2 du \le 2h \times \sup_{u\in[0,1]} \left(g_+^2(u)+g_-^2(u)\right) = O(h) = o_P(1).$$

B Bandwidth Choice

We can rewrite the model in terms of $\ell_t^\dagger = \ell_t/\lambda_t$ as follows:

$$\ell_t^\dagger = g(t/T) + g(t/T)\left(\zeta_t - 1\right), \tag{5}$$

where $\zeta_t - 1$ is a martingale difference sequence with finite unconditional variance. Under the assumption of weak stationarity of ζ_t, we have $E(\ell_t^\dagger) = g(t/T)$ and $\mathrm{var}(\ell_t^\dagger) = g(t/T)^2\sigma_\zeta^2$.

Under the regularity conditions given in the supplement, the pointwise mean squared errors of $\widetilde{\mathcal{J}}_i, \widetilde{g}_\pm(u_i)$, are respectively:

$$MSE_\Delta(u_i) = \frac{h^4}{4}\mu_2^2(K^+)\left(g_+^{(2)}(u_i) - g_-^{(2)}(u_i)\right)^2 + \frac{1}{Th}\left\|K^+\right\|^2\left(g_+(u_i)^2 + g_-(u_i)^2\right)\sigma_\zeta^2, \quad (6)$$

$$MSE_\pm(u_i) = \frac{h^4}{4}\mu_2^2(K^+)\left(g_\pm^{(2)}(u_i)\right)^2 + \frac{1}{Th}\left\|K^+\right\|^2 g_\pm(u_i)^2\sigma_\zeta^2. \quad (7)$$

The two estimands present different bias variance tradeoffs (for the first round estimates the formulas are the same except that σ_ζ^2 is replaced by the long run variance of $\lambda_t\zeta_t - 1$. The corresponding optimal bandwidths are respectively:

$$h_{\Delta,opt}(u_i) = C_K\left(\frac{\left(g_+(u_i)^2 + g_-(u_i)^2\right)\sigma_\zeta^2}{\left(g_+^{(2)}(u_i) - g_-^{(2)}(u_i)\right)^2}\right)^{1/5} T^{-1/5}, \quad h_{\pm,opt}(u_i) = C_K\left(\frac{g_\pm(u_i)^2\sigma_\zeta^2}{g_\pm^{(2)}(u_i)^2}\right)^{1/5} T^{-1/5},$$

where $C_K = (\left\|K^+\right\|^2/\mu_2^2(K^+))^{1/5}$ depends only on the kernel.[1]

Imbens and Kalyanaraman (2012) discuss and propose various methods for estimating the optimal bandwidth. Our approach is based on the so-called pilot method of Silverman (1986), Fan and Gijbels (1996). That is, we suppose that $g_+(u)$ is globally polynomial with parameters $\sum_{j=0}^p a_j^i u^j$, $p \geq 2$, on the interval $[u_i, u_{i+1})$, while $g_-(u)$ is globally polynomial with parameters $\sum_{j=0}^p a_j^{i-1} u^j$ on the interval $(u_{i-1}, u_i]$. We estimate the parameters a_j^i based on segmented least squares regression using data from the interval $[u_i, u_{i+1})$ and then plug in the estimated quantities.

C Standard Error Adjustment for overlapping windows

We discuss here our general strategy for taking account of cross correlation of estimated effects. We just present the idea for the $\widehat{ACAIL}(\tau)$ statistic from Section 3.3.

We have in general

$$\text{var}\left(\sum_{i=1}^N w_i \sum_{t=0}^\tau (\zeta_{it} - 1)\right) = w^\intercal \Sigma(\tau) w,$$

[1]One issue arises when $g_+^{(2)}(u_i) \simeq g_-^{(2)}(u_i)$; in that case it may be better to use $h_{\pm,opt}(u_i)$.

where $\Sigma_{ij}(\tau) = \mathrm{cov}\left(\sum_{t=0}^{\tau}(\zeta_{it}-1), \sum_{t=0}^{\tau}(\zeta_{jt}-1)\right)$. The time t here is in event time. We switch to calendar time where we assume that

$$\mathrm{cov}\left(\zeta_{it}, \zeta_{js}\right) = \begin{cases} \sigma_{ij} & \text{if } t = s \\ 0 & \text{if } t \neq s. \end{cases}$$

Therefore, $\Sigma_{ij}(\tau) = \sigma_{ij} n_{ij}^{o}(\tau)$, where $n_{ij}^{o}(\tau)$ is the number of periods of overlap of the event windows $\{0, \ldots, \tau\}$ for splits i and j. For $i = j$, $n_{ii}^{o}(\tau) = \tau + 1$, whereas for different splits for the same firm we usually have $n_{ij}^{o}(\tau) = 0$. We expect $\sigma_{ij} \geq 0$ for all pairs i, j. The corrected standard errors are

$$\sqrt{(\tau+1)\sum_{i=1}^{N} w_i^2 \widehat{\sigma}_i^2 + 2\sum_{j=i+1}^{N}\sum_{i=1}^{N} w_i w_j \widehat{\sigma}_{ij} n_{ij}^{o}(\tau)}, \tag{8}$$

where $\widehat{\sigma}_{ij} = T^{-1}\sum_{t=1}^{T}(\widehat{\zeta}_{it}-1)(\widehat{\zeta}_{jt}-1)$.

(a) Dow Jones index.

(b) S&P 500 index.

(c) S&P 400 index.

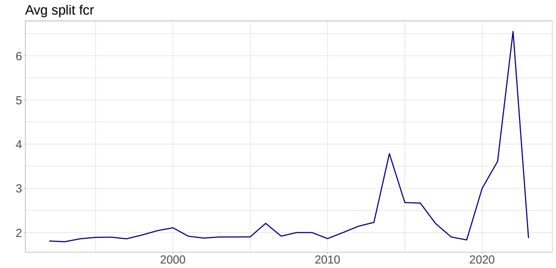
(d) S&P 600 index.

Figure 1: Average price by index.

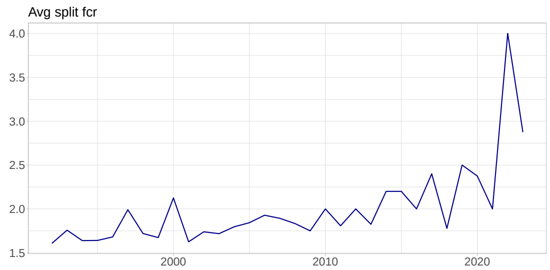
D Additional tables and figures

D.1 Preliminary analysis

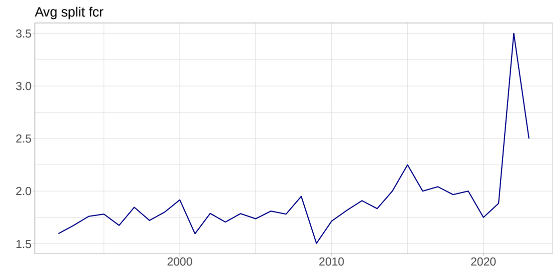
(a) Down Jones index.



(b) S&P 500 index.

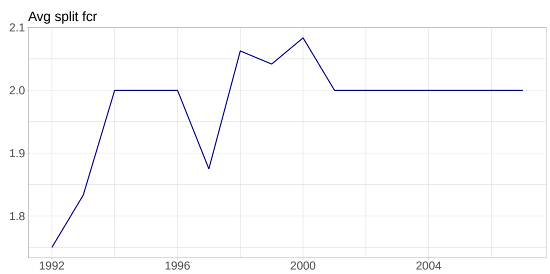


(c) S&P 400 index.

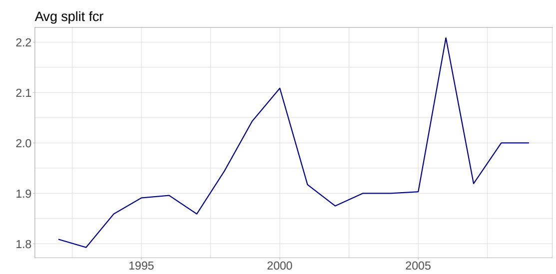


(d) S&P 600 index.

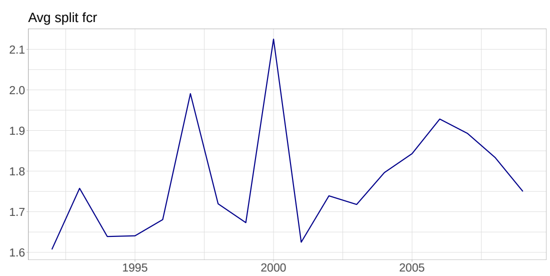
Figure 2: Average split factor by year.



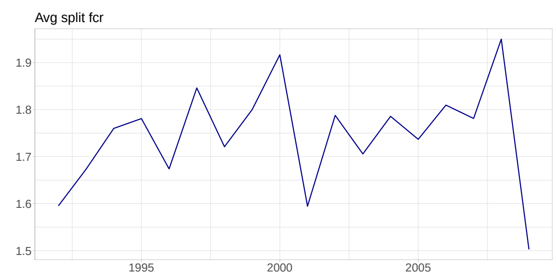
(a) Down Jones index.



(b) S&P 500 index.



(c) S&P 400 index.



(d) S&P 600 index.

Figure 3: Average split factor by year.

D.2 Permanent and temporary effects for each stock split

For each split of each stock, we plot in Figures 4 to 11 the results of testing for permanent breaks and temporary effects of stock splits. The upper panel of each figure presents the results for permanent effect. The test statistic is marked by a red dot while the two bars indicate the 2.5% and 97.5% quantiles. We can observe that most of the test statistics on stock split dates are outside the critical value bands. This suggests that, overall, stock splits have positive and significant effects on the long-run trend level of the illiquidity process.

The lower panel of each figure presents the results for temporary effects where the blue line represents the test statistic together with the 2.5% and 97.5% quantiles marked by the red lines.[2] The figures show that the effect of stock splits on the short-term dynamics of liquidity is rarely significant with only very few exceptions.

[2]Note that for the split events preceded by another one we only consider the period after the first stock split event for the computation of the quantiles.

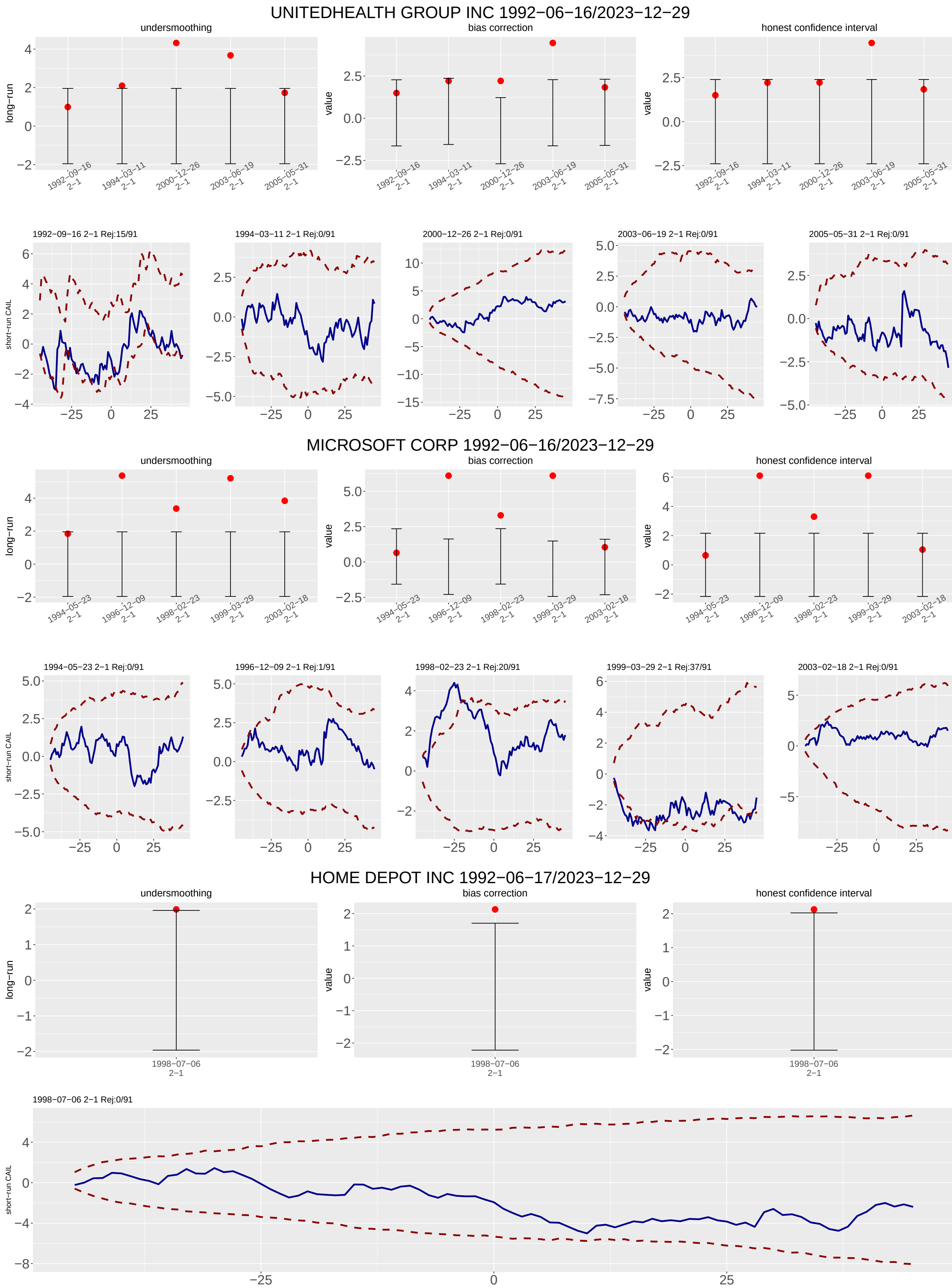


Figure 4: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

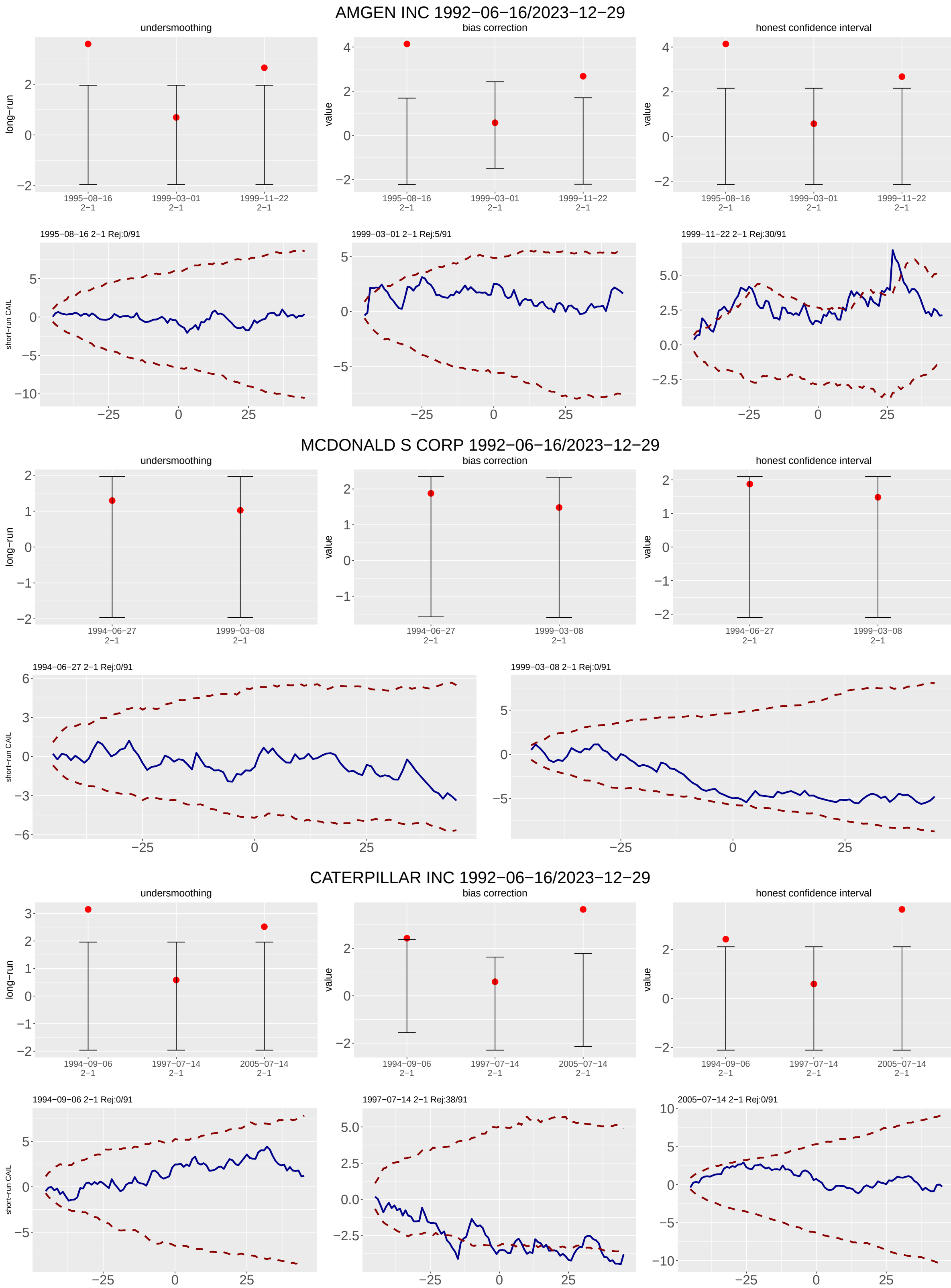


Figure 5: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

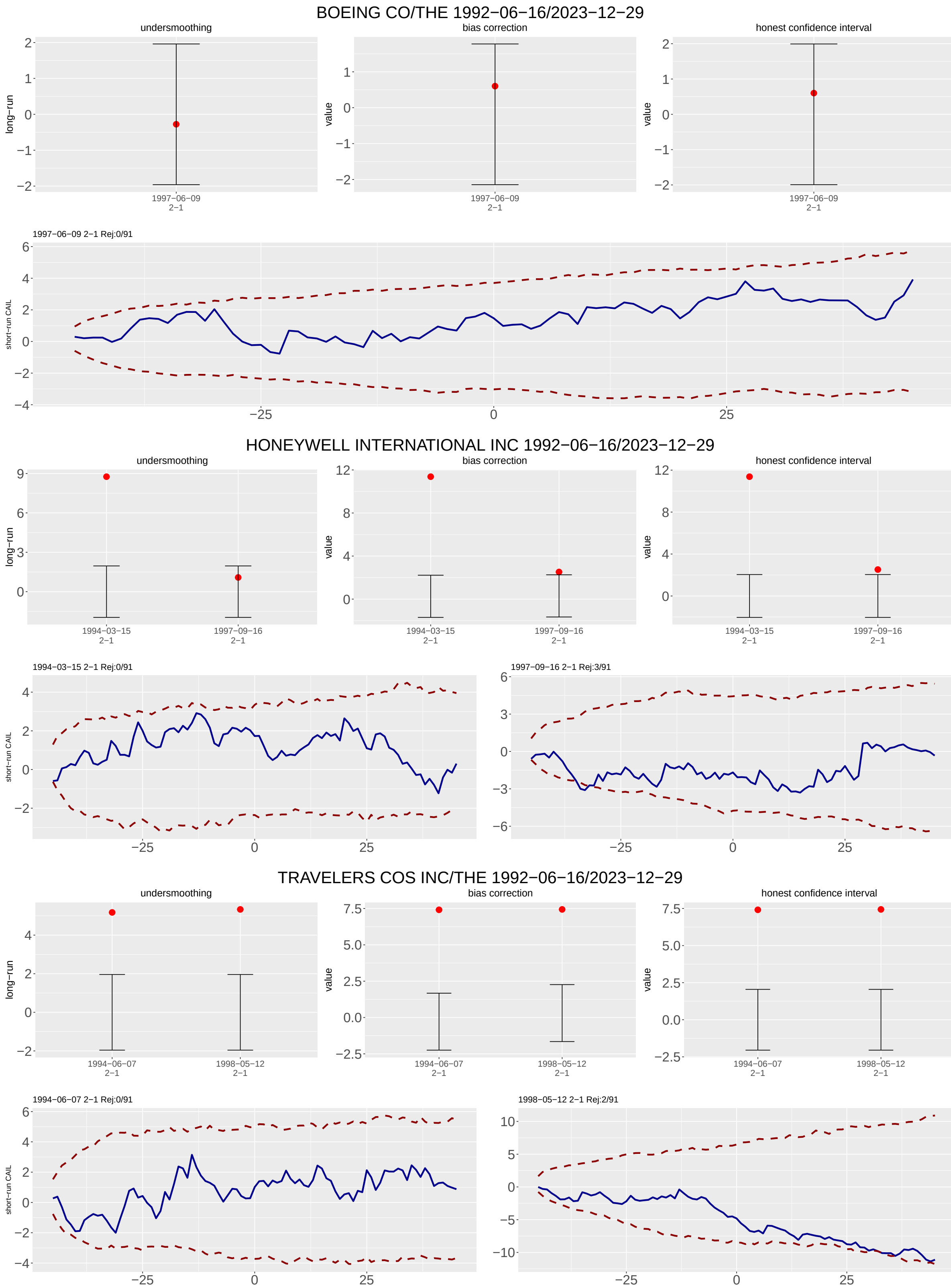


Figure 6: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

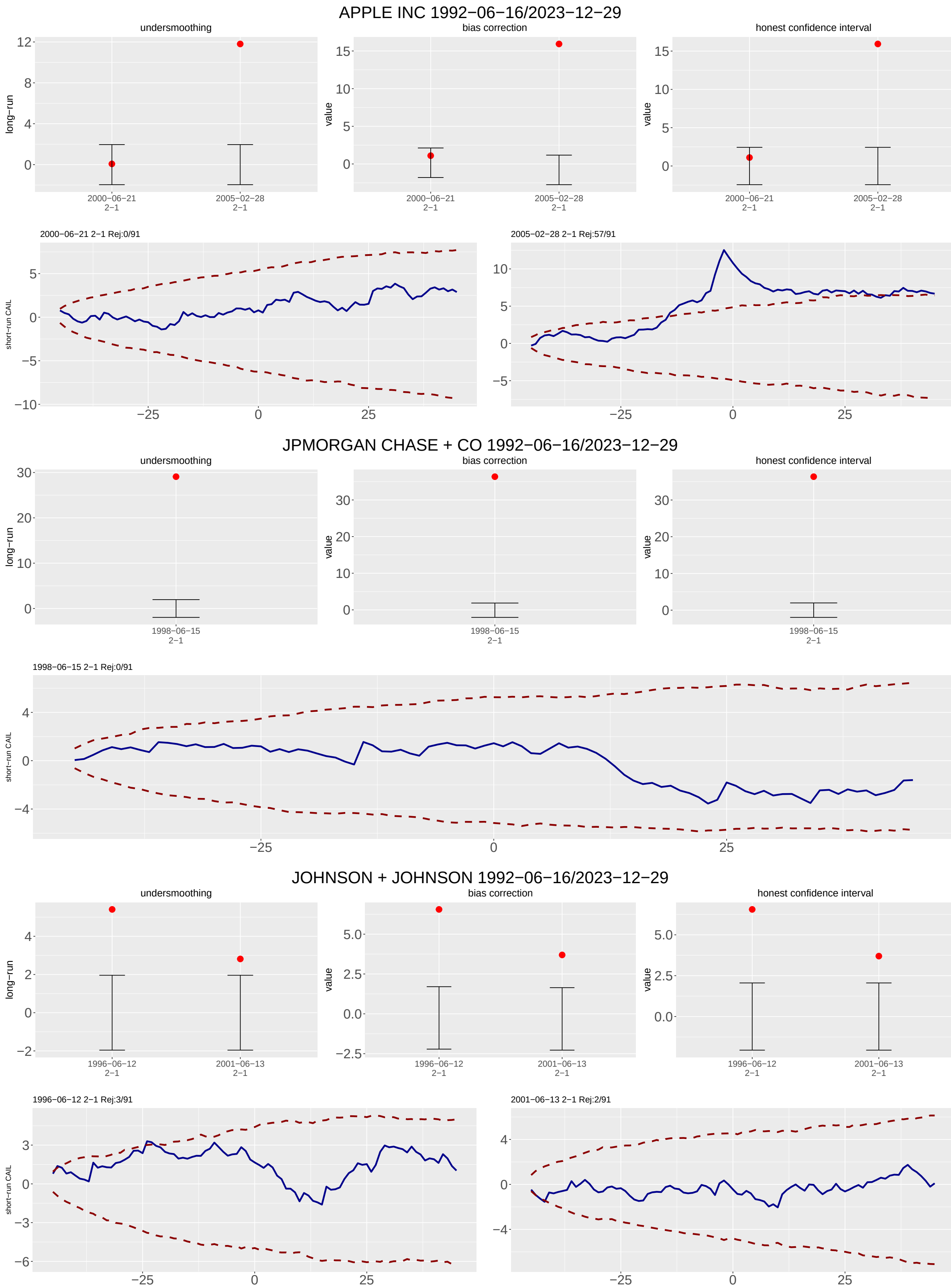


Figure 7: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

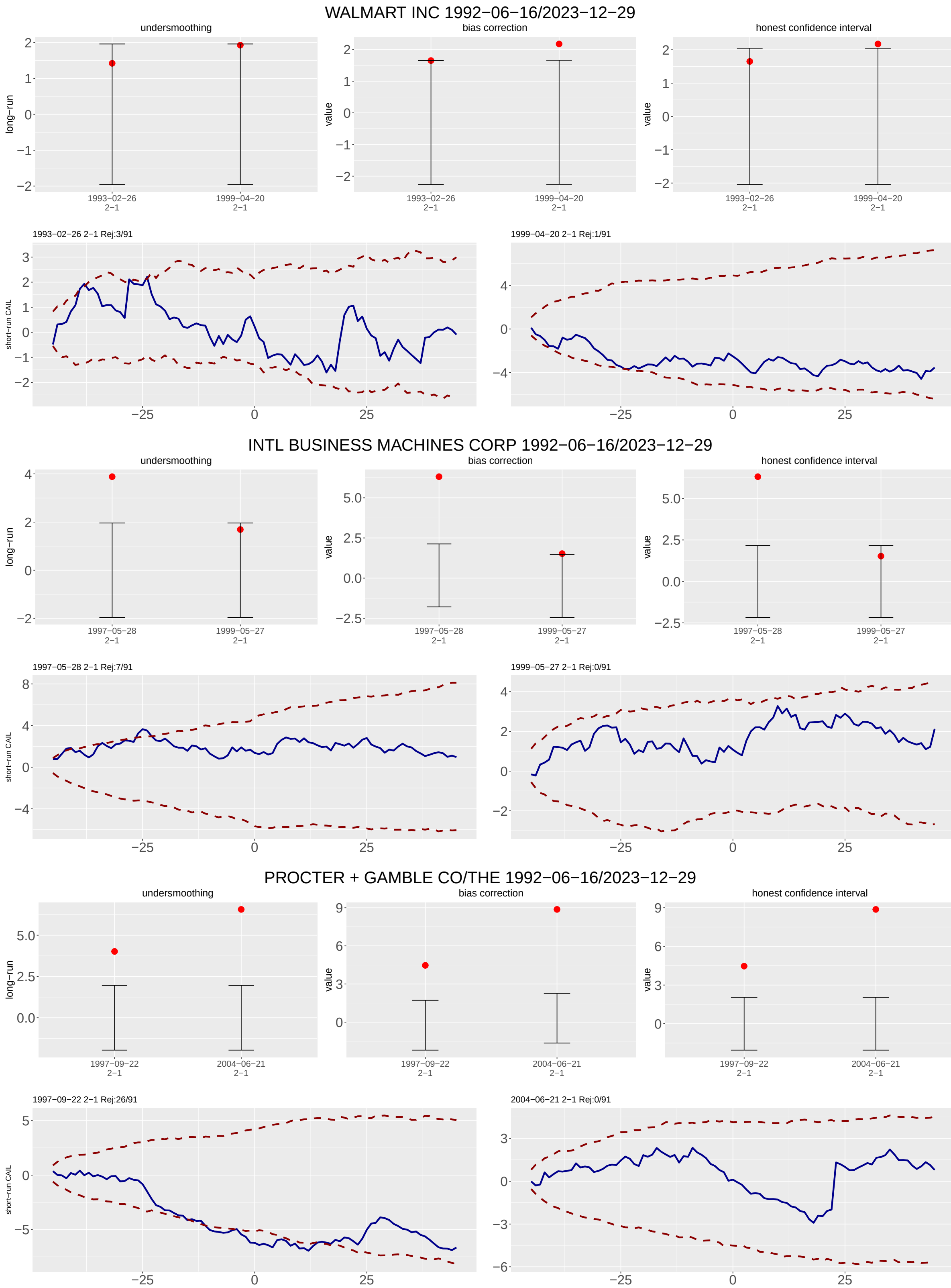


Figure 8: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

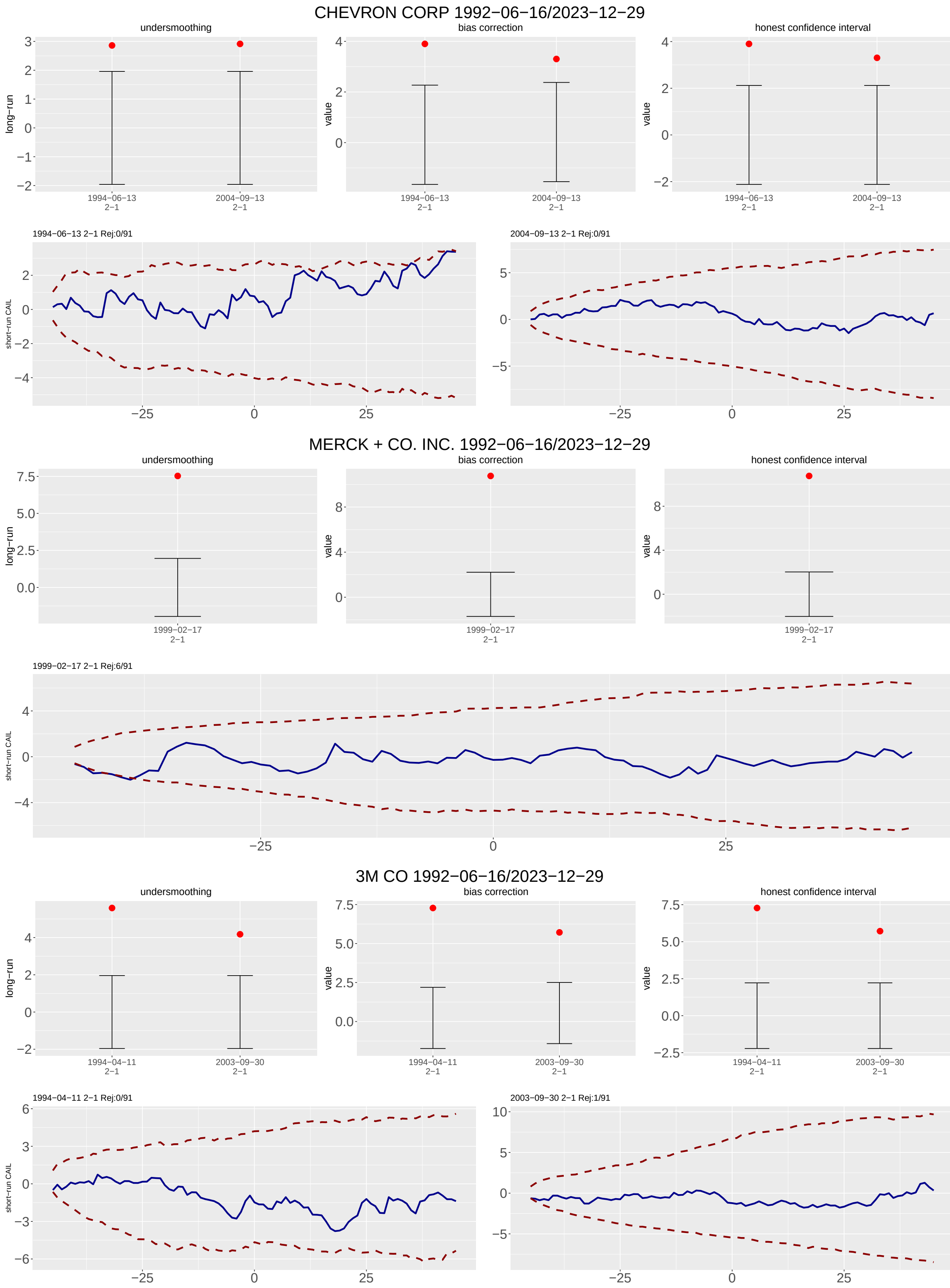


Figure 9: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

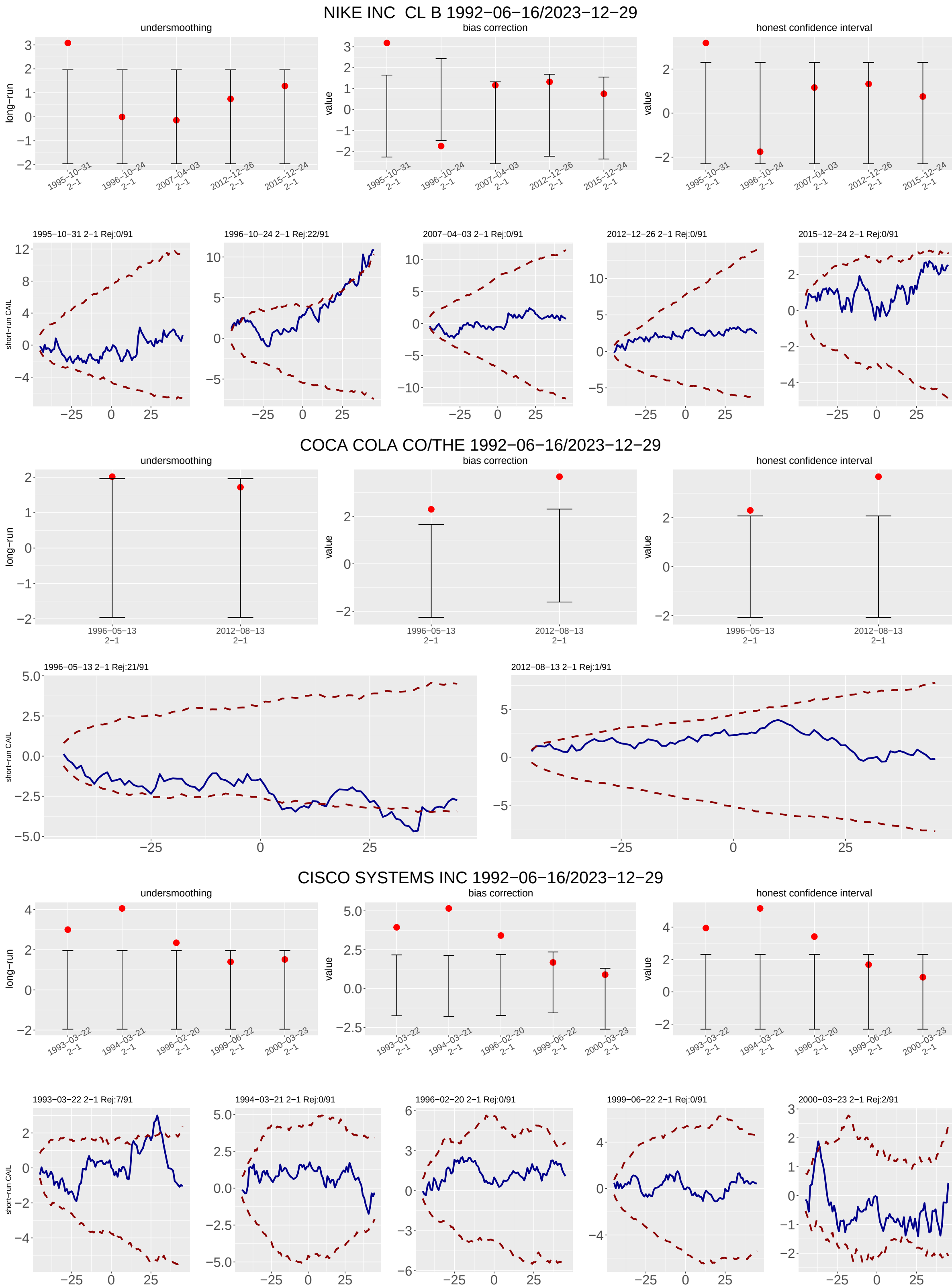


Figure 10: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

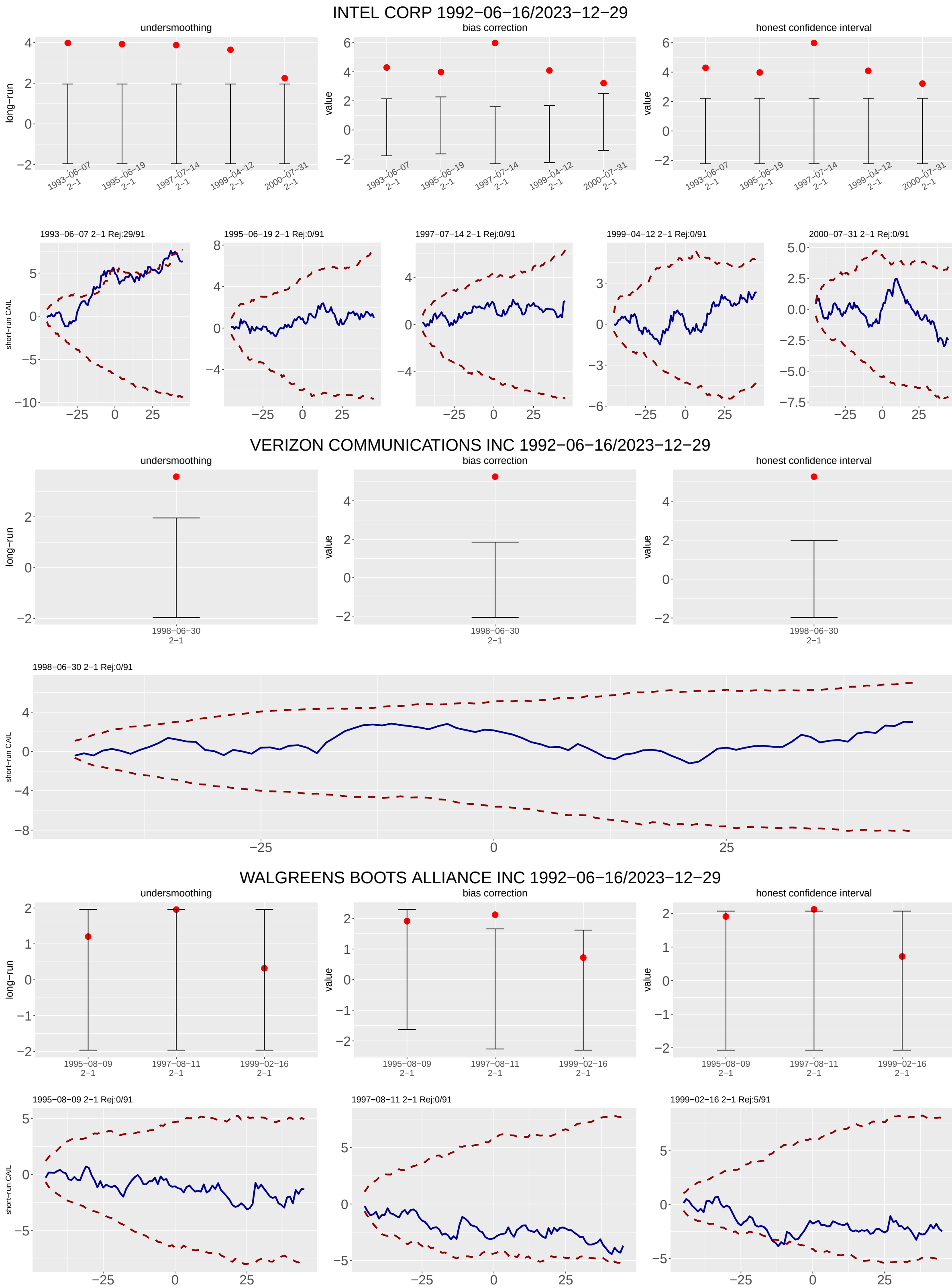


Figure 11: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

D.3 Temporary effects aggregated across splits for each firm

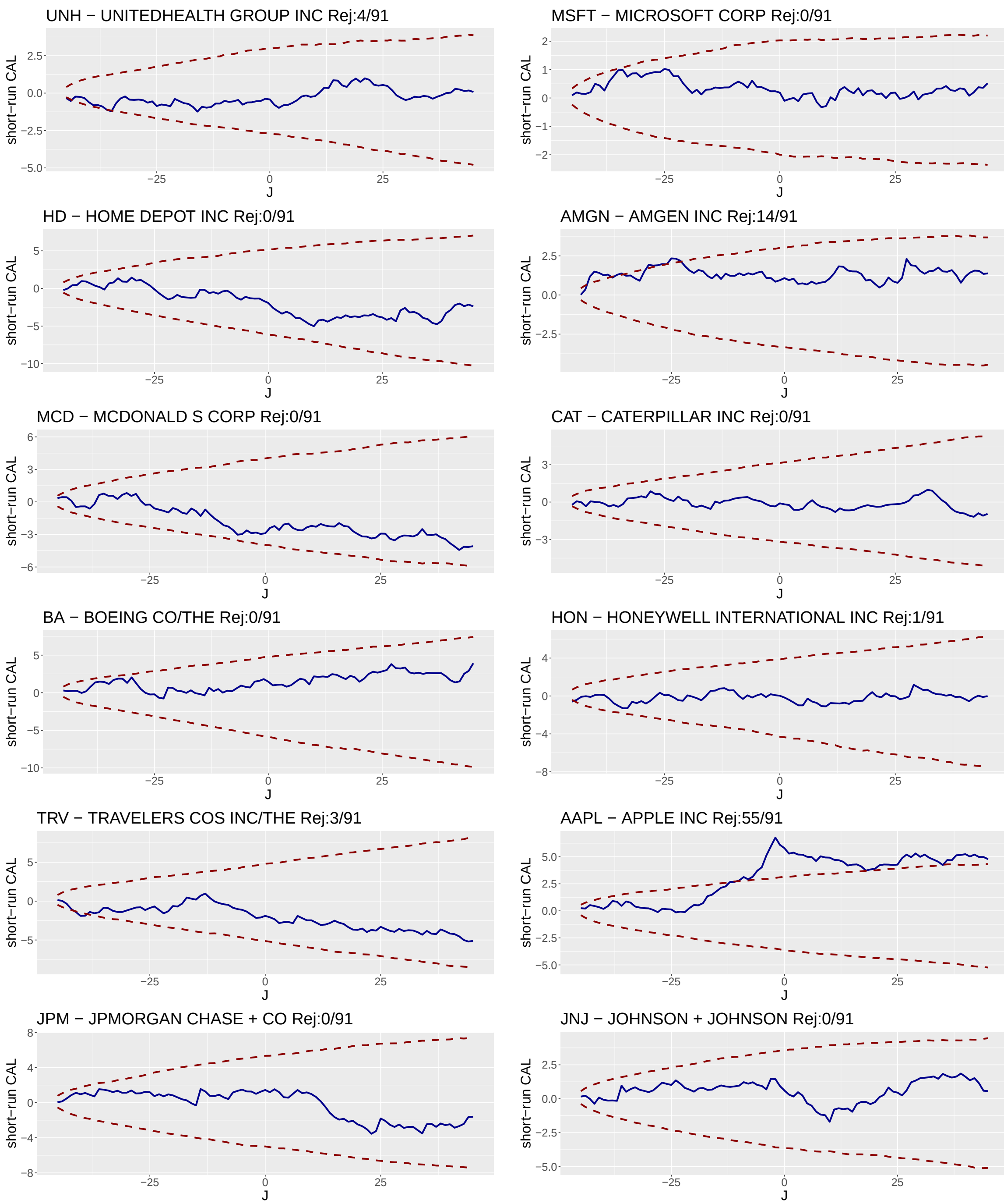


Figure 12: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

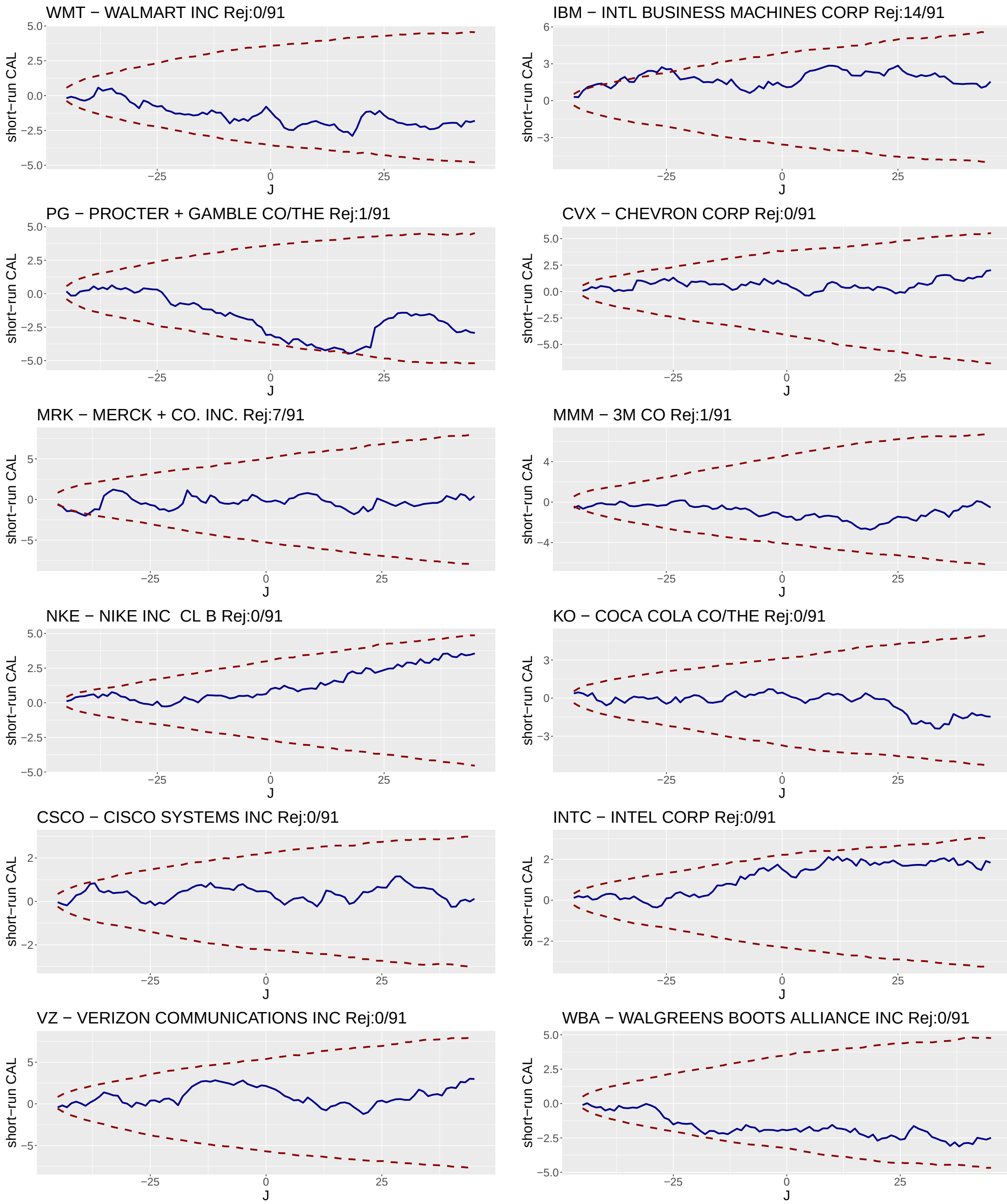


Figure 13: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

D.4 Permanent and temporary effects for each reverse split

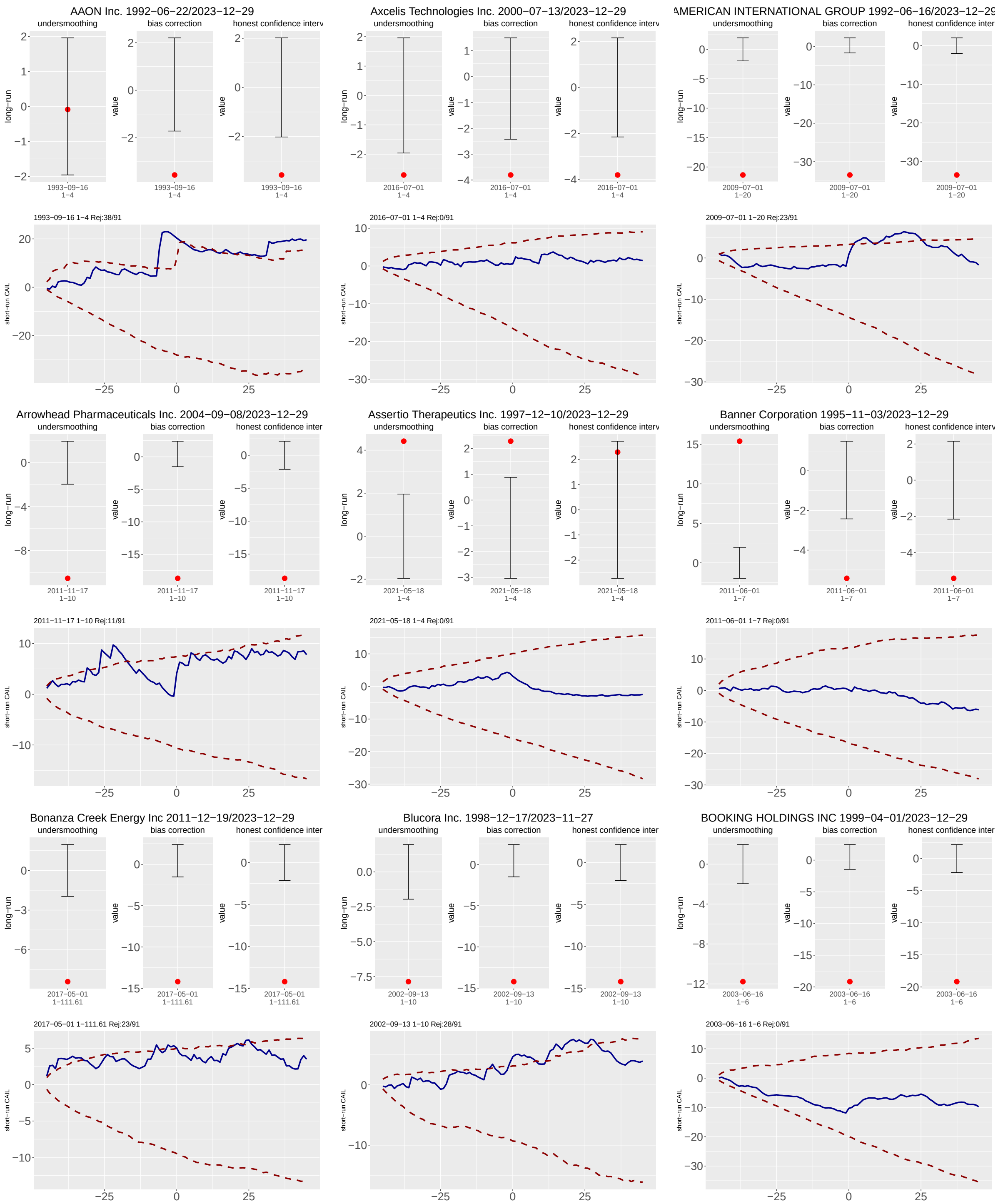


Figure 14: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

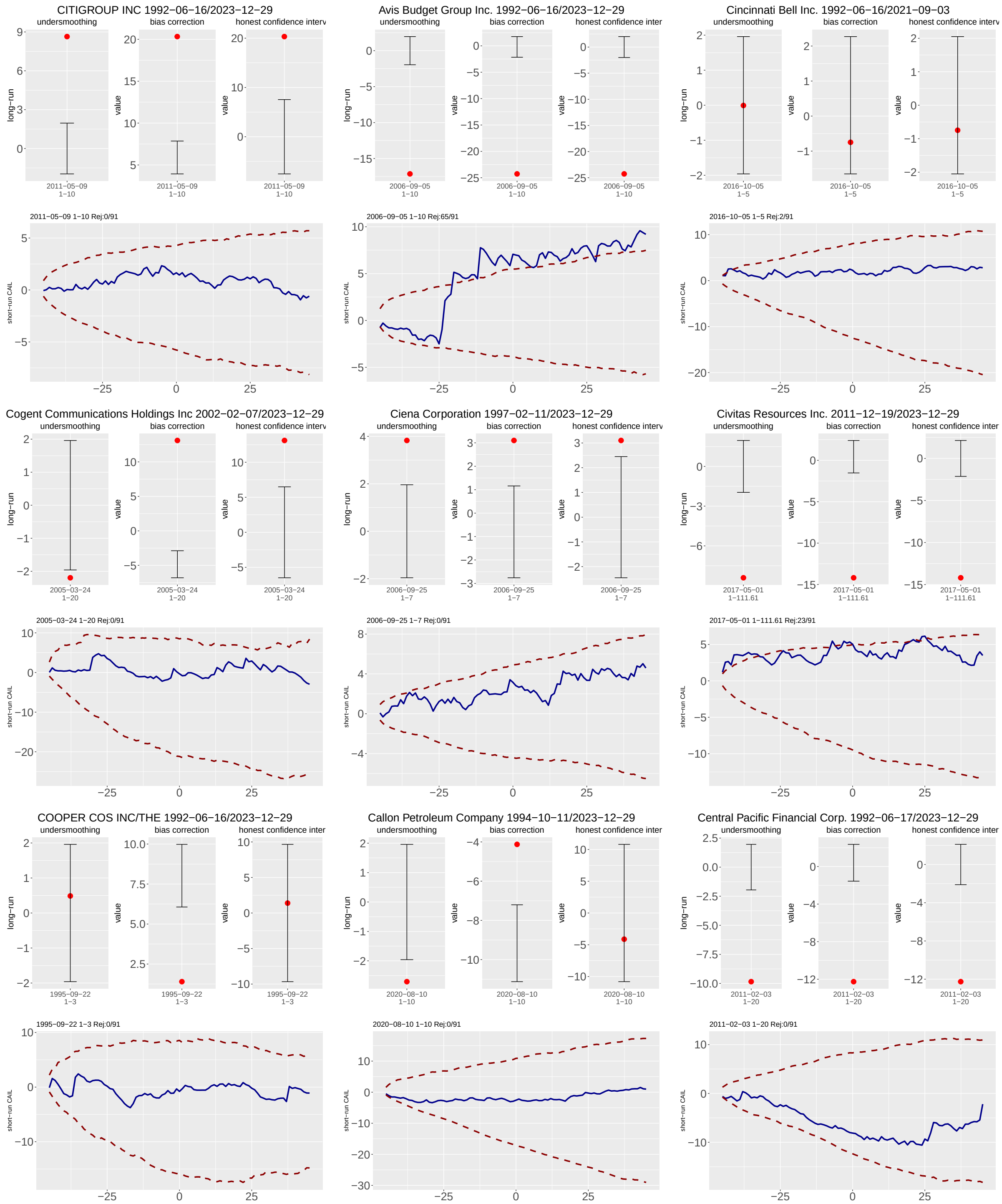


Figure 15: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

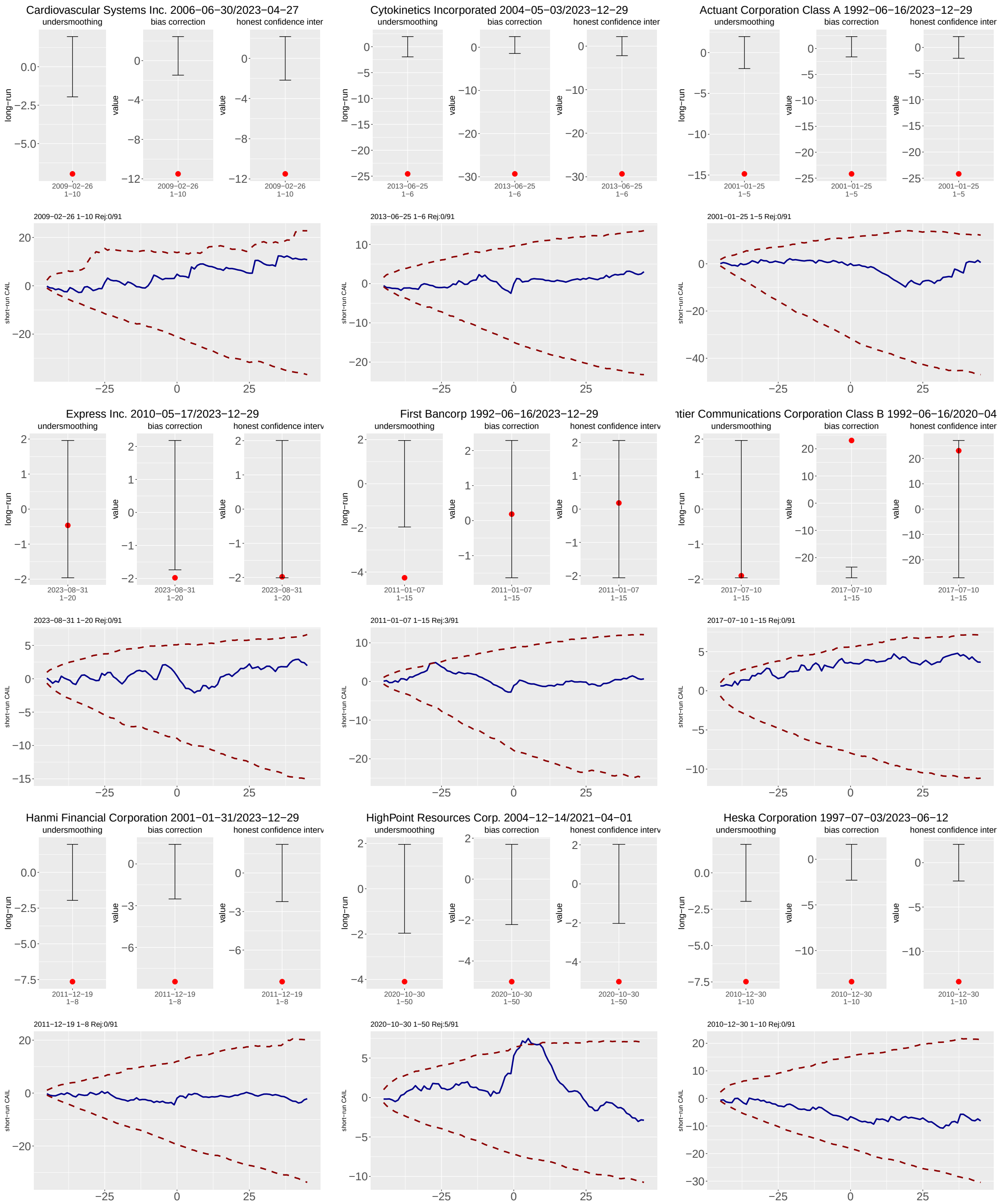


Figure 16: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

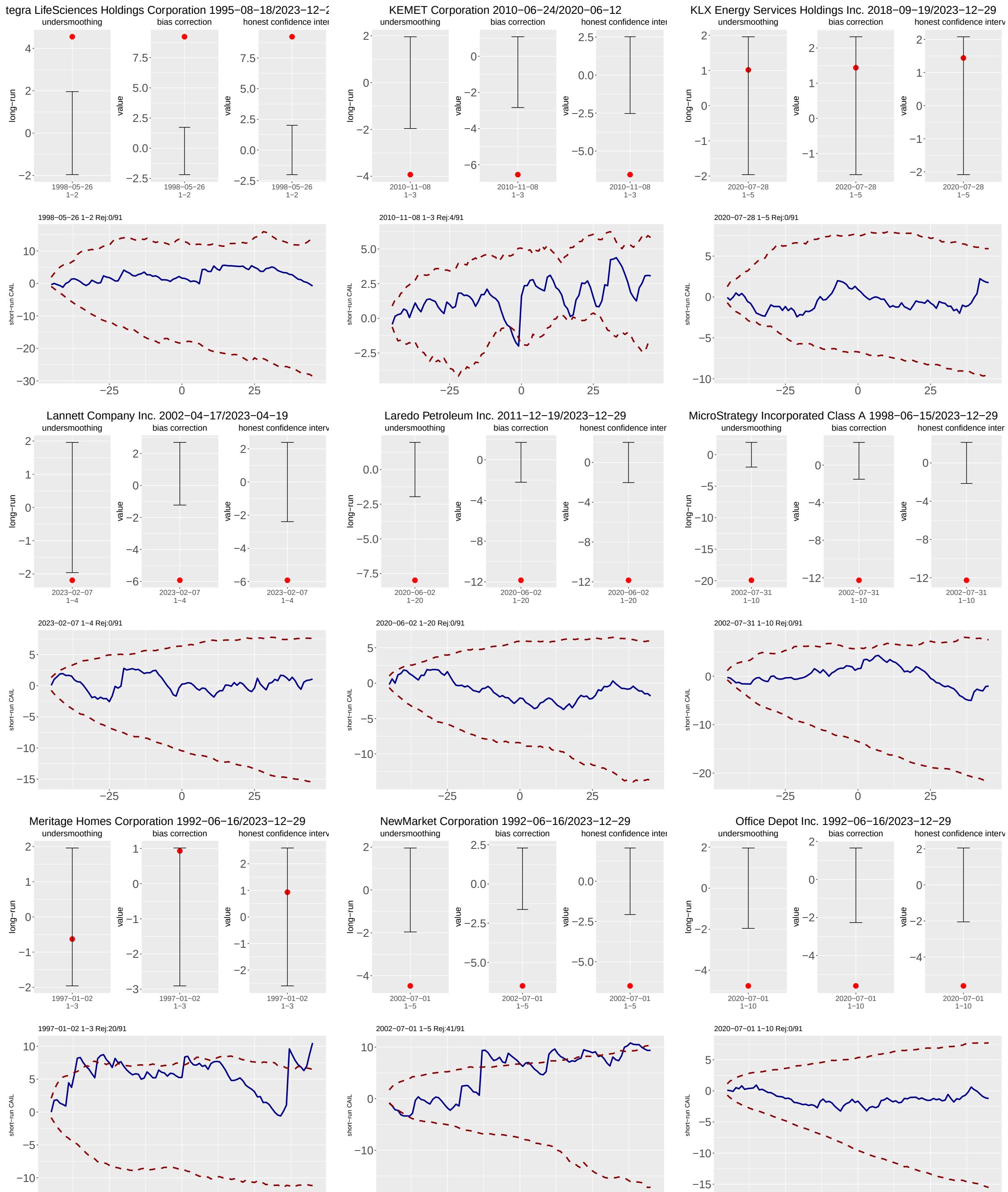


Figure 17: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

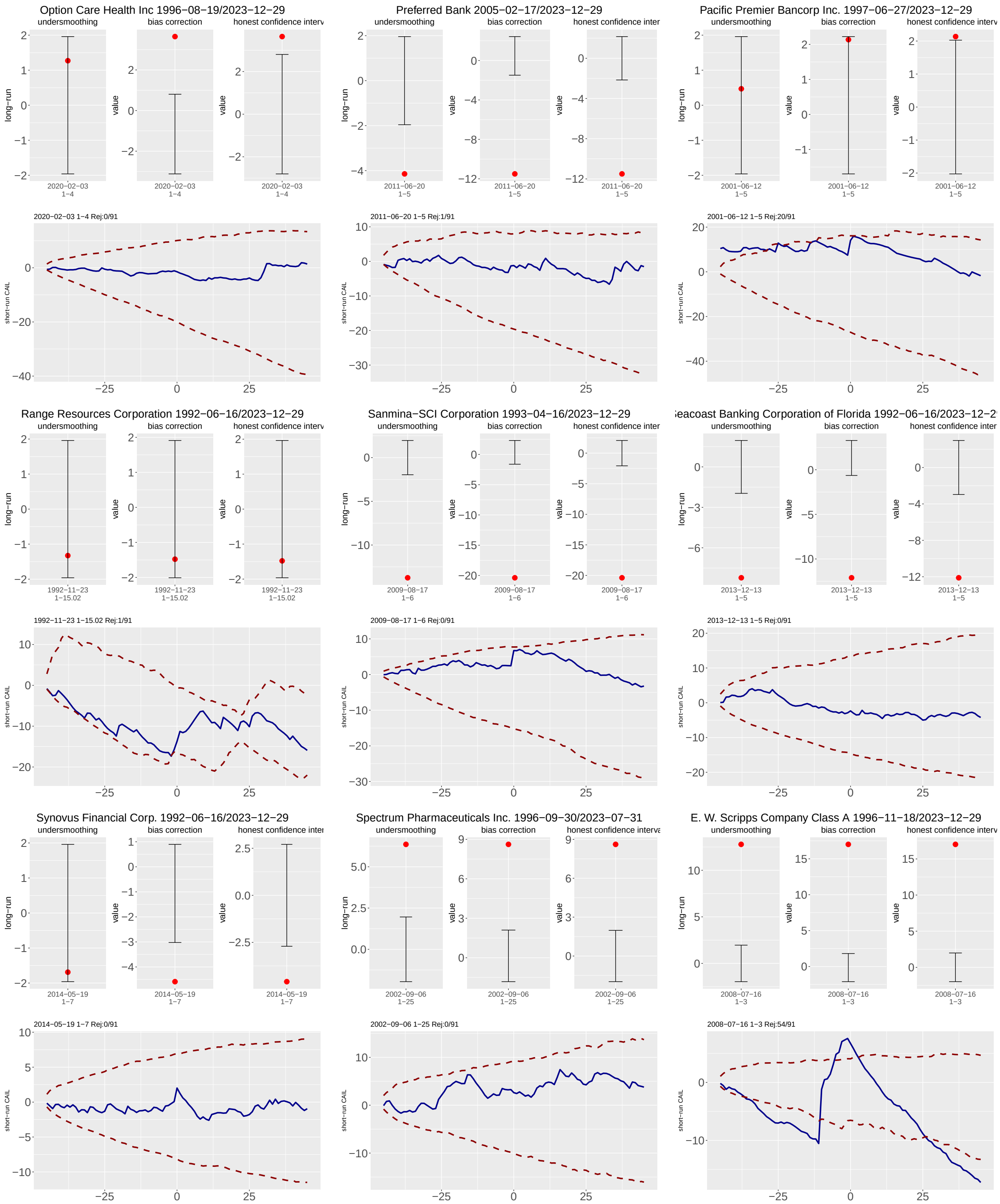


Figure 18: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

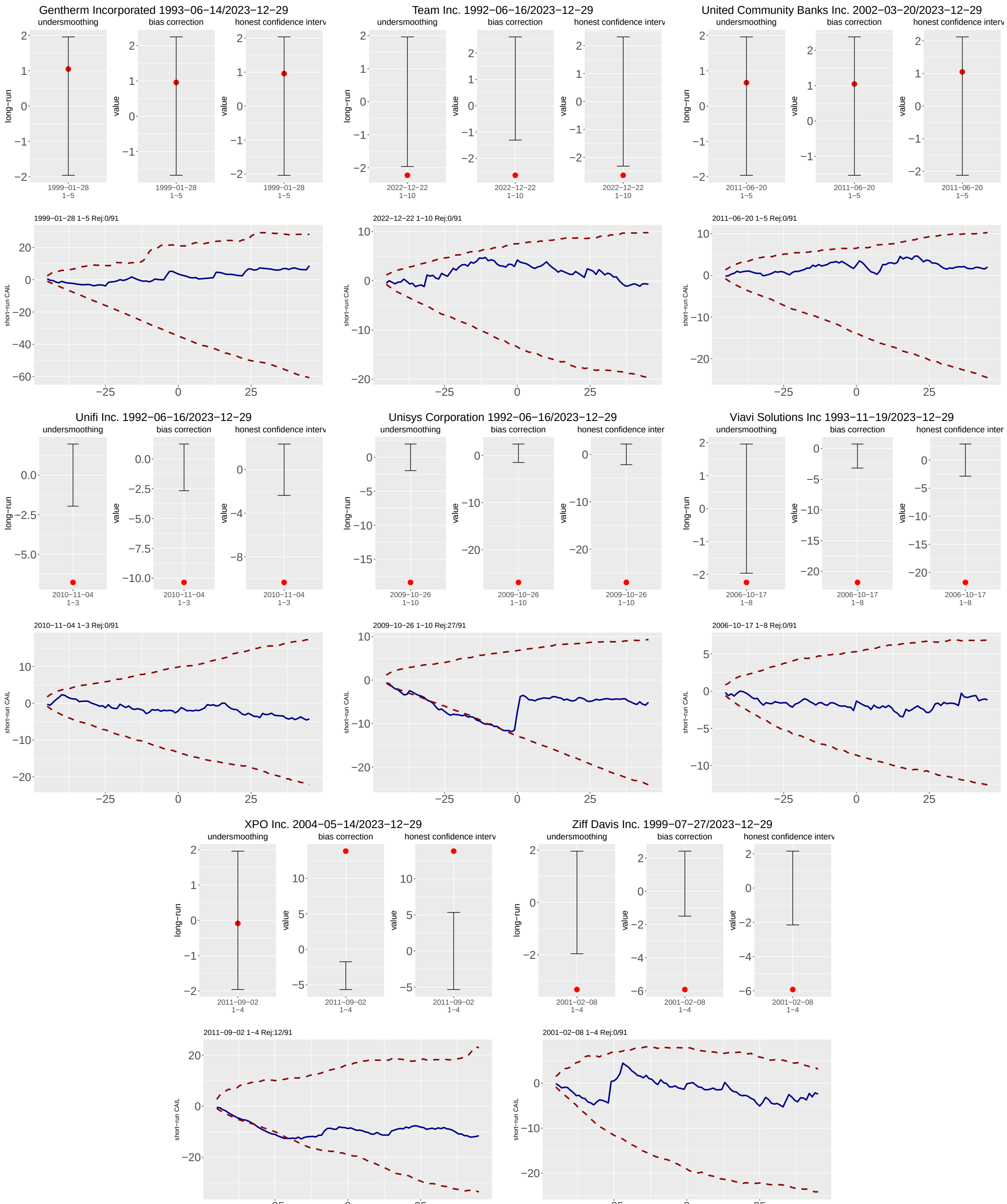


Figure 19: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

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