R&D Specialization and Productivity Slowdown

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Very Preliminary, Please don't circulate!
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Abstract

Patent data shows that US states that have more diversified patent portfolios and cite more complementary knowledge sources grow faster than other regions with more concentrated distribution of patents across industries. We build a two-sector open economy growth model where trade promotes specialization in both production and R&D. Regional specialization of production and research in a few RCA (relative comparative advantage) sectors may reduce long growth rate, when cross-sector knowledge diffusion is important to RCA sector's innovation and inter-national knowledge diffusion is too weak to supplement insufficient domestic non-RCA sectors' knowledge. Calibrated model parameters from US patent data show that both the self-sector knowledge contribution to innovation and the inter-state knowledge diffusion diminish continuously over time, which means growth rate is lower and the gain from trade is smaller or even reversed.

Keywords: Trade, R&D specialization, multi-sector model, cross sector knowledge diffusion

JEL Classification: J11, O33, O41, O47

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1 Introduction

Is trade always good for long run growth? Relative to the immense amount of research on trade and production, the literature on trade and innovation are still exploring all possible mechanisms. In standard Ricardian models, lower trade cost makes countries more specialized in their sectors with production relative comparative advantage (RCA). If we allow for endogenous R&D input allocation across sectors, we would find that R&D resources are as concentrated as production into RCA sectors, as predicted by Somale (2021) and Cai, Li, and Santacreu (2022).

Then the next question is: is specialization of R&D beneficial for long run growth? Under standard conditions, specialization of production promote exchange of labor in each region's RCA sectors and increase global welfare. Regional specialization of R&D in a few industries, however, generate a skewed local knowledge portfolio that could be different from the optimal bundle of knowledge input that fosters long run economic growth. This would generate the same impact as specialization of researchers commented by Austrian zoologist, ethologist, and ornithologist Konrad Lorenz:

'Scientists are people who know more and more about less and less until they know everything about nothing.'

What if great inventions do depend on a little bit of knowledge from every other sector? If innovation in one sector need the knowledge input from *all* sectors, knowledge diffusion is geographically *localized*, and that there is *decreasing return* to knowledge capital from each sector, then relative to the optimal input bundle, there is excessive supply of RCA sector's knowledge and insufficient supply of domestic non-RCA sector's knowledge to inventors. Therefore, regional specialization generates static benefit from trade but may compromise long run growth by impeding innovation.

Luckily, inter-regional knowledge spillovers can supplement local knowledge portfolio with external flow of non-RCA knowledge and partially alleviate the suboptimal supply of local knowledge. Hence, if inter-regional knowledge spillover is strong enough, trade induced specialization of research can still sustain gain from trade.

In the patent data, we look for answers to the follow questions: Does local patent portfolio affect state-sector level innovation rate? How do state-sector level innovation and production benefit from cross-state knowledge diffusion? Using NBER Patent data and BEA's regional industry level production data, we find that state level local knowledge portfolio matters for a state-sector's growth both in output and knowledge capital. First, States with a more diversified local knowledge portfolio grow faster.

Second, States grow faster in GDP and patent number if the external knowledge flow measured by citations to out of state patents is more complementary to domestic knowledge portfolio, especially so for the states with more skewed knowledge bundle. Therefore, this paper proposes another mechanism that knowledge diffusion promotes economic growth: to supplement local insufficient knowledge in non-RCA sectors.

In a two-country two-sector growth model, we abstract from the production input-output table and focus on the knowledge production input-output table. Innovation of new varieties in one sector combines knowledge inputs from both sectors using Cobb-Douglas production function, thus knowledge inputs from different sectors are imperfect substitutes.

Imagine that trade liberalization makes two regions more specialized in their own RCA sectors, both so for production and R&D. The deficiency of non-RCA knowledge deteriorates when trade cost decreases. Therefore, trade brings two offsetting impact on growth: on one hand, new products in the RCA sector are more profitable, which increases the return to innovation in the RCA sector; on the other hand, the uneven supply of sectoral knowledge inputs reduces the inter-sector knowledge diffusion to the RCA sector from non-RCA sector, which diminishes innovation rate in the RCA sector, especially so when the non-RCA sector's knowledge contributes a dominant share to the RCA sector's innovation . One solution to the negative impact of trade on innovation is to learn from the other region. Inter-regional knowledge diffusion can supplement each region with non-RCA sector's knowledge from the other region.

In a comparative statics exercise, we calibrate the model and exam global growth rate under various the key parameters: self-sector knowledge's contribution to innovation and the strength of inter-regional knowledge diffusion. We find that countries grow faster and benefit more from free trade when the self-sector knowledge's contribution to innovation or the strength of inter-regional knowledge diffusion are higher. Higher self-sector knowledge's contribution to innovation is complementary to free trade, because the concentration of knowledge stock in the RCA sector then nurtures future innovation in RCA sector, which dominates the negative impact from insufficient supply of non-RCA sector knowledge. Inter-regional knowledge diffusion helps alleviate the insufficient supply of non-RCA sector knowledge caused by free trade. In a special case with very low value of self-sector knowledge's contribution to innovation and inter-regional knowledge diffusion strength, growth rate even increases with trade cost, because the negative impact of trade on growth dominates.

Using US patent citation data, we measure the self-sector knowledge's contribution to innovation as the intensity to cite self-sector patents, i.e., self-sector citation share/self-sector knowledge share, and proxy the inter-regional knowledge diffusion strength as the intensity to cite out of state patent, i.e., out of state citation share/out of state patent share. Sadly, we discover that these two measures have declined over time since 1976 and now approach the aforementioned special case parameter values needed to generate a reversed relation between trade and growth.

To avoid such a situation, optimal innovation policy in the open economy need to encourage the research in non-RCA sectors, if those sectors' knowledge contributes heavily to RCA sector's innovation. Meanwhile fostering inter-regional diffusion of knowledge is beneficial, such policies include increasing inter-regional academic visitors and subsidies to inter-regional transportations.

2 Literature

This paper contributes to the flowing streams of literature. The first strand is the emerging literature on the dynamic gain from trade through innovation. The pioneers in this literature start from Eaton and Kortum (1996) and Eaton and Kortum (1999). Sampson (2023) is the first one to propose two-way interaction of trade and productivity, however, there is no role of knowledge spillovers. Buera and Oberfield (2020) study dynamic evolution of productivity with innovation and within industry knowledge spillovers. Sampson (2023) explains productivity gaps and income differences across countries with country-specific innovation efficiency and localization of within-industry knowledge spillovers.

The most related ones are dynamic open economy growth models with multiple sectors and inter-sectoral knowledge spillovers: Cai, Li, and Santacreu (2022) and Liu and Ma (2021). Cai, Li, and Santacreu (2022) incorporate heterogeneous IO tables, knowledge diffusion linkages and endogenous productivity in a multi-country multi-sector setting to study the bilateral relation between trade and productivity frontier. The knowledge production function is linearly additive in the knowledge input from different country-sectors, knowledge input from different sectors are perfectly substitutable, therefore what matters to global growth rate is the total amount of knowledge input accessible through diffusion network. In this paper, knowledge production function is Cobb-Douglass and multiplicative in knowledge inputs from different sectors, the ratio of sectoral knowledge input, instead of total knowledge stock, determines growth rate. Liu and Ma (2021) study the optimal R&D policy in an extended open economy model with international knowledge spillovers, they suggest that countries with higher self-sustained knowledge inputs allocate more research effort in the central sectors in the knowledge network. Our paper abstracts from the asymmetry of knowledge networks but adds asymmetry in sectoral comparative advantage in production, to study trade's impact on knowledge portfolio across sectors and its consequences in growth.

The second branch of literature is the empirical investigations on cross-regional knowledge diffusion. First, we add a new method to quantify the strength of cross-region knowledge diffusion. For example, Keller (2002) estimates how fast domestic R&D's impact on TFP decays over geographic distances. He found that knowledge has been more global since 1970s to 1990s. Comin, Dmitriev, and Rossi-Hansberg (2012) and Comin and Mestieri (2018) use the lag and penetration rate of new technology adoption to measure the extensive and intensive margins of knowledge diffusion for specific frontier technologies. They found that there has been a convergence of adoption lag but a divergence of penetration rate across countries. Cai, Li, and Santacreu (2022) approximate knowledge diffusion speed by the average citation lag between country-sector pairs. Liu and Ma (2021) use the share of citations given to domestic patents to measure the self-sustainability of a country's innovation, however the share of domestic citations is subject to the bias by the relative patent stock between home and foreign. In this paper, to match the model's specification, we want to measure the share of out of state patent stock diffused to home state to proximate the strength of inter-regional knowledge diffusion in US patent data. To adjust for the changing ratio between home and foreign patent stocks, we use the share of cross-state citation divided by the share of out of state patents to represent the intensity that inventors cite out of state patents. We find that this measure of inter-regional knowledge diffusion declines steadily from 1977 to 2021.

Second, previous research such as MacGarvie (2005) finds that two regions with similar industry structural cite more often to each other. However, diffusions measured by citations flows may not convert into growth of innovation and output. This paper finds that citation flows to states with complimentary knowledge portfolio has a stronger positive impact on state-sector growth in both patent number and employment. This fact supports the idea that inter-regional knowledge diffusion can supplement local knowledge portfolio with insufficient supply of non-RCA sector knowledge.

3 Empirics

3.1 Data

The state-industry level trade data is from US Trade Online in US Census Bureau, which contains data on state level total export value for 3 and 4 digit NAICS industries excluding services from 2002 to 2022. GDP and employment data are collected from BEA. It has information on state level GDP and employment for 2 and 3 digit NAIC industries from year from1998 to 2021. Patent data is collected from U.S. Patent and Trademark Office, providing detailed patent information including number of citation, state and technological classification from 1960 to 2021. We follow the crosswalk in Lybbert and Zolas (2014) to map the technology to industries under various NAICS codes.

To be consistent, we keep the year between 2002 and 2021 and regroup industries under 3 digit NAICS code. Table A.1 presents the detail on mapping consistent industry code. In particular, trade data are grouped into 23 sectors as it only has information on agricultural and manufacturing goods, while patent data has two more sectors–utilities and construction. GDP and employment has more sectors, for instance, the service sectors, and we keep them consistent when computing concentration ratio, while in the empirical analysis, we only keep the matching 23 sectors. In below, we present several state level and state-sector level facts related to production, innovation and knowledge diffusion.

3.2 State Level Facts

Fact 1. *In states where export share increases, concentration of patent portfolio and production across sectors rises.*

In this fact, we are expecting that concentration of R&D and change of production concentration are associated with increasing export over GDP at the sector level, that is, in regression (1) and (2), b^k and b^y are expected to be positive

$$\triangle HHI_{s,t}^k = b_0 + b^k ES_{s,t} + \log(Y_{s,t}) + D_s + D_t, \quad (1)$$

and

$$\triangle HHI_{s,t}^y = b_0 + b^y ES_{s,t} + \log(Y_{s,t}) + D_s + D_t, \quad (2)$$

where $ES_{s,t}$ is defined as export over output in state s time t, $Y_{s,t}$ is GDP in state s time t, $\triangle HHI_{s,t}^k$ is the change rate patent concentration

$$\triangle HHI_{s,t}^k = log(HHI_{s,t+1}^k) - log(HHI_{s,t}^k)$$

where $HHI_{s,t}^k$ is Herfindahl index of state $s's$ patent distribution over $J = 23$ sectors at year t being defined as

$$HHI_{s,t}^k = \sum_{j=1}^{J} (\frac{PS_{s,t}^j}{TPS_{s,t}})^2$$

where $PS_{s,t}^j$ is patent stock of state s sector j by time t, and $TPS_{s,t}$ is the total patent stock of state s by time t

$$TPS_{s,t} = \sum_{j=1}^{J} PS_{s,t}^j$$

Likewise, the change of production concentration is

$$\triangle HHI_{s,t}^y = log(HHI_{s,t+1}^y) - log(HHI_{s,t}^y)$$

and $HHI_{s,t}^y$ is Herfindahl index of state $s's$ production distribution over J sectors at year t.

$$HHI_{s,t}^y = \sum_{j=1}^{J} (\frac{Y_{s,t}^j}{TY_{s,t}})^2$$

$$TY_{s,t} = \sum_{j=1}^{J} Y_{s,t}^j$$

where $Y_{s,t}^j$ is the output at state s sector j time t, and $TY_{s,t}$ is the total output at state s time t. Table (1) presents regression results. In particular, column (1) and (2) support the fact that 1 percent increase of trade share will increase change rate of patent concentration by 0.36 percent and increase change rate of production concentration by 0.18 percent. This fact supports the theoretical predictions by Somale (2021) and Cai, Li, and Santacreu (2022) that trade liberalization makes regions more specialized not only in production but also in innovation.

Fact 2. *States with more concentrated patent portfolio grow slower in both patent number and production.*

In regressions (3) and (4), we expect b^k and b^y to be negative

$$g^k_{s,t+1} = b^k_{PS}\log(PS_{s,t}) + b^k\log(HHI^k_{s,t}) + \log(Y_{s,t}) + D_s + D_t; \tag{3}$$

and

$$g^y_{s,t+1} = b^y_{PS}\log(PS_{s,t}) + b^y\log(HHI^k_{s,t}) + \log(Y_{s,t}) + D_s + D_t; \tag{4}$$

where $g^k_{s,t+1}$ and $g^y_{s,t+1}$ are growth rates in patent number and GDP at state s year $t+1$. Column (3) and (4) in Table (1) support the fact that 1 percent increase of patent concentration will decrease patent growth rate by 0.904 percent and decrease GDP growth rate by 0.051 percent.

Fact 3. *States that cite more to other states with complementary patent portfolio grow faster.*

In regressions (5), we expect b_3 to be positive

$$g^y_{s,t+1} = b_{PS}\log(PS_{s,t}) + b_2\log(HHI^k_{s,t}) + b_3\text{log}(WCite^{TC}_{s,t}) + \log(Y_{s,t}) + D_s + D_t; \tag{5}$$

where $WCite^{TC}_{s,t}$ is the technology complementarity weighted summation of citations from state s to other states, specifically

$$WCite^{TC}_{s_1,t} = \sum_{s_2=1}^{J} Cite_{s_1,s_2,t}(1 - TP_{s_1,s_2,t})$$

where $Cite_{s_1,s_2,t}$ is the total citation from state s_1 to state s_2. Technology complementarity measure is $1 - TP_{s_1,s_2,t}$. Technology proximity between two states s_1 and s_2, $TP_{s_1,s_2,t}$, is defined as in Jaffe(1986)

$$TP_{s_1,s_2,t} = \frac{ps_{s_1,t}ps^{'}_{s_2,t}}{(ps_{s_1,t}ps^{'}_{s_1,t})^{1/2}(ps_{s_2,t}ps^{'}_{s_2,t})^{1/2}}. \tag{6}$$

Table 1: State level facts

VARIABLES	(1) $\triangle HHI^k_{s,t}$	(2) $\triangle HHI^y_{s,t}$	(3) $g^k_{s,t+1}$	(4) $g^y_{s,t+1}$	(5) $g^y_{s,t+1}$
log(PS)			0.310***	0.016***	0.020***
			(0.02)	(0.00)	(0.00)
HHI_PS			**-0.904*****	**-0.051***	-0.033
			(0.19)	(0.03)	(0.03)
log(GDP)	-0.08***	-0.10***	-0.417***	-0.086***	
	(0.02)	(0.02)	(0.03)	(0.01)	
trade share	**0.36*****	**0.18***			
	(0.09)	(0.10)			
log(WCiteTC)					**0.003*****
					(0.00)
Constant	1.14***	1.22***	2.636***	0.952***	0.158***
	(0.24)	(0.26)	(0.22)	(0.15)	(0.01)
Observations	928	928	928	928	928
R-squared	0.813	0.196	0.249	0.382	0.154
Year FE	YES	YES	NO	YES	NO
State FE	YES	YES	NO	YES	YES

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: This table presents the regression results of Fact 1-Fact 3. Column (1)-(2) support Fact 1, column (3)-(4) support Fact 2, and column (5) supports Fact 3 respectively.

where $ps_{s,t}$ is the vector of patent share across sectors in state s. Column (5) in Table (1) supports the fact that 1 percent increase of technology complementarity weighted sum of citations ($WCite^{TC}_{s_1,t}$) will increase per capita GDP growth rate by 0.003 percent.

3.3 State-Sector Level Facts

Fact 4. *If one state-sector exports more relative to production, its patent share in the state increases.*

We are expecting the specialization of production and export is related to specialization of R&D. In regression (7), b_{EX} and b_Y are expected to be positive

$$\log(PS^i_{s,t}) = b_{EX}\log(EX^i_{s,t}) + b_Y\log(Y^i_{s,t}) + D_{s,t} + D^i_t; \tag{7}$$

where superscript i denotes sector and subscript s denotes state, $PS^i_{s,t}$ is the patent share granted to sector i state s by year t, and $EX^i_{s,t}$ is defined as export over output, $Y^i_{s,t}$ is GDP level, D^i_t and $D_{s,t}$ are sector-year and state-year fixed effect. Column (1) of Table (2) supports this fact that 1 percent increase of trade share will increase 0.03 percent of patent number.

Fact 5. *One state-sector grow faster if its local state's patent portfolio is more proximate to the ideal knowledge input to this sector .*

We construct the ideal bundle of knowledge input to sector i at time t, cit^i_t, by the national average share of outward citation from sector i to other sectors at time t. It is a 1 by J vector, with the jth element being the share of citations from i sector to j sector among all citations from i at time t. $ps_{s,t}$ is state $s's$ patent stock share vector, with the jth element being the share of sector j patent in state $s's$ total patent stock. Technology proximity between $ps_{s,t}$ and cit^i_t, $TP^i_{s,t}$, is given by

$$TP^i_{s,t} = \frac{cit^i_t ps^{'}_{s,t}}{(cit^i_t cit^{i'}_t)^{1/2}(ps_{s,t}ps^{'}_{s,t})^{1/2}}.$$

Since inventors are constrained by local knowledge input supply, researcher should benefit from a local knowledge portfolio that is closer to the ideal knowledge input bundle of sector i; therefore, we expect the coefficient b_{TP} to be positive in regression (8)

$$g^i_{s,t+1} = b_{PS}\log(PS^i_{s,t}) + b_{TP}\log(TP^i_{s,t}) + b_Y\log(Y^i_{s,t}) + D_{s,t} + D^i_t; \tag{8}$$

where $g^i_{s,t+1}$ is GDP growth rate, $PS^i_{s,t}$ is the patent share granted, $TP^i_{s,t}$ is technology proximity, $Y^i_{s,t}$ is GDP level D^i_t and $D_{s,t}$ are industry-year and state-year fixed effect. Column (2) of Table (2) supports this fact that 1 percent increase of technology proximity will increase GDP growth rate by 0.04 percent.

Fact 6. *One state-sector (j,n) grows faster if there are more outward citations to state-sector (m,k), where k knowledge is more relatively abundant in state m than in state n.*

This fact captures inventors' motivation to cite patents across states: search for more abundant knowledge capital in a specific sector which is scare in local state. In regression (9), we expect b_{WC} to be positive and b_{TPWC} to be negative

$$g^i_{s,t+1} = b_{PS}\log(PS^i_{s,t}) + b_{TP}\log(TP^i_{s,t}) + b_{WC}\log(WCite^{TA,i}_{s,t}) + b_{TPWC}TP^i_{s,t}\log(WCite^{TA,i}_{s,t}) + b_Y\log(Y^i_{s,t}) + D_{s,t} + D^i_t; \tag{9}$$

Table 2: State-sector level facts

VARIABLES	(1) $\log(PS^i_{s,t})$	(2) $g^i_{s,t+1}$	(3) $g^i_{s,t+1}$
log(GDP)	0.14***	-0.01***	-0.23***
	(0.00)	(0.00)	(0.01)
log(trade share)	**0.03*** **		
	(0.00)		
log(TP)		**0.04*** **	0.08*
		(0.02)	(0.04)
log(PS)		0.00	0.01
		(0.00)	(0.01)
log(WCiteTA)			**0.02*** **
			(0.01)
TP*log(WCiteTA)			**-0.02*** **
			(0.01)
Constant	-4.74***	0.07***	1.53***
	(0.02)	(0.01)	(0.06)
Observations	20,901	18,565	10,224
R-squared	0.801	0.273	0.449
Year*State FE	YES	YES	YES
Year*Ind FE	YES	YES	YES
State*Ind FE	NO	NO	YES

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: This table presents the regression results of Fact 4-Fact 6. Column (1) supports Fact 4, column (2) support Fact 5, and column (3) supports Fact 6 respectively.

where $g^i_{s,t+1}$ is GDP growth rate, $PS^i_{s,t}$ is the patent share granted, $TP^i_{s,t}$ is technology proximity, $Y^i_{s,t}$ is GDP level D^i_t and $D_{s,t}$ are industry-year and state-year fixed effect, and

$$WCite^{TA,i}_{s_1,t} = \sum_{s_2=1}^{S}\sum_{j=1}^{J} Cite^{i\to j}_{s_1,s_2,t}[\log(PS^j_{s_2,t}) - \log(PS^j_{s_1,t})]$$

where $Cite^{i\to j}_{s_1,s_2,t}$ is the total citation from state $s_1$sector i to state $s_2$sector j. Technology abundance of sector j in state s is indicated by sector $j's$ patent share in state s, $\log(PS^j_{s,t})$. Column (3) of Table (2) supports this fact that 1 percent increase of $WCite^{TA}$ will increase GDP growth rate by 0.02 percent, and the coefficient of interaction term between this and technology proximity is -0.02.

In summary, state-sector and state level growth rate of innovation and output depend on local knowledge portfolio and external knowledge diffusion, more diverse local patent portfolio and knowledge diffusion from others states with complementary knowledge portfolio promote higher growth rate. In the next section, we build a two-country two-sector model to rationalize the above facts. Trade induced concentration of R&D in RCA sectors cause local knowledge portfolio to deviate from the ideal knowledge input bundle for innovation, hence hinder economic growth rate. Cross-country knowledge spillovers replenish skewed local knowledge pool with complementary external knowledge flows, and bring the total accessible knowledge pool closer to the ideal bundle of knowledge input. As a result, growth rate increases with intensity of cross-country knowledge diffusion. In this model, trade liberalization and cross-country knowledge diffusion strengthen each other's impacts on growth.

4 The Model

There are two symmetric countries: home and foreign and two sectors: 1 and 2. For all variables we use subscription $*$ to denote the correspondent foreign variables.

The representative household has a standard CRRA preference.

$$U = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta_c} - 1}{1-\theta_c} dt$$

The aggregate production function of final goods is a CES combination of two sectoral compound goods Y_1 and Y_2, which are non-tradable.

$$Y(t) = [\gamma_1 Y_1(t)^{\frac{\epsilon-1}{\epsilon}} + \gamma_2 Y_2(t)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}$$

Resource constraint for final goods is

$$Y(t) = C(t) + X(t) + Z(t), \tag{10}$$

where $X(t)$ and $Z(t)$ are inputs to intermediate goods and research, respectively.

4.1 Production

Sectoral compound goods in both sectors are produced competitively with labor and tradable intermediate goods.

$$Y_i(t) = \frac{1}{1-\beta}(\int_0^{N_i(t)} x_{i,d}(\upsilon,t)^{1-\beta}d\upsilon + \int_0^{N_i^*(t)} x_{i,e}^*(\upsilon,t)^{1-\beta}d\upsilon)L_i(t)^{\beta}, i \in (1,2) \tag{11}$$

$x_{i,d}(\upsilon,t)$ and $x_{i,e}^*(\upsilon,t)$ are domestically produced and imported intermediate goods used in sector i,$i \in (1,2)$; $N_i(t)$ and $N_i^*(t)$ are the number of sector i intermediate goods in home and foreign, respectively.

Each unit of home intermediate goods in sector i are produced using $\frac{1}{A_i}$, $i \in (1,2)$ units of final goods. A_i represents home's absolute advantage in sector i's production of intermediate goods. Home and foreign aggregate price indices are

$$P(t) = [\gamma_1^{\epsilon}P_1(t)^{1-\epsilon} + \gamma_2^{\epsilon}P_2(t)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}, \tag{12}$$

$$P^*(t) = [\gamma_1^{\epsilon}P_1^*(t)^{1-\epsilon} + \gamma_2^{\epsilon}P_2^*(t)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}. \tag{13}$$

The optimal prices of home intermediate goods at domestic and export markets are

$$p_{i,d}(\upsilon,t) = \frac{P(t)}{A_i}, \tag{14}$$

$$p_{i,e}(\upsilon,t) = \frac{P(t)\tau}{A_i}. \tag{15}$$

where τ is the symmetric iceberg type trade cost between home and foreign. Similarly, foreign intermediate goods producers set their domestic and export prices at

$$p_{i,d}^*(\upsilon,t) = \frac{P^*(t)}{A_i^*}, \tag{16}$$

$$p_{i,e}^*(\upsilon,t) = \frac{P^*(t)\tau}{A_i^*}. \tag{17}$$

The compound goods producers solve the following problem.

$$\max_{x_{i,d}(\upsilon,t),x_{i,e}^*(\upsilon,t),L_i(t)} P_i(t)Y_i(t) - W(t)L_i(t) - \int_0^{N_i(t)} p_{i,d}(\upsilon,t)x_{i,d}(\upsilon,t)d\upsilon - \int_0^{N_i^*(t)} p_{i,e}^*(\upsilon,t)x_{i,e}^*(\upsilon,t)d\upsilon$$

First order conditions of this problem are

$$W(t) = \frac{\beta}{1-\beta} P_i(t) L_i(t)^{\beta-1} \left(\int_0^{N_i(t)} x_{i,d}(\upsilon,t)^{1-\beta} d\upsilon + \int_0^{N_i^*(t)} x_{i,e}^*(\upsilon,t)^{1-\beta} d\upsilon \right), i \in (1,2), \quad (18)$$

$$P_i(t) x_{i,d}(\upsilon,t)^{-\beta} L_i(t)^{\beta} = p_{i,d}, i \in (1,2), \quad (19)$$

$$P_i(t) x_{i,e}^*(\upsilon,t)^{-\beta} L_i(t)^{\beta} = p_{i,e}^*, i \in (1,2). \quad (20)$$

Substitute the intermediate goods prices (14) to (17) into the first order conditions in (18) to (20), we have

$$W(t) = \beta P_i(t)^{\frac{1}{\beta}} \tilde{N}_i(t), i \in (1,2), \quad (21)$$

$$x_{i,d}(\upsilon,t) = \left(\frac{P_i(t) A_i}{P(t)}\right)^{\frac{1}{\beta}} L_i(t), i \in (1,2), \quad (22)$$

$$x_{i,e}^*(\upsilon,t) = \left(\frac{P_i(t) A_i^*}{P^*(t)\tau}\right)^{\frac{1}{\beta}} L_i(t), i \in (1,2). \quad (23)$$

Likewise, foreign compound goods producers choose their inputs according to following first order conditions.

$$W^*(t) = \beta P_i^*(t)^{\frac{1}{\beta}} \tilde{N}_i^*(t), i \in (1,2), \quad (24)$$

$$x_{i,d}^*(\upsilon,t) = \left(\frac{P_i^*(t) A_i^*}{P^*(t)}\right)^{\frac{1}{\beta}} L_i^*(t), i \in (1,2), \quad (25)$$

$$x_{i,e}(\upsilon,t) = \left(\frac{P_i^*(t) A_i}{P(t)\tau}\right)^{\frac{1}{\beta}} L_i^*(t), i \in (1,2). \quad (26)$$

When we substitute the FOCs back to (27) and the budget constraint of compound goods producers

$$Y_i(t) P_i(t) = W(t) L_i(t) + \int_0^{N_i(t)} x_{i,d}(\upsilon,t) p_{i,d}(\upsilon,t) d\upsilon + \int_0^{N_i(t)} N_i^*(t) x_{i,e}^*(\upsilon,t) p_{i,e}^*(\upsilon,t) \upsilon, i \in (1,2),$$

we can present sectoral and aggregate outputs as functions of the effective number of varieties in home sector i, $\tilde{N}_i(t) = \frac{A_i}{P(t)}^{\frac{1-\beta}{\beta}} N_i(t) + \left(\frac{A_i^*}{\tau P^*(t)}\right)^{\frac{1-\beta}{\beta}} N_i^*(t)$, $i \in (1,2)$, which is a productivity weighted average of domestic and foreign number of inputs, higher trade cost reduces $\tilde{N}_i(t)$ through fewer foreign inputs.

$$Y_i(t) = \frac{P_i(t)^{\frac{1-\beta}{\beta}} \tilde{N}_i(t) L_i(t)}{(1-\beta)^{\beta}}, i \in (1,2), \quad (27)$$

$$P_i(t) = (\frac{(1-\beta)W(t)}{\beta\tilde{N}_i(t)})^{\beta}, i \in (1,2). \tag{28}$$

$$Y(t) = [\gamma_1 P_1(t)^{\sigma}(\tilde{N}_1(t)L_2(t))^{\frac{\epsilon-1}{\epsilon}} + \gamma_2 P_2(t)^{\sigma}(\tilde{N}_1(t)L_2(t))^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \tag{29}$$

And the counterparts for the foreign country are:

$$Y_i^*(t) = \frac{P_i^*(t)^{\frac{1-\beta}{\beta}}\tilde{N}_i^*(t)L_i^*(t)}{(1-\beta)^{\beta}}, i \in (1,2), \tag{30}$$

$$P_i^*(t) = (\frac{(1-\beta)W^*(t)}{\beta\tilde{N}_i^*(t)})^{\beta}, i \in (1,2), \tag{31}$$

$$Y^*(t) = [\gamma_1 P_1^*(t)^{\sigma}(\tilde{N}_1^*(t)L_2^*(t))^{\frac{\epsilon-1}{\epsilon}} + \gamma_2 P_2^*(t)^{\sigma}(\tilde{N}_1^*(t)L_2^*(t))^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \tag{32}$$

where $\sigma = \frac{(1-\beta)(1-\epsilon)}{\beta\epsilon}$ and $\tilde{N}_i^*(t) = (\frac{A_i^*}{P^*(t)})^{\frac{1-\beta}{\beta}}N_i^*(t) + (\frac{A_i}{\tau P(t)})^{\frac{1-\beta}{\beta}}N_i(t)$, $i \in (1,2)$ is the effective number of varieties in sector i.

Therefore, home and foreign intermediate producers' total profit in time t are

$$\pi_i(\upsilon,t) = \pi_{i,d}(\upsilon,t) + \pi_{i,e}(\upsilon,t) = \beta[(\frac{A_i}{P})^{\frac{1-\beta}{\beta}}P_i(t)^{\frac{1}{\beta}}L_i(t) + (\frac{A_i}{P\tau})^{\frac{1-\beta}{\beta}}P_i^*(t)^{\frac{1}{\beta}}L_i^*(t)] \tag{33}$$

$$\pi_i^*(\upsilon,t) = \pi_{i,d}^*(\upsilon,t) + \pi_{i,e}^*(\upsilon,t) = \beta[(\frac{A_i^*}{P^*})^{\frac{1-\beta}{\beta}}P_i^*(t)^{\frac{1}{\beta}}L_i^*(t) + (\frac{A_i^*}{P^*\tau})^{\frac{1-\beta}{\beta}}P_i(t)^{\frac{1}{\beta}}L_i(t)] \tag{34}$$

One variety intermediate good in sector $i \in (1,2)$ generates a discounted total profit flow of $v_i(\upsilon,t)$ and $v_i^*(\upsilon,t)$ when the discount rate are r and r^* in home and foreign countries.

$$v_i(t) = v_i(\upsilon,t) = \frac{\pi_i(\upsilon,t)}{r} \tag{35}$$

$$v_i^*(t) = v_i^*(\upsilon,t) = \frac{\pi_i^*(\upsilon,t)}{r^*} \tag{36}$$

Combine (27) and (30) with the optimal input choices of final good producers, we can determine the relative price of compound goods.

$$\frac{P_1(t)}{P_2(t)} = \frac{\gamma_1}{\gamma_2}(\frac{Y_1(t)}{Y_2(t)})^{-\frac{1}{\epsilon}} = \frac{\gamma_1}{\gamma_2}\frac{\tilde{N}_1(t)}{\tilde{N}_2(t)}^{-\frac{\beta}{\epsilon}}\frac{L_1(t)}{L_2(t)}^{-\frac{1}{\epsilon}} \tag{37}$$

Since labor is mobile across the two sectors of production, Y_i producer's optimal choice of labor (18) gives us

$$1 = (\frac{P_1(t)}{P_2(t)})^{\frac{1}{\beta}} \frac{\tilde{N}_1(t)}{\tilde{N}_2(t)} = (\frac{\gamma_1}{\gamma_2})^{\frac{1}{\beta}} (\frac{\tilde{N}_1(t)}{\tilde{N}_2(t)})^{\epsilon-1} (\frac{L_1(t)}{L_2(t)})^{-\frac{1}{\epsilon\beta}}, \tag{38}$$

when we substitute (37) into (18). Hence, the labor allocation across sectors depends on the relative effective number of varieties.

$$\frac{L_1(t)}{L_2(t)} = (\frac{\gamma_1}{\gamma_2})^{\epsilon} (\frac{\tilde{N}_1(t)}{\tilde{N}_2(t)})^{\beta(\epsilon-1)} \tag{39}$$

$$\frac{L_1^*(t)}{L_2^*(t)} = (\frac{\gamma_1}{\gamma_2})^{\epsilon} (\frac{\tilde{N}_1^*(t)}{\tilde{N}_2^*(t)})^{\beta(\epsilon-1)} \tag{40}$$

4.2 Innovation

Inventors combine knowledge capital in all sectors to innovate new varieties in a given sector. The knowledge production function in country c sector i is a Cobb-Douglas function in both sectors' available knowledge stocks.

$$\dot{N}_i = \lambda_i K_{ii}^{\alpha_i} K_{ij}^{1-\alpha_i}, i, j \in (1,2) \tag{41}$$

α_i is the importance of self-sector knowledge in the innovation of sector i. K_{ij} is the amount of available sector j knowledge after scientists spend Z_{ij} units of final good searching in total knowledge pool in country c sector j PS_j, $i, j \in (1,2)$.

$$K_{ij} = PS_j^{1-\beta_s} Z_{ij}^{\beta_s}, i, j \in (1,2) \tag{42}$$

Equation (42) describes the matching function between researchers and patents, and β_s is the importance of research input Z_{ij} in the search for useful knowledge. $PS_j = N_j + \theta N_j^*$ is the summation of domestic knowledge stock $N_{c,j}$ and a fraction $0 < \theta < 1$ of foreign knowledge stock N_j^* that has diffused to home country and θ measures the strength of international knowledge diffusion. The assumption that knowledge diffusion is stronger within country than across countries follows Keller (2002) and .

Free entry condition for innovating firms means

$$P(Z_{ii}+Z_{ij})=\dot{N}_i v_i=\lambda_i(PS_i^{\alpha_i}PS_j^{1-\alpha_i})^{1-\beta_s}(Z_{ii}^{\alpha_i}Z_{ij}^{1-\alpha_i})^{\beta_s}v_i, i,j\in(1,2) \tag{43}$$

Since the optimal ratio between Z_{ii} and Z_{ij} is $\frac{\alpha_i}{1-\alpha_i}$ to maximize $Z_{ii}^{\alpha_i}Z_{ij}^{1-\alpha_i}$, we have

$$P(Z_{ii}+Z_{ij})=\frac{PZ_{ii}}{\alpha_i}=\dot{N}_i v_i=g_i N_i v_i \tag{44}$$

where $g_i=\frac{\dot{N}_i}{N_i}$ is the innovation growth rate in sector i.

Substituting (44) back to (43), we can write the growth rate of sector i's varieties as a function of knowledge value v_i and relative sizes of knowledge pools $\frac{PS_i}{N_i}$ and $\frac{PS_j}{N_i}$.

$$g_i=(\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i})^{\frac{\beta_s}{1-\beta_s}}\lambda_i(\frac{v_i}{P})^{\frac{\beta_s}{1-\beta_s}}(\frac{PS_i}{N_i})^{\alpha_i}(\frac{PS_j}{N_i})^{1-\alpha_i}, i,j\in(1,2), \tag{45}$$

4.3 General Equilibrium and Balanced Growth Path

4.3.1 Balanced Growth Path

Remember that the representative household's optimal consumption path satisfies

$$\frac{\dot{C}(t)}{C(t)}=\frac{1}{\theta_c}(r(t)-\rho).$$

and the transversality condition

$$\lim_{t\to\infty}[exp(-\int_0^\infty r(s)ds)N_i(t)v_i(t)]=0, i\in(1,2).$$

On BGP $N_i(t)$ grows at the same rate as $Y(t)$ and $C(t)$, hence

$$g=\frac{1}{\theta_c}(r(t)-\rho), i\in(1,2). \tag{46}$$

g and v_i are therefore jointly determined by (45) and (46).

The foreign counterparts of (45) and (46) are

$$g^*=\frac{1}{\theta_c}(r^*(t)-\rho), i\in(1,2). \tag{47}$$

$$g_i^* = (\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i})^{\frac{\beta_s}{1-\beta_s}}\lambda_i(\frac{v_i^*}{P^*})^{\frac{\beta_s}{1-\beta_s}}(\frac{PS_i^*}{N_i^*})^{\alpha_i}(\frac{PS_j^*}{N_i^*})^{1-\alpha_i}, i,j \in (1,2), \tag{48}$$

4.3.2 resource constraints

The total final good used to produce intermediate inputs is

$$X(t) = \sum_{i=1}^{2}\int_0^{N_i(t)} \frac{1}{A_i}(x_{i,d}(\upsilon,t) + \tau x_{i,e}(\upsilon,t))d\upsilon$$

$$X(t) = \sum_{i=1}^{2}\frac{N_i(t)}{A_i}[(\frac{P_i(t)A_i}{P(t)})^{\frac{1}{\beta}}L_i(t) + \tau^{\frac{\beta-1}{\beta}}(\frac{P_i^*(t)A_i}{P(t)})^{\frac{1}{\beta}}L_i^*(t)]d\upsilon$$

And, according to (44), the total research use of final good is

$$Z(t) = \sum_{i,j\in(1,2)}\frac{v_i(t)}{P(t)}\alpha_{ij}g_i(t)N_i(t) = \sum_{i\in(1,2)}\frac{v_i(t)g_i(t)N_i(t)}{P(t)}.$$

Therefore, the total resource constraints in both countries are

$$Y(t) = \sum_{i=1}^{2}\frac{N_i(t)}{A_i\beta}[(\frac{P_i(t)A_i}{P(t)})^{\frac{1}{\beta}}L_i(t) + \tau^{\frac{\beta-1}{\beta}}(\frac{P_i^*(t)A_i}{P(t)})^{\frac{1}{\beta}}L_i^*(t) + \frac{v_i(t)}{P(t)}g_i(t)] + C(t), \tag{49}$$

$$Y^*(t) = \sum_{i=1}^{2}\frac{N_i^*(t)}{A_i^*\beta}[(\frac{P_i^*(t)A_i^*}{P^*(t)})^{\frac{1}{\beta}}L_i^*(t) + \tau^{\frac{\beta-1}{\beta}}(\frac{P_i(t)A_i^*}{P^*(t)})^{\frac{1}{\beta}}L_i(t) + \frac{v_i^*(t)}{P^*(t)}g_i^*(t)] + C^*(t). \tag{50}$$

The total labor constraints in both countries are

$$L_1(t) + L_2(t) = L, \tag{51}$$

$$L_1^*(t) + L_2^*(t) = L^* \tag{52}$$

The free entry condition for innovation sector implies that the representative household's income comes purely from wage, which is then used in consumption.

$$P(t)C(t) = W(t)L, \tag{53}$$

$$P^*(t)C^*(t) = W^*(t)L^* \tag{54}$$

Lastly, The balance of trade between two countries means

$$\sum_{i=1}^{2} \frac{N_i^*(t)P^*(t)x_{i,e}^*(t)}{A_i^*} = \sum_{i=1}^{2} \frac{N_i(t)P(t)x_{i,e}(t)}{A_i}$$

$$\sum_{i=1}^{2} N_i^*(t)(\frac{A_i^*}{P^*(t)})^{\frac{1-\beta}{\beta}} \frac{W(t)L_i(t)}{\tilde{N}_i(t)} = \sum_{i=1}^{2} N_i(t)(\frac{A_i}{P(t)})^{\frac{1-\beta}{\beta}} \frac{W^*(t)L_i^*(t)}{\tilde{N}_i^*(t)} \tag{55}$$

In the general equilibrium, the 29 unknowns, including 12 prices, P, P^*, P_1, P_2, P_1^*, P_2^*, W, W^*, v_1, v_2, v_1^* and v_2^*, and 17 quantities, N_1, N_2, N_1^*, N_2^*, L_1, L_2, L_1^*, L_2^*, C, Y, Y_1, Y_2, C^*, Y^*, Y_1^*, Y_2^* and g, are jointly determined by the following 29 equations, (12), (13), (21), (24),(27), (29), (30), (32), (35), (36), (39), (40), (45), (46), (47), (48), (49), (50), (51), (52), (53), (54) and (55).

4.4 Comparative Statics

We explore different values of the importance of self-sector knowledge α_i and the strength of knowledge diffusion θ, to study trade cost τ's impact on growth rate g. We expect that trade liberalization could be more costly to growth when α_i and θ are smaller, that is when trade and specialization pushes innovation in the other sector to foreign country, which makes the more important other-sector knowledge harder to access.

In one scenario we choose $\alpha_i = 0.9$ and $\theta = 0.1$, in the other scenario we set $\alpha_i = 0.5$ and $\theta = 0.9$, to compare the relation between τ and g under different influences of innovation specialization.

To understand the contribution of different knowledge sources, we can rewrite the innovation growth rate g_i in (45) as

$$g_i = \lambda_i \underset{+}{(\frac{v^i}{P})^{\frac{\beta_s}{1-\beta_s}}} \underset{+}{(1+\theta\frac{N_j^*}{N_j})^{1-\alpha_i}} \underset{-}{(1+\theta\frac{N_i^*}{N_i})^{\alpha_i}} \underset{-}{(\frac{N_j}{N_i})^{1-\alpha_i}}, i, j \in (1,2), \tag{56}$$

where $1+\theta\frac{N_i^*}{N_i}$ and $1+\theta\frac{N_j^*}{N_j}$ represents the strength of international knowledge spillovers, while $\frac{N_j}{N_i}$ measures the inter-sectoral knowledge spillovers to sector i. v_i measures firm's incentive to innovate in sector i.

On the balanced growth path, if we consider internal solutions with positive production and innovation in both sectors only, then both sectors in two countries grow at the same

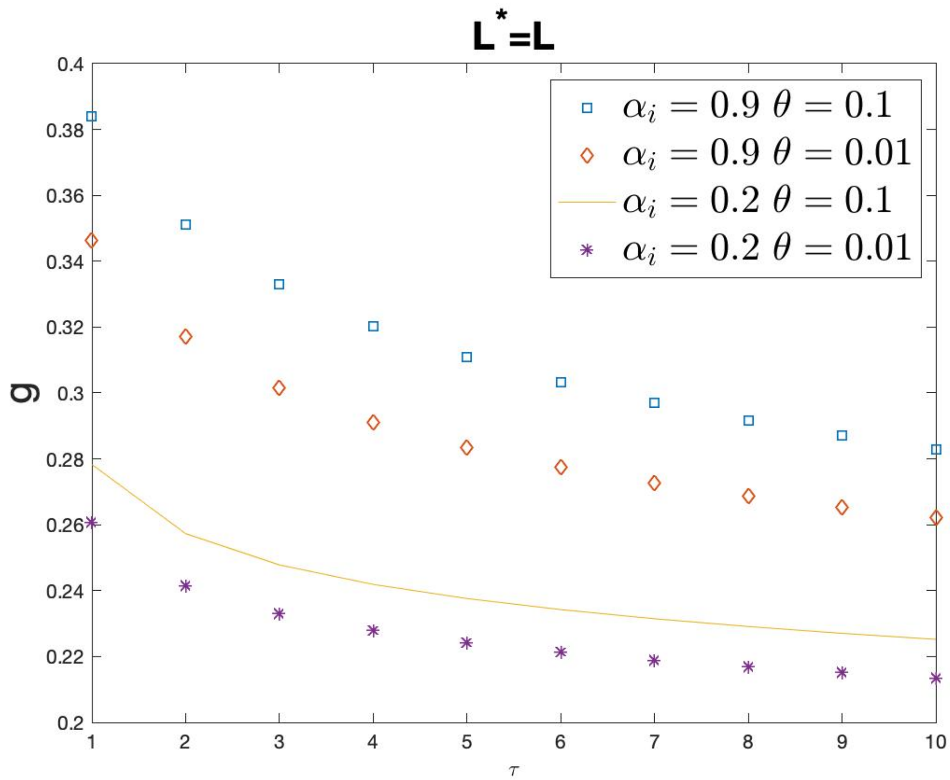

Figure 1: Symmetric Countries

rate. Therefore, if sector i is home's RCA sector, then trade liberalization brings a higher knowledge value v_i, a stronger knowledge diffusion from foreign sector j, but weaker knowledge spillovers from foreign sector i and home sector j, since N_j is smaller relative to N_j^*, but N_i is relatively larger than its foreign counterpart N_i^* and the other domestic sector N_j, $j \neq i$.

How much do countries benefit from free trade depends on the relative magnitude between the positive forces, the first two factors, and the negative forces, the last two factors in (56). We consider two scenarios. In the first case, we let the two countries to have equal population size, $L = L^*$. In another case, we set $L^* = 10L$ so that home is a small open economy relative to the foreign country. Then we vary the parameters that the growth rate is sensitive to, such as α_i and θ. In most cases, countries benefit from free trade, but under some extreme parameter values, we can find that free trade is a detriment to growth.

For example, when θ is close to zero, the second and third forces have limited influences, the game mainly happens between v_i and the last factor $(\frac{N_j}{N_i})^{1-\alpha_i}$. When α_i is small, or the other sectors' knowledge contributes more to RCA sector's innovation, the insufficient

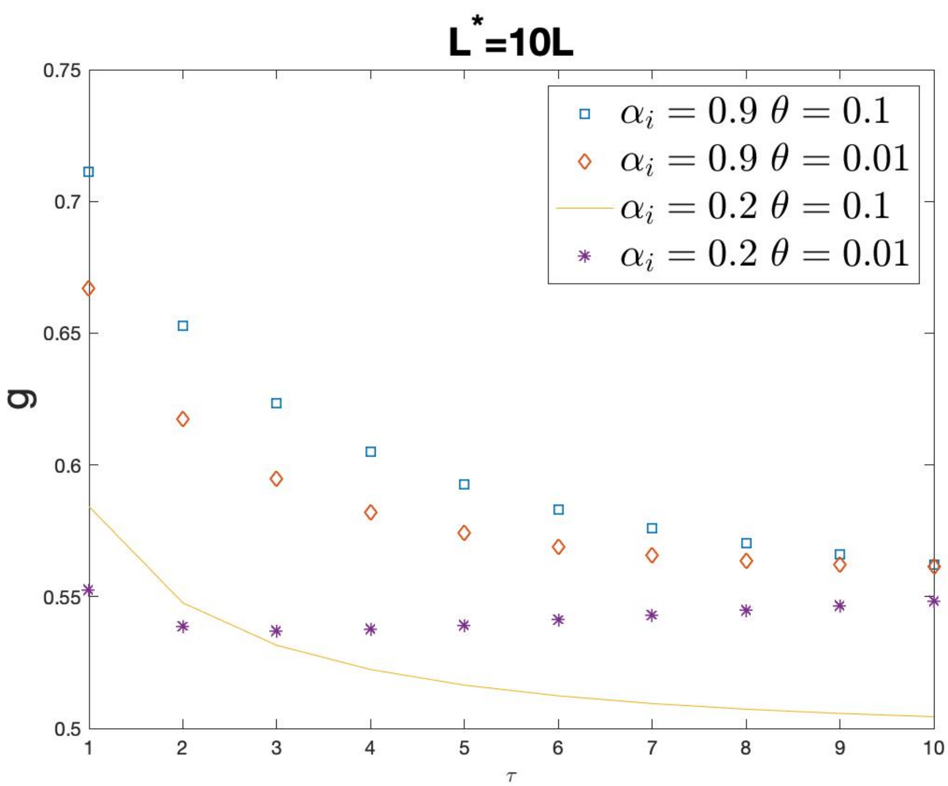

Figure 2: Home as a Small Open Economy

inter-sector knowledge diffusion dampens the gain from free trade with a greater strength. When $(\frac{N_j}{N_i})^{1-\alpha_i}$ is also extremely small, in the scenario that home country is a small open economy relative to foreign country and sector i is home's RCA sector as in Figure 2, growth rate increases with τ, which means free trade is harmful to growth, see the purple star line in Figure 2.

When *theta* is sufficiently large, the third positive factor $(1+\theta\frac{N_i^*}{N_i})^{\alpha_i}$ can compensate the last negative factor $\frac{N_j}{N_i})^{1-\alpha_i}$, so that the aggregate growth rate always benefits from free trade, even when α_i and $(\frac{N_j}{N_i})^{1-\alpha_i}$ are both very small, see the square and solid lines in Figure 2.

When the two countries have equal sizes in Figure 1, we won't see extreme low values of $(\frac{N_j}{N_i})^{1-\alpha_i}$, hence the last negative factor never dominates the positive factors, even when $alpha_i$ is small, therefore we always find that free trade boosts long run growth.

Even when growth always benefit from trade, there are quantitive differences under various parameter settings. On one hand, for a given value of international knowledge spillovers θ, a higher self-sector knowledge contribution share α_i implies faster growth rate and larger gain from free trade, for example, when we compare the square and solid lines or the diamond and the star lines in both Figures 1 and 2. This means if a country has comparative advantage in a sector whose innovation depends mainly on self-sector knowledge, then trade makes home country's innovation more specialized in RCA sector, more self-sector knowledge further nurtures future innovation in RCA sector, that is why growth rate increases more with lower trade cost.

On the other hand, for a given value of self-sector knowledge contribution share α_i, a stronger international knowledge diffusion θ means a faster growth rate and a greater gain from trade too, when we compare the square and diamond lines in both figures. The contrasts are more obvious in 2.

Overall, it is uncertain how much trade liberalization can boost growth and innovation. Especially, the stronger reliance on non-RCA sector's knowledge input in innovation and the lack of international knowledge spillovers may cause a dynamic loss from trade, even though there is a static gain from specialization.

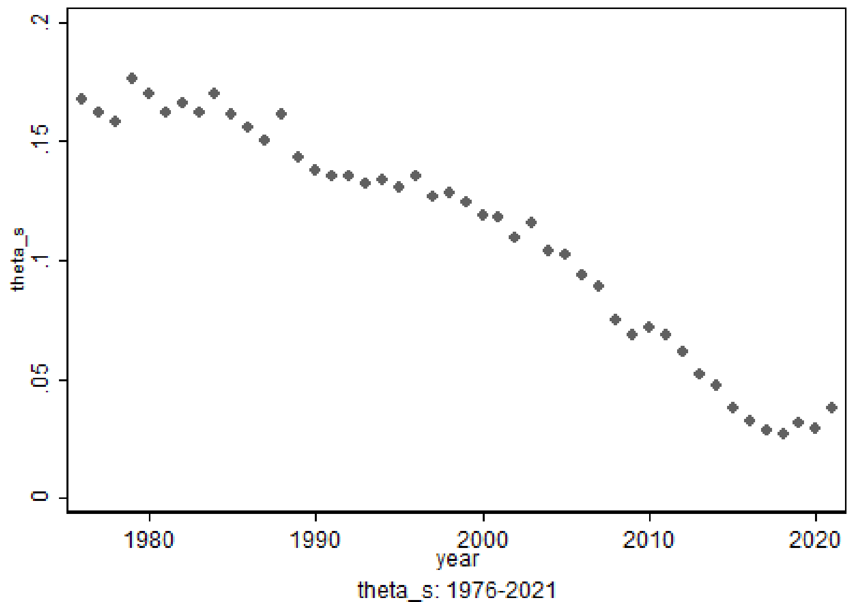

Figure 3: Inter-state Citation Intensity θ_{it}

4.5 Calibration

We estimate the key parameters in the model using US Patent dataset. We set one US state as one country and the sum of other states as its foreign country. In the model, θ is the share of foreign knowledge diffused to home country. We measure θ_{st} for state s year t using the number of citations to foreign patents $citation^*_{st}$ per foreign patent stock N^*_{st} divided by the number of citations to home patents $citation^h_{st}$ per home patent stock N_{st}, to adjust for the changing ratio between home and foreign patent stocks.

$$\hat{\theta}_{st} = \frac{\frac{\sum_f citation^f_{st}}{\sum_f N_{ft}}}{\frac{citation^h_{st}}{N_{st}}} \tag{57}$$

Figure 3 shows that $\theta_s t$ decreases overtime for most states. That means states rely more and more on domestic knowledge in innovation, and the 2nd and 3rd factors in (56) are diminishing overtime.

In a similar way, we calibrate α_{it} as the ratio of self-sector citations to other-sector citations relative the ratio of self-sector to other sector's patent stocks in sector i year t. On top of that, we construct the state level measure of self-sector citation intensity by a patent stock share weighted average of sectoral level α_{it} for every state.

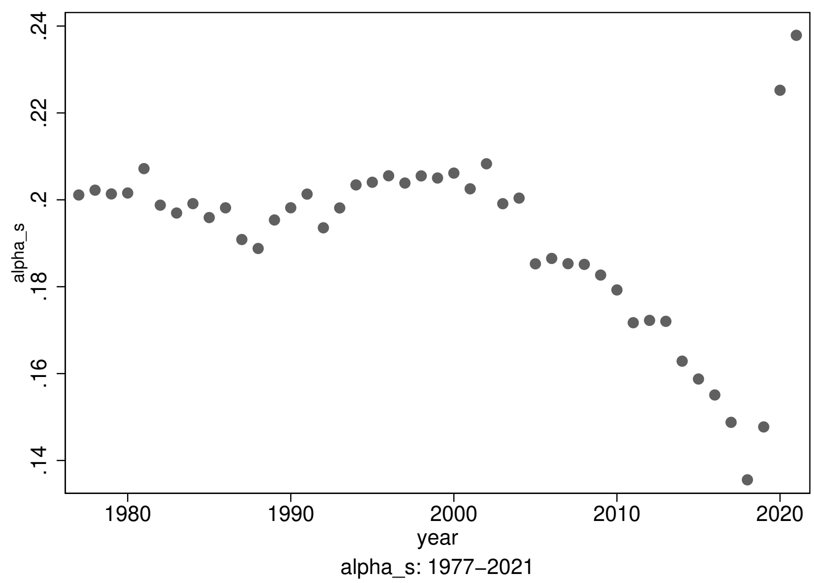

Figure 4: Self-sector Citation Intensity

$$\hat{\alpha}_{it} = \frac{\frac{citation_{iit}}{N_{it}}}{\frac{\sum_j citation_{ijt}}{\sum_j N_{jt}}} \tag{58}$$

$$\hat{\alpha}_{st} = \sum_i \frac{N_{sit}}{N_{it}} \hat{\alpha}_{it} \tag{59}$$

Figure 4 demonstrates that most sectors adopt less and less self-sector knowledge in their innovation over time, except for the last two years during the COVID pandemics. When the inter-sector knowledge diffusion is more important to innovation, the regional specialization in production and innovation strengthens the last negative factor in (56) and hence diminishes the dynamic gain from trade.

From the aforementioned two figures, we can summarize that the US inventors rely more and more on self-state and inter-sector knowledge diffusion in their innovation, which means that the US economy is approaching the world represented by the purple star lines in Figures 1 and 2, where aggregate growth rate is lower than before and the gain from trade is dwindling or even reversed.

5 Social Planner's Problem

To be finished.

6 Conclusion

Empirical evidences in US patent data and state-sector level production and trade data find that US states grow faster in GDP and innovation if their patent portfolios are more diversified, or if they are able to cite patents in other states whose set of knowledge is more complementary to self-state's knowledge portfolio.

We build a two-sector open economy growth model where the innovation in one sector utilizes knowledge from both sectors to show why . How much countries benefit from trade depends on the degree of inter-sectoral knowledge dependence in innovation and the strength of inter-national knowledge diffusion. Free trade brings static gain by the specialization of production and research in regional sectors with relative comparative advantage (RCA). However, such regional allocation of knowledge portfolio may harm the creation of new knowledge, when the innovation in one sector depends heavily on the knowledge from other sectors which are pushed more far away from domestic region by free trade. Without strong inter-regional knowledge diffusion, the insufficient knowledge in regional non-RCA sectors dampens aggregate growth rate and the gain from trade. Our calibrated model parameters from US patent data do find that inventors rely more and more intensively on knowledge from other sectors in innovation, and cite less and less intensively to patents in other states, which implies that we are approaching a situation where growth rate slows down and the gain from trade is smaller or even reversed.

References

Buera, Francisco J and Ezra Oberfield. 2020. "The global diffusion of ideas." *Econometrica* 88 (1):83–114.

Cai, Jie, Nan Li, and Ana Maria Santacreu. 2022. "Knowledge Diffusion, Trade, and Innovation across Countries and Sectors." *American Economic Journal: Macroeconomics* 14 (1):104–45.

Comin, Diego and Martí Mestieri. 2018. "If technology has arrived everywhere, why has income diverged?" *American Economic Journal: Macroeconomics* 10 (3):137–78.

Comin, Diego A, Mikhail Dmitriev, and Esteban Rossi-Hansberg. 2012. "The spatial diffusion of technology." Tech. rep., National Bureau of Economic Research.

Eaton, Jonathan and Samuel Kortum. 1996. "Trade in Ideas: Productivity and Patenting in the OECD." *Journal of International Economics* 40 (3-4):251–278.

———. 1999. "International Technology Diffusion: Theory and Measurement." *International Economic Review* 40 (3):537–570.

Keller, Wolfgang. 2002. "Geographic localization of international technology diffusion." *American Economic Review* 92 (1):120–142.

Liu, Ernest and Song Ma. 2021. "Innovation networks and r&d allocation." Tech. rep., National Bureau of Economic Research.

Lybbert, Travis J and Nikolas J Zolas. 2014. "Getting patents and economic data to speak to each other: An 'algorithmic links with probabilities' approach for joint analyses of patenting and economic activity." *Research Policy* 43 (3):530–542.

MacGarvie, Megan. 2005. "The determinants of international knowledge diffusion as measured by patent citations." *Economics Letters* 87 (1):121–126.

Sampson, Thomas. 2023. "Technology Gaps, Trade, and Income." *American Economic Review* 113 (2):472–513.

Somale, Mariano. 2021. "Comparative advantage in innovation and production." *American Economic Journal: Macroeconomics* 13 (3):357–396.

APPENDIX

A Data Description and Calculation

A.1 Industry classification

new code	description	Trade	GDP	Employment	Patent
111	Agricultural Products	111, 112	111-112	111-112	111, 112
113	Forestry and Marine Product; others	113, 114, 115	113-115	113-115	113, 114, 115
211	Oil & Gas	211	211	211	211
212	Minerals & Ores	212	212, 213	212, 213	212, 213
221	Utilities	NA	22	22	221
231	Construction	NA	23	236,237,238	233,234,235
311	Food & Kindred; Beverages & Tobacco	311,312	311-312	311, 312	311,312
313	Textiles & Fabrics	313, 314	313-314	313, 314	313, 314
315	Apparel & Accessories, Leather & Allied	315, 316	315-316	315, 316	315, 316
321	Wood Products	321	321	321	321
322	Paper	322	322	322	322
323	Printed Matter and related product	323	323	323	323
324	Petroleum & Coal Products	324	324	324	324
325	Chemicals	325	325	325	325
326	Plastics & Rubber Products	326	326	326	326
327	Nonmetallic Mineral Products	327	327	327	327
331	Primary Metal Mfg	331	331	331	331
332	Fabricated Metal Products	332	332	332	332
333	Machinery, except Electrical	333	333	333	333
334	Computer & Electronic Products	334	334	334	334
335	Electrical Equipment, Appliances & Components	335	335	335	335
336	Transportation Equipment	336	3361-3363, 3364-3369	3361-3363, 3364-3369	336
337	Furniture & Fixtures	337	337	337	337
339	Miscellaneous Manufactured Commodities	339	339	339	339
511	Newspapers, Book & other published matter	511	511	511	511

Notes: This table presents the crosswalk of industry code between CENSUS(trade), BEA(GDP, employment) and Patent Office(patent).