Striking While the Iron Is Cold: Fragility after a Surge of Lumpy Investments[†]

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Abstract

I theoretically and quantitatively show that the cross-sectional ranking of the interest elasticities of investment between large and small firms is counterfactually flipped in the models with fixed and convex adjustment costs. Then, I develop a heterogeneous-firm real business cycle model where the semi-elasticities of large and small firms' investments are matched with the empirical estimates. In the model, following a negative TFP shock, the timings of large firms' lumpy investments are significantly synchronized due to the low elasticity to the general equilibrium effect. After a surge of large firms' lumpy investments, TFP-induced recessions are especially severe, and the semi-elasticity of the aggregate investment drops significantly.

Keywords: Business cycle, lumpy investment, interest elasticity, nonlinear dynamics.

JEL codes: E32, E22, D25.

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1 Introduction

Large surges in the fraction of large firms making large-scale investments occurred during 1980, 1998, and 2007.[1] These three years were followed by recessions within two years. Is it merely a coincidence that investment surges of large firms precede recessions?

This paper studies a mechanism that makes an economy more fragile to a negative TFP shock after a surge in lumpy investments of large firms. I develop and analyze a business cycle model with heterogeneous firms in which the semi-elasticities of large and small firms' investments are matched with the empirical estimates. Then using the model, I qualitatively and quantitatively analyze the amplification of productivity-driven aggregate fluctuations. Due to the low interest elasticity, the nonlinearity in the large firms' investments is not washed out by the general equilibrium effect, leaving large firms' large-scale investment timings synchronized after a negative aggregate TFP shock. These synchronized investments of large firms generate macro-level nonlinearity, and the response of aggregate investment to an aggregate TFP shock depends on the large firms' past investment history.

Large firms are a particular focus of this paper for three reasons. First, large firms are insensitive to fluctuations in macroeconomic conditions, including the interest rate. Therefore, they generate a significant nonlinearity in the aggregate investment dynamics. Second, large firms are the most observable group of firms, as most of them are listed and subject to financial disclosure regulations mandated by the U.S. Securities and Exchange Commission (SEC). Therefore, any forward-looking information contained in the large firms' investment dynamics can be traced in a timely manner and be conducive to designing contemporaneous policies. Last, large firms account for a substantial portion of the aggregate investment. Therefore, the large firms' investment fluctuations significantly impact the aggregate investment dynamics.

The contribution of this paper can be broadly summarized into three dimensions: 1) model, 2) measurement, and 3) policy implications. First, on the model side, I theoretically and quantitatively show that the cross-sectional ranking of the interest elasticities between large and small firms' investments are counterfactually flipped in the existing models. Then, I develop and analyze a heterogeneous-firm real business cycle model that can correctly capture the cross-sectional ranking of the interest elasticities. Using the model, I show that aggregate investment displays substantial state dependence due to large firms' interest-inelastic capital adjustment patterns. Second, this paper develops a fragility index based on the large firms' recent capital adjustment history. The fragility index has predictive power on the one-period-ahead investment growth and serves as a sufficient statistic on the responsiveness of the aggregate investment to a TFP shock. In practice, this index is easy to trace contemporaneously compared to other indices in the literature, as the index is based on large firms' observable variables. Third, I show that aggregate investment's interest elasticity depends on the level of the fragility index over the business cycle, which implies that the effectiveness

[1] Following Cooper and Haltiwanger (2006), I define an investment beyond 20% of existing capital stock as a large-scale capital adjustment. Firms that hold capital stocks greater than the 90th percentile of the capital distribution in each industry based on the two-digit NAICS code are defined as large firms.

of the monetary policy can be low after a surge of large firms' lumpy investments.[2]

Using the U.S. Compustat data and monetary policy shocks, I document that the interest elasticities of large firms are substantially lower than those of small firms. Also, the extensive margin variation in the fraction of making lumpy investments is significantly less sensitive to an interest rate change in large firms than in small firms. Then, using a two-period firm-level investment model, I theoretically show that the cross-sectional ranking of the elasticities in the models with standard convex adjustment cost is counterfactually flipped. Also, I compare the empirical patterns with the interest elasticities computed from the full dynamic models incorporating capital adjustment costs. As predicted by the theoretical results, the cross-sectional ranking of the elasticities between large and small firms is inconsistent with the empirical result. Therefore, a new model framework is necessary to study how the surges in the large firms' lumpy investments affect the business cycle.

I develop and analyze a heterogeneous-firm real business cycle model in which a firm-level investment is subject to a size-dependent fixed cost and convex adjustment cost. A firm-level size-dependent fixed cost is grounded on the production line (establishment) level fixed costs that increase exponentially in firm size. The speed of the increase in the cost depends on the interdependence across the production lines within the firm. The size-dependent fixed cost is parametrized to capture this interdependence across the production lines. This parameter controls the large and small firms' interest elasticities in the extensive margin. I calibrate this parameter to match the ratio of interest elasticities between large and small firms.

The calibrated baseline model with size-dependent fixed cost can correctly capture the average interest elasticity for all firms and the cross-sectional ratio of the elasticities between large and small firms. Also, the elasticities of the fraction of firms making large-scale investments are consistent with the empirical estimates.

Using this model, I study how the large firms' synchronized large-scale investments affect the business cycle. When an aggregate TFP shock hits the economy in the model, the timing of lumpy investments is synchronized across all firms. This is because firms tend to hold their investment project until the economy recovers sufficiently close to a normal level. After this initial synchronization in the investment timings, firm-level lumpy investments display substantially different recovery patterns over the post-shock periods depending on the firm size. The synchronized large-scale investment leads to a surge of large firms' lumpy investments, while small and medium firms do not display such patterns: the investment timings of small and medium firms are smoothed out over the post-shock periods. This is because the large firms' investments in the extensive margin are insensitive to the general equilibrium effect: a rising interest rate due to the increasing investments does not incentivize large firms to spread out their investment timings. However, the surge in the interest rate makes small and medium firms flatten investment timings over the post-shock periods as they are relatively nimble in terms of investment timing adjustment.

Next, I construct a fragility index that measures the fraction of large firms that have

[2]The policy implication is limited to a positive implication, as the model does not include a monetary policy block.

recently completed a large-scale capital adjustment. This index is distinguished from other indices in the literature as it is constructed from readily observable allocations instead of firm-level productivities (Bachmann et al., 2013; Caballero and Engel, 1993). The fragility index gives information on how large a fraction of large firms is ready to make a large-scale investment. After a surge of large firms' lumpy investments, the fragility index increases, as many firms have recently finished large-scale investments after the surge. Then, in the following period, due to a lack of large firms willing to make a large-scale investment, the response of aggregate investment to a TFP shock deviates from the one in the steady-state, resulting in a state-dependence in the investment dynamics. When the fragility index increases by one standard deviation, the aggregate investment responds 0.56 percentage points stronger to the same negative one-standard-deviation TFP shock. This shows that the fragility dynamics amplify the productivity-driven aggregate fluctuations due to the history-dependence of aggregate investment.

The fragility index has predictive power on the aggregate investment dynamics, as it is significantly correlated with the one-period-ahead fraction of firms making large-scale investments. From the regression of the aggregate investment on the exogenous output shocks and the fragility index, I document that the fragility index is significantly negatively correlated with the future investment growth both in the model and the data at a similar magnitude. Especially from the predicted investment growth solely using the fragility dynamics, I report that around 43 per cent of the drop (out of the entire seven percentage point drop) in the investment growth during the dot-com bubble crash is accounted for by the fragility index dynamics. On the other hand, the investment plunge during the Great Recession is not explained well by the fragility fluctuations. Despite the preceding surge of lumpy investments of large firms, it turns out that the surge happened during a particularly short period, resulting in a fragility index lower than those of other recessions. Also, because the Great Recession has been asymmetrically driven by the crash of small firms, the large firms' investment dynamics is relatively less capable of explaining the Great Recession (Fort et al., 2013).

The fragility dynamics also significantly affect the interest elasticity of the aggregate investment. For this analysis, I measure the semi-elasticity of aggregate investment at each time on the business cycle. As the fragility index increases by one standard deviation, the semi-elasticity of the aggregate investment drops by 0.27 percentage points. This is because there are not many large firms that can flexibly participate in and out of the large-scale investment plan after a surge of large-scale investments (high fragility period). The main caveat of this result is that the effectiveness of the conventional monetary policy is state-dependent due to the elasticity fluctuations. Especially after a surge of large firms' lumpy investments, the effectiveness of the monetary policy decreases because the corporate investment channel is less responsive than during the normal period.

Related literatures This paper contributes to the literature that studies how firm-level lumpy investments affect the business cycle. Abel and Eberly (2002) empirically showed that there are statistically and economically significant nonlinearities in firm-level investments.

They point out that it is necessary to track the cross-sectional distribution of firm-level investments to account for aggregate investment. Cooper et al. (1999) and Gourio and Kashyap (2007) found aggregate investment is largely driven by establishment-level capital adjustment in the extensive margin. Especially, Cooper et al. (1999) found synchronized lumpy investments can generate an echo effect of aggregate shocks in partial equilibrium. Gourio and Kashyap (2007) pointed out that if a fixed cost is drawn from a highly concentrated non-uniform distribution, aggregated lumpy investments show different impulse responses than frictionless models in partial equilibrium. In contrast, Khan and Thomas (2008) found that lumpiness in investment at the establishment level is washed out after aggregation due to a strong general equilibrium effect.

I contribute to the literature by theoretically and quantitatively showing that the cross-sectional ranking of the interest elasticities in the existing model frameworks is counterfactual. This fact is particularly important as the growing body of empirical research in the literature points out the importance of large firms' investment in the business cycle and their inelasticities to the macroeconomic environment (Crouzet and Mehrotra, 2020). Therefore, to structurally analyze the role of large firms in the business cycle, a new model framework is necessary. In my model, the interest elasticities of large firms and small firms become consistent with the empirical estimates. Using the model, I conclude that large firms' lumpy investments generate substantial nonlinearity in the aggregate investment fluctuations. This is a consistent result with Koby and Wolf (2020), which shows the observed dampening effect of factor price is not as strong as the implied level in models with a fixed cost, using the semi-elasticity estimates to the bonus depreciation from Zwick and Mahon (2017).

In this paper, the aggregate investment endogenously fluctuates due to large firms' synchronized patterns from the past aggregate TFP shocks. Therefore, the focus of this paper is shared with Carvalho and Grassi (2019), which highlights the endogenous source of aggregate fluctuations from the large firms' dynamics. While one of the most important ingredients for the aggregate fluctuations in their paper is the granularity of large firms, which breaks the law of large numbers, my paper focuses on the low interest elasticity of large firms to the general equilibrium effect. Therefore, once jointly accounted, these two empirically supported but not tightly related characteristics of large firms can potentially account for the greater importance of large firms on the business cycle.

There are several papers that are closely related to the modeling contribution of this paper. House (2014) points out that a conventional model with fixed cost cannot capture inelastic lumpy investments due to a strong general equilibrium effect; model-implied lumpy investments are highly price-elastic. To overcome this limitation in the fixed cost model, Bachmann et al. (2013) introduce maintenance and replacement investments under the high fixed cost parameter. In their model, micro-level lumpiness does not wash away after aggregation, leading to state-dependent sensitivity of aggregate investment in general equilibrium. Winberry (2021) includes habit formation in the household's utility function so that aggregate TFP sensitivity of real interest rate becomes counter-cyclical. Combined with convex adjustment cost, the counter-cyclically responsive real-interest rate does not strongly dampen aggregated lumpy investments over the business cycle.

In contrast to these approaches, I introduce a size-dependent fixed adjustment cost in the model on top of the convex adjustment cost. The size-dependent fixed adjustment cost is micro-founded in the production-line (establishment) level fixed cost and the interdependence across the lines. This feature helps the model correctly capture inelastic large firms and elastic small firms.

The fragility index of this paper is closely related to several papers measuring the responsiveness of an economy to exogenous aggregate shocks. Caballero et al. (1995) develops a micro-level adjustment hazard function that captures heterogeneous price adjustment probability. Dynamics in the cross-section of the hazard rates generates substantial nonlinearity in the economy's aggregate dynamics. Bachmann et al. (2013) defines a responsiveness index as a function of the aggregate productivity and sufficient statistics of the joint distribution of the capital stocks and the idiosyncratic productivities. They show that the responsiveness index is significantly driven by the fraction of capital-adjusting firms. Relatedly, Baley and Blanco (2021) shows that two sufficient statistics can characterize aggregate investment dynamics: 1) the capital to productivity ratio's dispersion and 2) its covariance with the duration of inaction. Compared to these papers, my paper highlights the role of the marginal distribution of large firms' inaction duration over the business cycle, which is readily observable in the data in a timely manner due to their mandated financial disclosure.

Also, this paper is related to the literature studying the state-dependent effectiveness of monetary policy. The most closely related paper is Tenreyro and Thwaites (2016), which shows that business investment and durables expenditure are less responsive to monetary policies during recessions. I document that the investment drop during the recession of the dot-com bubble crash is substantially accounted for by the rising fragility index. At the same time, I show that the interest elasticity of aggregate investment significantly decreases in the fragility index. According to this result, monetary policy could not have functioned effectively during the dot-com bubble crash. Likewise, my paper gives a micro-founded explanation of why monetary policy is not effective during a recession. Going one step further, it gives a testable implication: monetary policy in a recession not preceded by a surge of large firms' lumpy investments might be as effective as in normal years.

Last, this paper contributes to nonlinear business cycle literature. A large body of research has focused on the nonlinearity in aggregate fluctuations that arise when heterogeneous agents are subject to micro frictions. Berger and Vavra (2015) concludes that lumpiness in households' durable adjustment results in pro-cyclical responsiveness of aggregate durable expenditures to an aggregate shock. Fernandez-Villaverde et al. (2020) found that financial frictions can generate endogenous aggregate risk under the heterogeneous household model. In this setup, the aggregate allocations display state-dependent responsiveness to an aggregate TFP shock. Volatility shocks in real interest rate studied in Fernandez-Villaverde et al. (2011) and uncertainty shocks in Bloom et al. (2018) are also highlighted as an important source of the nonlinearity in the business cycle. To this literature, this paper contributes by analyzing interest-inelastic large firms' lumpy investments as a significant source of nonlinearity in the aggregate investment dynamics.

Roadmap Section 2 shows motivating facts about surges of large firms' lumpy investments before and after the recessions. Section 3 analyzes the firm-level interest elasticities in the existing models and compares them with the data. Section 4 develops a heterogeneous-firm business cycle model in which the cross-section of the interest elasticities is matched with the empirical estimates. Section 5 analyzes the macroeconomic implications of the calibrated model. Section 6 concludes. Proofs and other detailed figures and tables are included in appendices.

2 Motivating fact

In this section, I empirically analyze the cyclical behavior of the lumpy investments of large firms.

I use U.S. Compustat data for the firm-level empirical analysis. While Compustat data covers only public firms, its coverage is relatively less an issue in this analysis because the focus is on firms with large capital stocks. Throughout the empirical analysis, large firms are defined as firms that hold capital stocks greater than the 40th percentile of the capital distribution in each industry of the two-digit NAICS code. The choice of the 40th percentile is to define large firms in Compustat space consistent with large firms in Zwick and Mahon (2017), which estimated the interest elasticities of firm-level investments.[3] The sample period covers 1980 to 2016. Firms with negative assets and zero employment are excluded from the sample. All the firm-level variables except capital stock and investment are deflated by the GDP deflator. Investment is deflated by the nonresidential fixed investment deflator available from National Income and Product Accounts data (NIPA Table 1.1.9, line 9). The firm-level real capital stock is obtained by applying the perpetual inventory method to net real investment. The industry is categorized by the first two-digit NAICS code.[4]

Table 1 reports summary statistics for large and small firms during the sample periods. Under the given definition of large firms, greater than 90% of aggregate sales and employments belong to large firms. On average, large firms are around 20 times greater than small firms in sales and employment. Large firms are old firms on average, listed around five years longer than small firms. Large firms' ratio of total liability out of the total asset is around 53.1% and is smaller than the small firms' fraction 121%.

[3] In Zwick and Mahon (2017), large and small firms are defined as the top 30% and bottom 30% of sales distribution. From the size cutoffs (15.4M, 48.8M) in terms of sales in years 1998 through 2000 and 2005 through 2007 (Table B.1, panel (d)), I compute the corresponding capital size cutoffs in Compustat.

[4] If only the SIC code is available for a firm, I imputed the NAICS code following online appendix D.2 of Autor et al. (2020). If both NAICS and SIC are missing, I filled in the next available industry code for the firm.

Table 1: Summary statistics of large and the other firms

	Large	Non-large
Total		
Aggregate Sales ($1 bil.)	11,317.1	629.6
Aggregate Employment (1 mil.)	39.2	3
Firm-level		
Avg. Sales ($1 mil.)	1,420.1	67.8
Avg. Employment (1K)	5.1	0.3
Avg. Age after IPO (yrs.)	11.1	5.7
# of Firms.	7,984	9,294
Avg. Liability / Total Asset (%)	53.1	121

Notes: Large firms are defined as top 60% firms in terms of the size of capital stock, and the other bottom 40% firms are defined as non-large firms. Aggregate level statistics are obtained from the time-series average of the cross-sectional sum of firm-level variables separately for large and small firms. Firm-level statistics are obtained from the time-series average of the cross-sectional mean of firm-level variables separately for large and non-large firms. All the firm-level variables are from U.S. Compustat data except for age after IPO. Age after IPO is obtained from a firm's current financial year minus the financial year of the first observation in Compustat data.

2.1 Surges of large firms' lumpy investments and recessions

In the following analysis, I empirically analyze the relationship between large firms' lumpy investments and the timing of recessions. I define an investment spike as a firm-specific event where a firm makes a large-scale investment greater than 20% of the firm's existing capital stock.[5] I refer to this investment spike as a lumpy investment or capital adjustment in the extensive margin interchangeably. Then, I define spike ratio as follows:

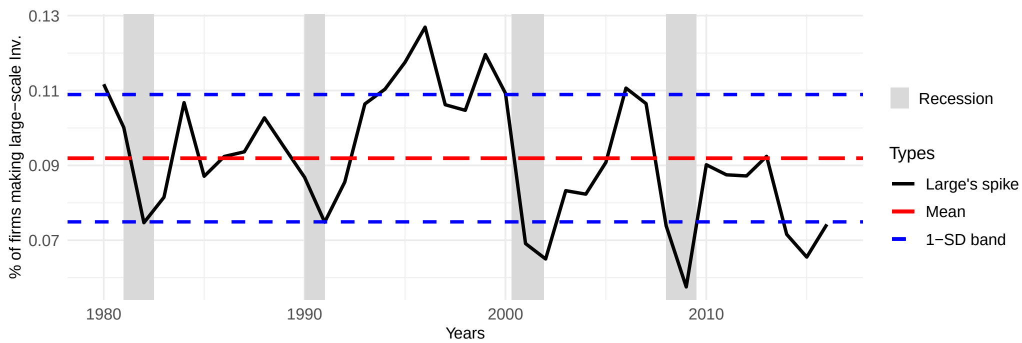
$$\text{Spike ratio}_{j,t} := \frac{\sum_{i \in j} \mathbb{I}\{i_{it}/k_{it} > 0.2\}}{\#\text{ of } j\text{-type firms at } t}, \quad j \in \{small, large\}$$

The numerator counts all the incidences of investment spikes from firm type $j \in \{small, large\}$ at time t, and it is normalized by the total number of j-type firms. Figure 1 plots the time series of spike ratio of large firms. On average, 9.2% of large firms adjust their existing capital stocks in the extensive margin in a year. As can be seen from Figure 1, since 1980, there have been only four periods (1980, 1996, 1998, and 2007) during which the fraction of large firms making spiky investments surged beyond one-standard deviation. Three out of the four events were followed by recessions within two years.

Conversely, there were four recessions in the U.S. over the same period, and three out of the

[5]The 20% cutoff is from the non-convex adjustment cost literature (Cooper and Haltiwanger, 2006; Gourio and Kashyap, 2007; Khan and Thomas, 2008). If a firm's acquired capital stock is greater than 5% of existing capital stock in a certain year, I rule out the observation from the sample due to noisy accounting during the acquisition year.

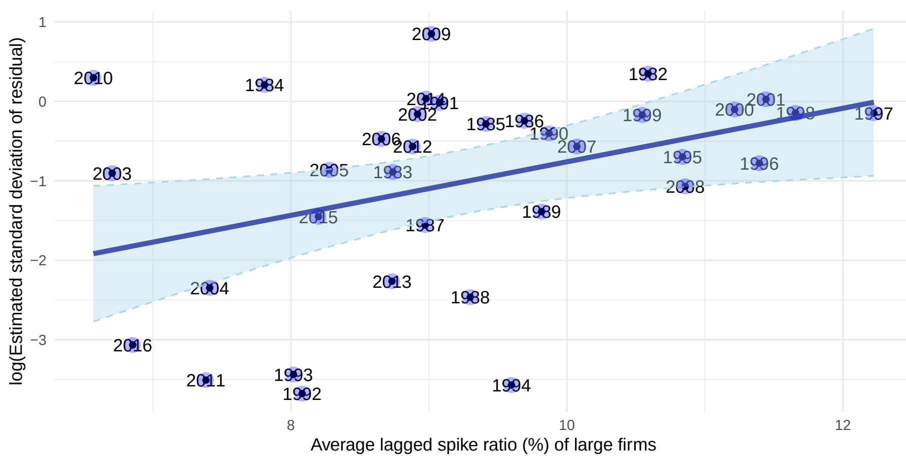
Figure 1: Three surges of large firms' lumpy investments before recessions



Notes: The firm-level large-scale investment is defined as an investment greater than 20% of the existing capital stock. The solid line plots the time series of the fraction of large firms making large-scale investments. The grey areas highlight the NBER recession periods.

four recessions were preceded by the surge of large firms' lumpy investments. The exception was the recession in 1990, and it was the mildest recession among the four recessions.

Figure 2: Conditional heteroskedasticity of aggregate investment



Notes: The estimated standard deviation of the residual (y-axis) is obtained from fitting the aggregate investment-to-capital ratio (%) into an autoregressive process with four lags. The average lagged spike ratio of large firms (%) is obtained from averaging the most recent past two spike ratios for each observation of residualized investments. The years overlaid on the dots are the observation year of the residualized investment-to-capital ratios.

Relatedly, in the following analysis, I show that the aggregate investment rate is conditionally heteroskedastic on the average lagged spike ratio of large firms. That is, the residualized volatility of the aggregate investment rate is high if a great fraction of large firms have made lumpy investments in recent years.

For this analysis, I use aggregate data on non-residential investment (NIPA Table 1.1.5, line 9) and aggregate capital (Fixed Asset Accounts Table 1.1, line 4) from BEA. The thick line in Figure 2 plots logged estimates of the standard deviation of residuals from autoregression of aggregate investment rates as a function of the recent average of large firms' spike ratio.[6] The recent average is based on the average spike ratio of the past three years. As can be seen from this figure, aggregate investment rates are heteroskedastic conditional on the lagged average spike ratio. Table A.1 reports the regression coefficients for the fitted line. According to the regression result, a one-standard-deviation increase (1.47%) in the large firms' past spike ratio is associated with a one-standard-deviation increase (0.50%) in the aggregate investment's residualized volatility. Consistent with the patterns in Figure 1, the three recession years of interest are located at the top-right corner in Figure 2.

Motivated by these facts, I analyze the role of large firms' lumpy investments on the aggregate investment fluctuations. Before I move on to the analysis, I discuss several more reasons for the importance of studying the large firms on the business cycle.

2.2 Why large firms?

This paper focuses on the role of large firms' lumpy investments in the business cycle. Such focus derives naturally from the motivating fact in the previous section: surges of large firms' lumpy investments have been followed by recessions. On top of this, there are three important reasons for studying large firms.

First, large firms are insensitive to fluctuations in macroeconomic conditions, including the interest rate. According to Crouzet and Mehrotra (2020), large firms are cyclically less sensitive than small firms. Relatedly, Zwick and Mahon (2017) points out that large firms are twice less sensitive to a tax change than small firms. In the next section, I also show that the extensive margin elasticities of large firms' large-scale investments are significantly lower than those of small firms. Then, the large firms' synchronized timings of large-scale investments are not easily mitigated by macroeconomic counterforces such as the general equilibrium effect. Therefore, the interest-inelastic investments of large firms add a substantial nonlinear component to the aggregate investment dynamics.

Second, large firms are readily traceable in a timely manner. Most large firms are listed in the U.S., subject to financial disclosure regulations mandated by the U.S. SEC. Therefore, any forward-looking information contained in the large firms' investment dynamics can be almost contemporaneously observed and be conducive to designing contemporaneous policies.

Finally, large firms account for a substantial portion of the aggregate investment. According to Zwick and Mahon (2017), the investments of the top 5% of firms in their sample cover more than 60% of entire investments. Therefore, the large firms' investment fluctuations strongly affect the aggregate investment cycle.

[6] This empirical analysis is motivated from the conditional heteroskedasticity analysis in Figure 1 of Bachmann et al. (2013).

3 Firm-level interest elasticity in the existing model and the data

In this section, I theoretically and quantitatively investigate how the existing models predict the interest elasticities of large and small firms. Especially, I study the role of convex and fixed adjustment costs on the cross-section of the interest elasticities.

3.1 A two-period model with a convex adjustment cost

Consider a firm that is given capital stock k and productivity z. For simplicity, I assume a firm lives only for two periods. A firm's investment is subject to a standard convex adjustment cost. A firm produces business output using a concave production function, $f(z,k) = zk^{\alpha}$.[7] The idiosyncratic productivity follows a Markov chain, $z'|z \sim \mathit{\Gamma}$. Then, the problem of firm-level investment can be summarized as follows:

$$\max_{I} \quad -I - \frac{\mu}{2}\left(\frac{I}{k}\right)^{2} k + q\mathbb{E}_{z}z'((1-\delta)k + I)^{\alpha}$$

where I is the investment; μ is the convex adjustment cost parameter; z' is the future productivity; $\alpha \in (0,1)$ is the span of control parameter; q is the discount factor. A variation in q is equivalent to the change in the interest rate. The first-order condition with respect to investment I leads to the following inter-temporal optimality condition:

$$1 + \mu\left(\frac{I^{*}}{k}\right) = q\mathbb{E}z'\alpha((1-\delta)k + I^{*})^{\alpha-1} \tag{1}$$

Taking a log on both sides and using an approximation of $log(1+x) \approxeq x$ for small x, Equation (1) can be reduced into the following form:[8]

$$\mu\left(\frac{I^{*}}{k}\right) \approxeq log(q) + log(\mathbb{E}z'\alpha) + (\alpha-1)log(k) + (\alpha-1)\left(\frac{I^{*}}{k} - \delta\right)$$

Then, I re-arrange the terms to obtain the following equation:

$$\frac{I^{*}}{k} \approxeq A(\mu)log(q) + B(\mu,k) \tag{2}$$

where $A(\mu) = \frac{1}{\mu+(1-\alpha)}$ and $B(\mu,k) = A(\mu)(log(\mathbb{E}z'\alpha) + (\alpha-1)log(k) - (\alpha-1)\delta)$. It is worth noting that the second term on the right-hand side, $B(\mu,k)$ does not play any role in the response of investment to the change in q. Equation (2) provides rich implications about the response of investment to the change in q.

[7] For simplicity, I assume the optimal labor demand is implicitly considered in the production function.

[8] I use the following substep: $log(k(1-\delta) + I^{*}) = log\left(k\left((1-\delta) + \frac{I^{*}}{k}\right)\right) = log(k) + log\left((1-\delta) + \frac{I^{*}}{k}\right) \approxeq log(k) - \delta + \frac{I^{*}}{k}$

First, Equation (2) implies that the investment-to-capital ratio positively (negatively) responds to an increase in q (decrease in r). This is because an increase in q makes future production more profitable to a firm, leaving greater investment motivation for the firm. This is formally proven without an approximation in Lemma 3 in Appendix B.1.

Second, Equation (2) implies that the firm-level interest elasticity increases in size. To clearly present the implication, I multiply k on both sides of Equation (2), and I take partial derivatives with respect to k and q on both sides to get

$$\frac{\partial^2}{\partial q \partial k} I^* \approxeq A(\mu) \frac{\partial^2}{\partial q \partial k} klog(q) = \underbrace{A(\mu)}_{>0} \underbrace{\frac{1}{q}}_{>0} > 0.$$

The inequality above holds for any $\mu > 0$. In a model without the convex adjustment cost, the same equation could be derived with $\mu = 0$. In the following statement, I formally show that the interest sensitivity of investment, $\frac{\partial logI^*}{\partial q}$, increases in size k.

Proposition 1 (Size-monotonicity in the interest elasticity).
Given $\mu > 0$, the following inequalities holds:

$$\begin{aligned}
(i) \quad & \frac{\partial}{\partial k}\left(\frac{\partial k^*}{\partial q}\right) > 0 \text{ for } \forall k > 0 \\
(ii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial logk^*}{\partial q}\right) > 0 \text{ for } \forall k > 0 \\
(iii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right) > 0 \text{ for } \forall k > 0 \\
(iv) \quad & \frac{\partial}{\partial k}\left(\frac{\partial logI^*}{\partial q}\right) > 0 \text{ if } I^* > 0.
\end{aligned}$$

Proof. See Appendix B.1 ■

The result of Proposition 1 is contradictory to the empirical findings in Zwick and Mahon (2017). According to the paper, the large firms' interest elasticities are significantly smaller than those of the small firms.[9] From the fact that $A(\mu)$ decreases in μ, a large μ can mitigate the counterfactually diverged elasticity ranking, but it cannot flip the order. Therefore, a model with convex adjustment cost only cannot be a proper model to study the role of large firms' investments on the business cycle. This theoretical prediction will be quantitatively verified in the following section's comparison of the elasticities in the infinite period problem.

Third, Equation (2) implies that the elasticity of firm-level investment decreases in μ, as $A(\mu)$ decreases in μ. This prediction is consistent with the computational outcomes in Winberry (2021) and Koby and Wolf (2020), which argue that the convex adjustment cost helps

[9] Zwick and Mahon (2017) defines large firms as the top 30% firms in the sales distribution and the small firms as the bottom 30% in the sales distribution. Under this definition, the elasticity ratio between small and large firms is around 2.

the elasticity of the average investment to be lowered to the empirical estimate. Intuitively, the higher the convex adjustment cost parameter, the higher the marginal cost of adjustment, leaving the marginal response to a change in q costlier. This prediction is formally proved in the following proposition without the approximation:

Proposition 2 (Elasticity dampening effect)**.**
Given $\mu > 0$, if $I^* > 0$, the following statements hold:

$$
\begin{aligned}
(i) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial k^*}{\partial q}\right) < 0 \\
(ii) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial log k^*}{\partial q}\right) < 0 \\
(iii) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial I^*}{\partial q}\right) < 0 \\
(iv) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial log I^*}{\partial q}\right) \begin{cases} \leq 0 & if\ \frac{1}{1-\delta} \geq \mu \\ > 0 & if\ \frac{1}{1-\delta} < \mu \end{cases}.
\end{aligned}
$$

Proof. See Appendix B.1 ■

In the last statement of Proposition 2, the response of investment to q in per cent can increase in μ if μ is sufficiently large. This is due to the convex adjustment parameter's dominant shrinking force on the level of the denominator in $\frac{1}{I^*}\frac{\partial I^*}{\partial q} = \frac{\partial log I^*}{\partial q}$.

To sum up, the convex adjustment cost is helpful for controlling the average elasticities of firm-level investment. However, it does not help flip the counterfactual ranking of elasticities between large and small firms.

3.2 A two-period model with a fixed adjustment cost

Now I consider a two-period model in which a firm needs to pay a fixed adjustment cost $\xi \sim Unif[0, \overline{\xi}]$ to invest.[10] If a firm does not pay the fixed cost, the firm's capital stock simply depreciates at the rate of δ. Except that the convex adjustment cost is replaced by the fixed cost, the model is the same as the one in the previous section.

I define $\xi^*(k, q)$ as the threshold of adjustment with respect to the shock realization, ξ as follows:

$$
\xi^*(k, q) := \underbrace{-I^* + q\mathbb{E}_z z'((1-\delta)k + I^*)^\alpha}_{\text{Net benefit of capital adjustment}} - \underbrace{q\mathbb{E}_z z'((1-\delta)k)^\alpha}_{\text{Net benefit of inaction}}.
$$

Thus, a firm invests if $\xi^*(k, q) > \xi$. Then, I define $\psi(k, q)$ as a probability of adjustment as

[10]The random shock assumption is following Khan and Thomas (2008).

follows:

$$\psi(k,q) := \frac{\min\{\xi^*(k,q),\overline{\xi}\}}{\overline{\xi}}.$$

The ex-ante investment, $\widehat{I}$ can be characterized in the following form:

$$\widehat{I} = \psi(k,q)I^*$$

where I^* is unconstrained optimal level of investment that satisfies the first-order condition (1) under $\mu = 0$.[11] The interest elasticity of the ex-ante investment $\widehat{I}$ depends on how both $\psi(k,q)$ and I^* respond to a change in q. Then, I define a cutoff $\widehat{k}(q) = \frac{k^*}{1-\delta}$, where a firm with k greater than this threshold makes a negative investment. In Lemma 5 of Appendix B.3, I formally show that such $\widehat{k}$ uniquely exists given q. The following decomposition holds for $\forall k \in (0, \widehat{k}(q))$:[12]

$$\begin{aligned}\frac{\partial}{\partial q}log(\widehat{I}) &= \frac{\partial}{\partial q}log(\psi(k,q)) && \text{[Extensive margin responsiveness]}\\ &+ \frac{\partial}{\partial q}log(I^*) && \text{[Intensive margin responsiveness]}\end{aligned}$$

The average response of firm-level investment per cent is additively separable into extensive and intensive margin responsiveness. As the intensive margin has been studied in the previous section, I focus on the extensive margin responsiveness in this section.

In Lemma 6, I show that $\psi(k,q)$ increases in q. This is because a higher discount factor leads to a greater discounted future profit, leaving the marginal benefit of investment greater. Therefore, firms respond to an interest rate change in both extensive and intensive margin in the same direction. However, when it comes to the rankings of extensive margin elasticity over the size, the theoretical prediction in the extensive margin diverges from the one in the intensive margin.

To understand the cross-sectional ranking of the extensive margin interest elasticities, I decompose the partial derivative of $log(\psi(k,q))$ with respect to q and k for $\forall k \in (0, \widehat{k}(q))$ as

[11] For the simplicity of the proofs, I assume $\mu = 0$ for the model with a fixed cost.

[12] We focus only on firms that make positive investments as in the empirical specification in Zwick and Mahon (2017). Therefore, the extensive margin transition from non-adjuster to adjuster is ignored. However, the transition in the opposite direction is counted.

follows:

$$\frac{\partial}{\partial k}\frac{\partial}{\partial q}log(\psi(k,q)) = \frac{\partial}{\partial k}\frac{\partial}{\partial q}log(\xi^*(k,q))$$

$$= \frac{\partial}{\partial k}\frac{\frac{\partial}{\partial q}\xi^*(k,q)}{\xi^*(k,q)} \tag{3}$$

$$= \underbrace{-\frac{\frac{\partial\xi^*(k,q)}{\partial k}\frac{\partial\xi^*(k,q)}{\partial q}}{\xi^*(k,q)^2}}_{\text{Denominator effect } (>0)} + \underbrace{\frac{\frac{\partial^2\xi^*(k,q)}{\partial q\partial k}}{\xi^*(k,q)}}_{\text{Direct effect } (<0)} \tag{4}$$

In the following proposition, I determine the sign of each component in the decomposition.

Proposition 3 (The effect of the firm size and the price on the adjustment probability)**.** *For* $\forall k$ *s.t.* $\xi^*(k,q) < \overline{\xi}(q)$*,*

$$\frac{\partial\xi^*(k,q)}{\partial k}\frac{\partial\xi^*(k,q)}{\partial q} < 0 \text{ and } \frac{\partial}{\partial k}\frac{\partial}{\partial q}\xi^*(k,q) < 0.$$

Proof. See Appendix B.3. ■

According to Proposition 3, the first term of the right-hand side in Equation (4) is positive while the second term is negative. In other words, as the size of a firm increases, the magnitude of the change in the adjustment probability (the numerator of (3)) decreases, but at the same time, the adjustment probability also decreases (the denominator of (3)). Therefore, the ranking of the investment response in the extensive margin in per cent across the firm size cannot be determined.

To sum up, the fixed cost affects the ex-ante investment response through the extensive margin. When measured in the absolute value, the ranking of the interest elasticity in the extensive margin decreases in firm size. However, when measured in per cent, the ranking becomes unclear due to the countervailing force from the interest rate effect on the level of the adjustment probability. The ex-ante investment elasticity is determined by both the intensive and extensive margin responsiveness. From the previous section, the ranking of the intensive margin responsiveness is counterfactually flipped in the model. Therefore, to correct the counterfactual ranking by including the fixed cost, the extensive margin elasticity needs to be substantially lower for large firms in the model with both convex and fixed adjustment costs. In the next section, I quantitatively investigate the ranking of the large and small firms' interest elasticities under the infinite-period models with different adjustment costs.

3.3 Comparison of the semi-elasticities across models

This section compares the semi-elasticities of firm-level investment across different models. I consider three different models: 1) a model with fixed cost (Khan and Thomas, 2008); 2) a model with convex adjustment cost; 3) a model with both fixed and convex adjustment cost (Winberry, 2021). As each model is based on the description of the reference paper, I abstract the detailed explanation of each model. The models are calibrated to match the cross-sectional average of the investment-to-capital ratio and the cross-sectional average spike ratio.[13] Additionally, for the model with both fixed and convex adjustment costs, I matched the cross-sectional dispersion of the investment-to-capital ratio.

Table 2 reports the semi-elasticities of firm-level investments for different groups across different models. The elasticities are measured by the average contemporaneous change in the firm-level investment in per cent from the steady-state when the interest rate changes by 1%.[14] In particular, I calculate the average between the elasticity measured when the interest rate increases by 1% and the one measured when the interest rate decreases by 1% to address the asymmetry in the responses to the positive and negative interest rate shocks. The average interest elasticity of group $j \in \{All, Small, Large\}$ is defined as follows:

$$Elasticity_{jt} = \frac{\int_{\{I_{ijt}>0\}} \Delta log(I_{ijt}\psi_{ijt} + I^c_{ijt}(1-\psi_{ijt}))d\Phi_j}{\Delta r_t}$$

where ψ_{ijt} is the extensive margin adjustment probability; I_{ijt} is then investment after fixed cost is paid and I^c_{ijt} is the investment when the fixed cost is unpaid; Φ_j is the joint distribution of firms conditional on group j.

The elasticity of the spike ratio of group j is defined as the average contemporaneous change in the fraction of firms investing greater than 20% of the existing capital stock when the interest rate changes by 1%.

$$Elasticity^{SpikeRatio}_{jt} = \frac{\int_{\{I_{ijt}>0\}} \Delta \mathbb{I}\left\{\frac{I_{ijt}\psi_{ijt}+I^c_{ijt}(1-\psi_{ijt})}{k_{ijt}} > 0.2\right\} d\Phi_j}{\Delta r_t}$$

According to Table 2, in any of the three models, the interest elasticity of investment is greater in large firms than in small firms. By including both fixed and convex adjustment costs, the counterfactual elasticity divergence is slightly mitigated, as can be seen from the lowest small-to-large ratio, 0.62. Still, the ratio is substantially lower than the empirical ratio of 1.95, as reported in the fourth column. This is due to a dominant intensive margin impact that is proved in Proposition 1.

Consistent with findings from the literature, the average interest elasticity is in the empirically supported range when convex adjustment cost is included. When both fixed and convex adjustment costs are included, the average elasticity is around 5, satisfying an empirical upper bound of 7.2 from Zwick and Mahon (2017).

I also analyze the spike ratio's elasticity as this elasticity can be directly measured in the

[12]The model with convex adjustment cost is a simpler version of the model with both fixed and convex adjustment cost, where the fixed cost is discarded. The models do not include the habit formation in the household utility differently from Winberry (2021).

[13]The target moment is the same as in the baseline model calibration, which is reported in Table 4

[14]The elasticity is measured in the partial equilibrium as in Winberry (2021) and Koby and Wolf (2020).

Table 2: Semi-elasticity comparison across models

	Fixed	Convex only	Convex + Fixed	Data
Investment				
All	382.73	18.18	5.01	7.2
Small	313.76	14.8	4.32	
Large	481.93	21.79	6.99	
S/L ratio	0.65	0.68	0.62	1.95
Spike ratio				
All	25.61	1.97	1.04	
Small	37.97	0.74	1.24	
Large	16.39	1.35	1.14	
S/L ratio	2.32	0.55	1.09	

Notes: The semi-elasticities of investment variables are computed from contemporaneous investment response to an interest rate change in the partial equilibrium. To address the asymmetry between responses to the positive and negative interest rate shocks, I report the average responses to the positive 1% and negative 1% interest rate changes.

data and can guide us on the missing component in the model for capturing the cross-section of the empirically supported interest elasticities. Based on the comparison of the model-implied elasticities and the data estimates in the next section, I discuss how the models need to be improved.

When a model includes only a fixed cost, the large firms' spike ratio elasticities are 2.6 times lower than those of small firms. In contrast, the convex adjustment cost flips the ranking of elasticity of the spike ratio, leaving large firms to become relatively more elastic than small firms. When both adjustment costs are considered, small and large firms' elasticities of the spike ratios are a similar level.

3.4 Firm-level interest elasticities of investments in the data

In this section, I empirically estimate the elasticity of firm-level investment using firm-level balance sheet data and monetary policy shocks in the literature. Prior research papers in the literature have provided the well-identified interest elasticities of firm-level investments, but those estimates are not informative enough to pin down the missing component in the existing model frameworks. For this, I estimate the elasticities of small and large firms' spike ratios to develop a model with realistic firm-level investment.

I construct an exogenous monetary policy shock following Ottonello and Winberry (2020) and Jeenas (2018). The monetary policy shock is obtained by time aggregating high-frequency monetary policy shock identified from the unexpected jump (drop) in the federal funds rate during a 30-minutes window around the announcement of the Federal Open Market Committee (FOMC).[15] To capture the unexpected component in the federal funds rate, I use the change in the rate implied by the current-month federal funds futures contract. All the data on the timings of the FOMC announcement and the high-frequency surprise are from Gurkaynak et al. (2005) and Gorodnichenko and Weber (2016). The sample period covers from March 1990 until December 2009. I follow the convention that the positive monetary policy shock is an unexpected increase in the federal funds futures rate, so it implies the contractionary monetary policy.

To match the data frequency between the firm-level data and the monetary policy shock, I time aggregate the monetary policy shocks. Specifically, I compute the one-year backward weighted average monetary policy shock at each firm's financial year end. The weight of each surprise is determined by the number of days between the corresponding FOMC announcement and the next FOMC announcement.[16] If the next FOMC announcement was made after the financial year end, the days are counted until the financial year end. This data joining process matches a firm's balance sheet information and the monetary policy shock at the same financial year. The weighted moving average monetary policy shock is plotted in Figure B.1.

Large firms and small firms are defined respectively as top 60% and bottom 20% of capital distribution at each year in Compustat data. This choice is to match the definition of large and small firms in Zwick and Mahon (2017).[17] I consider log of firm-level investment, $log(I_{it})$ and binary indicator of investment greater than 20% of existing capital stock, $\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$ as dependent variables in the regression.

To study the heterogeneous firm-level investment responses to the monetary policy shock, I estimate the following regression separately for large firms and small firms:

$$f(k_{it}, k_{it+1}) = \beta MP_t + \alpha_i + \alpha_{sy} + Controls_{it} + \epsilon_{it}$$

where MP_t is the monetary policy shock; α_i is firm fixed effect; α_{sy} is sector-year fixed effect. The control variables include lagged current account (ACT_{t-1}), lagged total debt (DT_{t-1}), and operating profit ($OIBDP_t$) normalized by lagged total asset (AT_{t-1}), log of lagged capital stock, and log of employment (EMP_t). The standard errors are two-way clustered across firms and years.

Table 3 reports the coefficient of monetary policy shock (MP_t) for large and small firms across different choices of dependent variables.[18] As can be seen from the first two columns, the elasticity of the investment is significantly lower in large firms than in small firms. This is consistent with the empirical results in the literature and contradictory to the model-implied elasticities in the previous section. Also, the sensitivity of the spike ratio is significantly lower

[15]The result is robust over the choice of a wider window (one-hour window) as reported in Table B.3.

[16]A higher weight is assigned for a monetary policy shock when there was greater amount of time for a firm to respond to the shock (Ottonello and Winberry, 2020).

[17]The closest regression result in Zwick and Mahon (2017) is Table B.2. Following the paper, I use the capital expenditure $CAPX_t$ as an investment variable.

[18]I check the robustness of result using a different cutoff 10% than 20% in Table B.2.

Table 3: Investment sensitivities to the monetary policy shocks

	Dependent variables:			
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$	
	L	S	L	S
$MP_{Tight,t}$	-2.201 (0.606)	-7.025 (2.41)	-0.870 (0.366)	-2.072 (0.676)
Obs.	29,400	7,903	29,400	7,903
R^2	0.929	0.791	0.603	0.558
Firm FE	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes

Notes: The independent variables include monetary policy shocks, fixed effects (firm and sector-year), and firm-level control variables (lagged current account (ACT_{t-1}), lagged total debt (DT_{t-1}), and operating profit ($OIBDP_t$) normalized by lagged total asset (AT_{t-1}), log of lagged capital stock, and log of employment (EMP_t)). The numbers in the bracket are the standard errors. The standard errors are clustered two-way by firm and year.

in large firms than small firms, as reported in the third and fourth columns.[19]

The differences in the elasticities in Table 2 and Table 3 sharply indicate that the existing models with fixed and convex adjustment costs cannot correctly capture the ranking of interest elasticities between large and small firms. Therefore, a new model is needed to study the role of large firms' investments over the business cycle. Then, a question still remains about which component of the existing model needs to be improved to capture the empirical relationship. There are broadly two directions: lowering either intensive or extensive margin elasticities of large firms.

On this issue, the elasticity of spike ratio gives an answer. I set the model with both fixed and convex adjustment costs as a benchmark model. From the comparison of the interest elasticities of spike ratios between the benchmark model and the data, the large firms' spike ratio needs to be less elastic, and small firms' spike ratio needs to be more elastic than in the benchmark model to match the empirical counterpart. Therefore, the extensive margin elasticity needs to be improved from the benchmark model. In the following section, I develop a heterogeneous-firm real business cycle model where the elasticities of investments and spike ratios are at the empirically supported level through the modification in the extensive margin investment patterns of the benchmark model.

[19]Two estiamtes are statitically different under the significance level of 0.05.

4 Model

I develop and analyze a heterogeneous-firm real business cycle model in which the cross-section of the semi-elasticities of firm-level investment is matched with the empirical estimates. In the model, time is discrete and lasts forever. There is a continuum of measure one of firms that own capital, produce business outputs, and make investments. The business output can be reinvested as capital after a firm pays adjustment costs.

4.1 Technology

A firm owns capital. It produces a unit of goods that can be converted to a unit of capital after paying an adjustment cost. The production technology is a Cobb-Douglas function with decreasing returns to scale:

$$z_{it}A_t f(k_{it}, l_{it}) = z_{it}A_t k_{it}^{\alpha} l_{it}^{\gamma}, \quad \alpha + \gamma < 1$$

where k_{it} is firm i's capital stock at the beginning of period t; l_{it} is labor input; z_{it} is idiosyncratic productivity; A_t is aggregate TFP. Idiosyncratic productivity, z_{it}, and aggregate TFP, A_t, follow the stochastic processes as specified below:

$$\begin{aligned} ln(z_{it+1}) &= \rho_z ln(z_{it}) + \epsilon_{z,t+1}, \quad \epsilon_{z,t+1} \sim_{iid} N(0, \sigma_z) \\ ln(A_{t+1}) &= \rho_A ln(A_t) + \epsilon_{A,t+1}, \quad \epsilon_{A,t+1} \sim_{iid} N(0, \sigma_A) \end{aligned}$$

where ρ_i and σ_i are persistence and standard deviation of $i.i.d$ innovation in each process $i \in \{z, A\}$, respectively. Both stochastic processes are discretized using the Tauchen method in computation.

4.1.1 Investment and adjustment cost

I assume a firm-level large-scale investment could be made only after paying a total adjustment cost C_{it}, which varies over firm-level allocations. The total adjustment cost is a function of capital stock k_{it}, investment size I_{it}, and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \overline{\xi}]$ as in Winberry (2021). And this total adjustment cost is composed of two additively separable parts: a convex adjustment cost and a fixed adjustment cost. The convex adjustment cost is a function of the current capital stock k_{it} and the investment I_{it} as assumed in the literature. The fixed cost F_{it} is a function of the current capital stock k_{it} and a fixed cost shock $\xi_{it} \sim_{iid} Unif[0, \overline{\xi}]$. The fixed cost does not incur if a firm adjusts capital within a moderate range ($I_{it} \in \Omega(k_{it}) := [-\nu k_{it}, \nu k_{it}]$). A firm needs to pay a fixed cost for investment beyond this range. The fixed cost is assumed to be overhead labor cost, so it varies over the business cycle due to wage fluctuations.[20]

[20] This setup is following Khan and Thomas (2008) and Winberry (2021).

To summarize, I assume the following total adjustment cost structures:

$$
\begin{aligned}
C_{it} &= C(k_{it}, I_{it}, \xi_{it}; w_t) \\
&= \mu \left(\frac{I_{it}}{k_{it}} \right)^2 k_{it} + F(k_{it}, \xi_{it}) w_t
\end{aligned}
$$

$$
F(k_{it}, \xi_{it}) = \begin{cases} \xi_{it} k^{\zeta} & \text{if } I_{it} \notin \Omega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \\ 0 & \text{if } I_{it} \in \Omega(k_{it}) = [-\nu k_{it}, \nu k_{it}] \end{cases}
$$

The difference of this model from the existing literature is the size-dependent fixed cost parametrized by the extensive margin elasticity dispersion parameter, ζ. As ζ increases, the extensive margin elasticity gap between small and large firms broadens, leaving the cross-section of the interest elasticity consistent with the empirical level in Zwick and Mahon (2017) and Koby and Wolf (2020). In Section 5, I quantitatively investigate how the ζ parameter affects the dispersion of interest elasticity.

4.1.2 Size-dependent fixed cost: A theoretical explanation

In this section, I provide a theoretical explanation for the presence of size-dependent fixed cost. The presence of fixed cost in the investment has been widely accepted in the micro-level investment literature. However, it has been relatively less investigated whether the fixed cost occurs at the establishment level or at the firm level. Depending on the model specification and the granularity of the data, each paper flexibly defines the fixed cost.

In this paper, the fixed cost is modeled at the firm level, but its functional form is grounded on the establishment-level fixed cost. I argue that if a firm decides to make a large-scale investment by expanding establishments, fixed cost occurs at each existing establishment due to interdependence across the establishments. For example, if a new establishment is constructed, the production lines in the existing establishments have to be adjusted to coordinate with the new one, and managers have to be reallocated across the different production units. Therefore, intuitively, firm-level fixed cost increases in the number of establishments and the degree of interdependence across the establishments.

To sharpen the theoretical points clear, let's assume a firm has n number of establishments and plans to expand a new factory. If establishments are coordinated pairwise, and if the fixed cost of each coordinated pair is ξ, the total firm-level fixed cost F is as follows:

$$
F_2 = \binom{n}{2} \times \xi = \frac{n(n-1)}{2} \xi
$$

which features quadratic growth in the number of establishments. This was when each establishment is interdependent pairwisely. Then, if an establishment's operation is dependent on

$\zeta - 1$ number of other establishments on average, the firm-level fixed cost becomes as follows:

$$F_\zeta = \binom{n}{\zeta} \times \xi = \frac{n(n-1)(n-2)\dots(n-\zeta+1)}{\zeta!}\xi$$

The firm-level fixed cost F_ζ exponentially increases in the number of establishments to the power of ζ. For a higher interdependence across the establishments, the fixed cost increases faster. Even if the source of the fixed cost is not at the establishment level, the intuitive explanation is that the interdependence across the basic operation unit (e.g., department or team) convexly raises the complexity inside the firm. And this increases the firm-level fixed cost when the firm makes a large-scale capital adjustment.

In this paper, the number of establishments (or basic production units) is proxied by the total capital stock k_{it}. This is consistent with Cao et al. (2019). Using the US administrative data, Cao et al. (2019) points out that the firm growth is substantially driven by the expansion in the number of establishment. Therefore, the number of establishments is well-proxied by the size of the capital stock k_{it}.

4.2 Household

A stand-in household is considered. The household consumes, supplies labor, and saves in a complete market. In the beginning of a period, the household is given with an equilty portfolio a, information on the contemporaneous distribution of firms Φ, and the aggregate TFP level A. The household problem is as follows:

$$\begin{aligned} V(a;S) = \max_{c,a',l_H} \quad & log(c) - \eta l_H + \beta \mathbb{E} V(a';S') \\ \text{s.t. } c + & \int \Gamma_{S,S'} q(S,S') a'(S') dS = w(S) l_H + \int a(S) dS \\ & G_\phi(S) = \Phi', \quad G_A(A) = A', \quad S = \{\Phi, A\} \end{aligned}$$

where V is the value function of the household; Φ is a distribution of firms; A is an aggregate productivity; $\Gamma_{S,S'}$ is the aggregate state transition probability; c is consumption; a' is a state-contingent future saving portfolio; l_H is labor supply; w is wage; and r is real interest rate. Household is holding the equity of firms as their asset.

From the household's first-order condition and the envelope condition, I obtain the following chracterization of the stochastic discount factor $q(S, S')$:

$$q(S,S') = \beta \frac{C(S)}{C(S')}$$

I define $p(S) := \frac{1}{C(S)}$. In the recursive formualation of a firms' problem in the next section, I use $p(S)$ to normalize the firm's value function following Khan and Thomas (2008).

4.3 A firm's problem: Recursive formulation

In this section, I formulate a firm's problem in the recursive form. A firm is given with capital k, an idiosyncratic productivity z, in the beginning of a period. Also, they are given with the knowledge on the contemporaneous distribution of firms Φ and the aggregate TFP level A. For each period, firm determines investment level I and labor demand l_d. A firm's problem is formulated in the following recursive form:

$$
\begin{aligned}
J(k,z;S) = \quad & \pi(k,z;S) + (1-\delta)k \\
& + \int_0^{\bar{\xi}} \max\left\{R^*(k,z;S) - F(k,\xi)w(S), R^c(k,z;S)\right\} dG_\xi(\xi) \qquad (5) \\
R^*(k,z;S) = & \max_{k' \geq 0} \; -k' - c(k,k') + \mathbb{E}q(S,S')J(k',z';S') \\
R^c(k,z;S) = & \max_{k^c \in \Omega(k)} \; -k^c - c(k,k^c) + \mathbb{E}q(S,S')J(k^c,z';S')
\end{aligned}
$$

The following lines explain the details of each component in the value function.

$$
\begin{aligned}
\text{(Operating profit)} \quad & \pi(z,k;S) := \max_{n_d} zAk^\alpha n_d^\gamma - w(S)n_d \;\; (n_d\text{: labor demand}) \\
\text{(Convex adjustment cost)} \quad & c(k,k') := \left(\mu^I/2\right)\left((k'-(1-\delta)k)/k\right)^2 k \\
\text{(Size-dependent fixed cost)} \quad & F(k,\xi) := \xi k^\zeta \\
\text{(Constrained investment)} \quad & I^c \in \Omega(k) := [-k\nu, k\nu] \quad (\nu < \delta) \\
\text{(Idiosyncratic productivity)} \quad & z' = G_z(z) \text{ (AR(1) process)} \\
\text{(Stochastic discount factor)} \quad & q(S,S') = \beta\left(C(S)/C(S')\right) \\
\text{(Aggregate states)} \quad & S = \{A, \Phi\} \\
\text{(Aggregate law of motion)} \quad & \Phi' := H(S),\; A' = G_A(A) \text{ (AR(1) process)},
\end{aligned}
$$

Then, I multiply $p(S) = 1/C(S)$ on the both sides of line (5) to obtain

$$
\begin{aligned}
p(S)J(k,z;S) = \quad & p(S)(\pi(k,z;S) + (1-\delta)k) \\
& + \int_0^{\bar{\xi}} \max\left\{p(S)R^*(k,z;S) - p(S)w(S)F(k,\xi), p(S)R^c(k,z;S)\right\} dG_\xi(\xi)
\end{aligned}
$$

I define the normalized value functions as follows:

$$
\begin{aligned}
\widetilde{J}(k,z;S) &:= p(S)J(k,z;S) \\
\widetilde{R}^*(k,z;S) &:= p(S)R^*(k,z;S) \\
\widetilde{R}^c(k,z;S) &:= p(S)R^c(k,z;S)
\end{aligned}
$$

It is necessary to check whether the recursive form is preserved for the normalized value functions. Using $p(S)q(S,S') = \beta p(S')$,

$$\begin{aligned}\widetilde{R}^* &= \max_{k'\geq 0} \ (-k' - c(k,k'))p(S) + \mathbb{E}p(S)q(S,S')J(k',z';S') \\ &= \max_{k'\geq 0} \ (-k' - c(k,k'))p(S) + \mathbb{E}\beta p(S')J(k',z';S') \\ &= \max_{k'\geq 0} \ (-k' - c(k,k'))p(S) + \beta\mathbb{E}\widetilde{J}(k',z';S')\end{aligned}$$

Similarly,

$$\widetilde{R}^c = \max_{k^c \in \Omega(k)} \ (-k^c - c(k,k^c))p(S) + \beta\mathbb{E}\widetilde{J}(k^c,z';S').$$

Therefore, the recursive form is preserved for the normalized value functions. As in Khan and Thomas (2008), the recursive form based on the normalized value function eases computation of the dynamic stochastic general equilibrium because the price p depends only on the current aggregate state variable S.

A firm makes a large scale investment only if $R^*(k,z;S) > R^c(k,z;s)$. Therefore, a firm-level extensive margin investment decision can be characterized by the threshold rule, g_{ξ^*}, as follows:

$$g_{\xi^*}(k,z;S) = \min\left\{\frac{\widetilde{R}^*(k,z;S) - \widetilde{R}^c(k,z;S)}{w(S)p(S)k^{\zeta}}, \overline{\xi}\right\}.$$

This threshold rule is distinguished from the threshold rules in other existing models in that the threshold weakly decreases in the size of a firm. In other words, the required marginal benefit of large-scale investment is greater for large firms to make the extensive margin investment than for small firms. This generates an empirically-supported cross-section of interest elasticities. I quantitatively show this in Section 5.

I denote g_{k^*} as the optimal future capital stock conditional on the extensive margin investment, g_{k^c} as the optimal future capital stock conditional on the small-scale investment, and g_k as the unconditional optimal investment.

Then, the following relationship holds:

$$g_k(k,z;S) = \begin{cases} g_{k^*}(k,z;S) & \text{if } \xi < g_{\xi^*}(k,z;S) \\ g_{k^c}(k,z;S) & \text{if } \xi \geq g_{\xi^*}(k,z;S). \end{cases}$$

That is, if a fixed cost shock, ξ is less than the threshold, a firm makes a large-scale investment.

4.4 Recursive competitive equilibrium

In this section, I define the recursive competitive equilibrium in the economy.
$(g_c, g_a, g_{l_H}, g_{k^*}, g_{k^c}, g_{\xi^*}, g_{n_d}, \widetilde{V}, \widetilde{J}, \widetilde{R}^*, \widetilde{R}^c, p, w)$ is a recursive competitive equilibrium if the following conditions are satisfied.

1. $g_c, g_{lH}, \widetilde{V}$ and g_a, solves the household's problem.

2. $g_{k^*}, g_{k^c}, g_{\xi^*}, g_{n_d}, \widetilde{J}, \widetilde{R}^*$, and $\widetilde{R}^c$ solve a firm's problem.

3. Market Clearing:

$$\text{(Labor Market)} \quad g_{lH}(\Phi; S) = \int \left(g_{n_d}(k,z;S) + \left(\frac{g_{\xi^*}(k,z;S)}{\bar{\xi}} \right) \left(\frac{g_{\xi^*}(k,z;S)}{2} \right) k^{\zeta} \right) d\Phi$$

$$\begin{aligned}\text{(Product Market)} \quad g_c(\Phi; S) = \int \Bigg(& zAk^{\alpha} g_{n_d}(k,z;S)^{\gamma} \\ & - \Big((g_{k^*}(k,z;S) - (1-\delta)k) + c(k, g_{k^*}(k,z;S)) \Big) \frac{g_{\xi^*}(k,z;S)}{\bar{\xi}} \\ & - \Big((g_{k^c}(k,z;S) - (1-\delta)k) + c(k, g_{k^c}(k,z;S)) \Big) \frac{1 - g_{\xi^*}(k,z;S)}{\bar{\xi}} \Bigg) d\Phi\end{aligned}$$

4. Consistency Condition[21]:

$$\text{(Consistency)} \quad G_{\Phi}(\Phi) = H(\Phi) = \Phi', \text{ where for } \forall K' \subseteq \mathbb{K} \text{ and } z' \in \mathbb{Z},$$

$$\begin{aligned}\Phi'(K', z') = \int \Gamma_{z,z'} \Bigg(& \mathbb{I}\{g_{k^*}(k,z;S) \in K'\} \frac{g_{\xi^*}(k,z;S)}{\bar{\xi}} \\ & + \mathbb{I}\{g_{k^c}(k,z;S) \in K'\} \frac{1 - g_{\xi^*}(k,z;S)}{\bar{\xi}} \Bigg) d\Phi\end{aligned}$$

4.5 Solution method: The repeated transition method

This section explains the solution method I use to compute the recursive competitive equilibrium. I use the repeated transition method, which I concurrently developed for the computation of nonlinear aggregate dynamics under aggregate uncertainty in Lee (2021). As highlighted in Bachmann et al. (2013), the strong general equilibrium effect significantly contributes to the linearity in the dynamics of aggregate allocations. However, once the model captures realistic interest elasticity, the general equilibrium effect is necessarily weakened, leaving the aggregate dynamics highly nonlinear. Due to this highly nonlinear aggregate dynamics in general equilibrium, there are two layers of difficulties in using the algorithm of Krusell and Smith (1998). The first is difficulty in choosing a sufficient statistics for the aggregate dynamics. The model's nonlinear aggregate dynamics might not be sufficiently explained by the moves in aggregate capital stocks, unlike Khan and Thomas (2008). The second difficulty is in setting the parametric form in the law of motion. This problem interacts with the former difficulty because even correctly chosen sufficient statistics would not give accurate computation results due to the wrong functional specification. Therefore, it is

[21] $\mathbb{K}$ and $\mathbb{Z}$ are the supports of the marginal distributions of capital and productivity induced from Φ.

almost impossible to jointly identify the correct sufficient statistics and functional form in the law of motion.

In the repeated transition method, I simulate a long series of aggregate shocks and compute the dynamics of prices and aggregate allocations at each point in the simulation, in the spirit of Boppart et al. (2018). However, in contrast to Boppart et al. (2018), the repeated transition method departs from the certainty equivalence.[22] As the prices and predicted future path of allocations are directly computed at each point on the path, the method does not require a parametric form of the law of motion.

Using this method, I compute the predicted aggregate allocations, which the time series of the simulated aggregate allocations almost perfectly converges to. And this time series of the predicted aggregate allocations is not based on a parametric form of the law of motion in a state-space representation. Figure E.4 compares the time series of the predicted allocations and the simulated allocations.[23] In the figure, panel (a) shows the predicted aggregate dynamics and the simulated dynamics of the marginal utility, p_t. These two dynamics converged to each other with an extremely small error, as can be seen in the solid line in panel (c). However, if the dynamics of simulated marginal utility are fitted into the log-linear law of motion in the contemporaneous capital stock K_t, the prediction error can become substantially large as in the dashed line in panel (c). A similar pattern is observed in the aggregate dynamics of aggregate capital stock K_t in panel (b). The simulated and predicted paths for K_t are computed at extremely high accuracy with the repeated transition method, while the log-linear fitting leads to a significant prediction error as in panel (d).

Then, I compare the fitness of different specifications of the law of motion by fitting the equilibrium dynamics into each of them.[24] Table E.7 and Table E.8 report the fitness of the different laws of motion of p_t and K_t, respectively. When the law of motion includes only a log of contemporaneous capital stock K_t (specification (1)), the prediction errors remain large, indicating the nonlinear nature of the equilibrium dynamics.[25] However, once the law of motion includes the fragility index in the law of motion (specification (2)), which I define in Section 5.3, the fitness significantly improves for the dynamics of p_t. However, it does not make a significant change in the fitness for the dynamics of K_t. Finally, if the law of motion includes contemporaneous and lagged capital stocks up to three lags in a non-parametric form (specification (3)), the fitness substantially improves from the basic log-linear specification for both p_t and K_t.

[22] The algorithm of Boppart et al. (2018) can be applied for the case of an MIT shock and the perfect foresight.

[23] This figure is the fundamental accuracy plot suggested in Den Haan (2010).

[24] I compare only the fitness of the law of motion to the converged dynamics of equilibrium allocations. Therefore, if the model is solved based on each of the laws of motion, the implied dynamics might display even greater prediction errors than the reported level.

[25] Den Haan (2010) points out that a slight deviation in R^2 from unity such as $R^2 = 0.995$ can imply a substantially large prediction error and significant nonlinearity.

5 Quantitative analysis

This section quantitatively analyzes the macroeconomic implications of large firms' lumpy investments. First, I train the baseline model to fit the data moments using the method of simulated moments. Especially, the different interest elasticities between small and large firms are the key moments to be fitted, which are hardly captured in alternative models. Second, I study the nonlinear dynamics of lumpy investments using impulse response analysis. The nonlinear dynamics arise from the synchronization of large-scale investment timing. Lastly, I quantitatively analyze how the large firms' synchronization pattern affects the response of aggregate investment to a one-standard-deviation TFP shock and the aggregate interest elasticity.

5.1 Calibration

In this section, I elaborate on how the model is fitted to the data and compare the fitness with alternative models. Table 4 reports the target and untargeted moments from the data and the simulated moments in the model. Table 5 reports the calibrated parameters given the fixed parameters reported in Table B.4. In the simulation step, I use the non-stochastic method in Young (2010).

Table 4: Fitted Moments

Moments	Data	Model	Reference
Targeted moments			
Semi-elasticity of investment (%)	7.20	6.63	Zwick and Mahon (2017)
Cross-sectional semi-elasticity ratio (%)	1.95	2.13	Zwick and Mahon (2017)
Cross-sectional average of i_t/k_t ratio	0.10	0.10	Zwick and Mahon (2017)
Cross-sectional dispersion of i_t/k_t $(s.d.)$	0.16	0.16	Zwick and Mahon (2017)
Cross-sectional average spike ratio	0.14	0.14	Zwick and Mahon (2017)
Positive investment rate	0.86	0.86	Winberry (2021)
$sd(log(Y_t))$	0.06	0.07	NIPA data (Annual)
Untargeted moments (all in yrs.)			
Average inaction periods	6.38	7.72	Compustat data
Dispersion of inaction periods	4.87	5.50	Compustat data
Average of lag difference of inaction periods	0.27	0.67	Compustat data
Dispersion of lag difference of inaction periods	6.47	8.36	Compustat data

Notes: The data moments are from the sources specified in the reference column. The same sample restriction as in the empirical analysis applies to Compustat data. I use linearly detrended real GDP from the National Income and Product Accounts at the annual frequency for the aggregate output volatility.

The target semi-elasticity of average investment is from Zwick and Mahon (2017). This target is interpreted as an upper bound rather than a point estimate, as not all the possible idiosyncrasies or frictions are accounted for in the calculation as pointed out by Koby and Wolf (2020). The simulated aggregate semi-elasticity in the baseline model is also similar to the level in Winberry (2021). The cross-sectional semi-elasticity ratio is from Zwick and Mahon (2017), which documents that small firms' investments are around twice elastic as large firms towards the interest rate change. In the paper, large and small firms are defined as the top 30% and bottom 30% of firms in terms of size, respectively. I define large and small firms in the model consistent with their definition. The cross-sectional average and dispersion of the investment-to-capital ratio and the average spike ratio are targeted to match the levels in Zwick and Mahon (2017) as in Winberry (2021). Consistent with the literature, I define the spike ratio as the fraction of firms investing greater than 20% of the existing capital stock. The target of positive investment rate is from Winberry (2021). The positive investment rate is defined as the fraction of firms with an investment that is greater than 1% but smaller than 20% of existing capital stock. Only a negligible fraction of firms make a negative investment in both data and the model as the average spike ratio and positive investment sum to unity. To discipline the aggregate TFP-driven fluctuations in the model, I target the output volatility calculated from annual National Income and Product Accounts (NIPA) data.

Table 5: Calibrated Parameters

Parameters	Description	Value
Internally calibrated parameters		
ζ	Fixed cost curvature	3.500
$\bar{\xi}$	Fixed cost upperbound	0.440
μ^I	Capital adjustment cost	0.780
ν	Small investment range	0.041
σ	Standard deviation of idiosyncratic TFP	0.130
σ_A	Standard deviation of aggregate TFP shock	0.025
Externally estimated parameters		
ρ	Persistence of idiosyncratic TFP	0.750

Notes: Parameters in the upper part of the table are calibrated to match the moments in Table 4. The persistence of idiosyncratic TFP is directly computed from fitting the estimated firm-level TFP (Compustat) into AR(1) process. The firm-level TFP is estimated following Ackerberg et al. (2015) using US Compustat data.

In the model, variations in the fixed cost parameter and convex adjustment cost parameter lead to a sharply divergent effect on the dispersion of investment rate (investment-to-capital ratio), while both lowers the average investment rate. For a higher fixed cost parameter (still in a moderate range), the dispersion of investment rate is higher as the difference in the investment rate between extensive margin adjusters and non-adjusters increases.[26] On the other hand, a higher convex adjustment cost uniformly mutes down the investment rate,

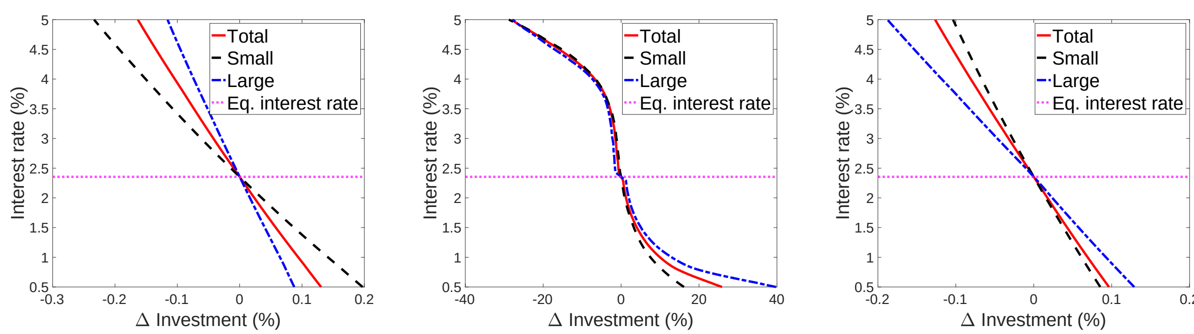
[26]If a fixed cost is too high, the fraction of adjusters become too small to have a meaningful contribution to the investment rate dispersion.

leading to a lower dispersion in the investment rate. These two divergent effects, together with the average investment rate, identify the fixed and convex adjustment cost parameters.

The fixed cost curvature parameter ζ is identified from the cross-sectional semi-elasticity ratio between small and large firms. As ζ increases beyond unity, the large firms' interest elasticity decreases through both the extensive and the intensive margins. The extensive margin channel operates by making it harder for larger firms to make a large-scale investment even if the interest rate decreases. The intensive margin channel operates through the selection effect on the adjusting large firms: those who remain to adjust the capital even when a fixed cost increases are on average less interest elastic firms than those marginal firms that change their adjusting stance when a fixed cost increases. The calibrated level of ζ is 3.5, which I interpret 3.5 establishments are involved per production line on average.

As can be seen from Table 4, the baseline model (column 1) can correctly capture the cross-sectional elasticity ratio between small and large firms. Therefore, the baseline model provides an appropriate framework for analyzing the role of large firms' investment on the dynamic stochastic general equilibrium. This is a novel contribution to the literature as the cross-section of the interest elasticity is not well-captured in the existing model framework.

Figure 3: Semi-elasticities of investments across different models



(a) Baseline (b) Fixed cost only (c) Convex + Fixed

Notes: The figure plots the deviation of investment from the steady-state level when the interest rate changes for each different model. The vertical axis is the interest rate in per cent, and the horizontal axis is the percentage deviation from the steady-state investment. The horizontal dotted line indicates the equilibrium interest rate.

Figure 3 visualizes the large and small firms' interest elasticities for the baseline model (panel (a)), for a model with fixed cost only (panel (b)), and for a model with convex and fixed cost (panel (c)).[27] In each panel, the vertical axis is the interest rate in per cent, and the horizontal axis is the percentage deviation from the steady-state investment. The horizontal dotted line indicates the equilibrium interest rate. As the interest rate decreases, all models' average deviation of investment from the steady-state increases. However, in the model with convex and fixed adjustment cost (panel (c)), the large firms' average deviation of investment

[27] The model with convex and fixed adjustment cost is a prototype of the models in Winberry (2021) and Koby and Wolf (2020).

from the steady-state increases faster than small firms as the interest rate decreases. In the model with a fixed cost only, the interest elasticities of all groups are significantly higher than the ones in the other two models, as can be checked from the large-scale variation along the horizontal axis.

Table 6 compares the semi-elasticities of investment to the interest rate change for different models. The first column is the result from the calibrated baseline model; the second is from the model with fixed adjustment cost only; the third is from the model with convex adjustment cost only; the fourth is from the benchmark model with convex and fixed adjustment costs; the fifth is from the model with convex and linearly size-dependent fixed adjustment costs; the last is from the model with convex and linearly size-dependent fixed adjustment costs.[28]

The first four rows of the table report the elasticities of investment conditional on $I_{ijt} > 0$ where i is a firm index, j is a size group indicator, and t is the time subscript; the next four rows report the extensive margin elasticities; the following four rows report the intensive margin elasticities; the last four rows report the spike ratio elasticities.[29] The elasticity of investment is as defined in section 3.3. I calculate the average between the elasticity measured when the interest rate increases by 1% and the one measured when the interest rate decreases by 1% to address the asymmetry in the responses to the positive and negative interest rate shocks.

In the table, there are two additional interest elasticities to be defined. The extensive margin elasticity of group $j \in \{All, Small, Large\}$ is defined as the average contemporaneous change in the firm-level investment driven by extensive margin probability changes in per cent from the steady-state when the interest rate changes by 1%. Therefore, the investment policy functions are fixed at the steady-state level, while the extensive margin probabilities deviate from the steady-state:

$$Elasticity_{jt}^{ext} = \frac{\int_{\{I_{ijt}>0\}} \Delta log(I_{ijt}^{ss}\psi_{ijt} + I_{ijt}^{ss,c}(1-\psi_{ijt}))d\Phi_j}{\Delta r_t}$$

where ψ_{ijt} is the extensive margin adjustment probability; I_{ijt} is then investment after fixed cost is paid and I_{ijt}^c is the investment when the fixed cost is unpaid; Φ_j is the joint distribution of firms conditional on group j. The intensive margin elasticity of group j is defined as the average contemporaneous change in the firm-level investment driven by investment magnitude changes in per cent from the steady-state when the interest rate changes by 1%. Therefore, the extensive margin probability is fixed at the steady-state level, while the investment policy

[28]Each model is calibrated to match the same moments as in the baseline calibration, except for the cross-sectional elasticity ratio. For the models with fixed adjustmet cost only and with a convex adjustment costs only, I did not match the cross-setional dispersion of i_{it}/k_{it} as these models have one fewer parameter than the others.

[29]Following Zwick and Mahon (2017), I define the elasticty conditional on $I_{ijt} > 0$ as investment elasticity.

functions deviate from the steady-state.

$$Elasticity_{jt}^{int} = \frac{\int_{\{I_{ijt}>0\}} \Delta log(I_{ijt}\psi_{ijt}^{ss} + I_{ijt}^{c}(1-\psi_{ijt}^{ss}))d\Phi_j}{\Delta r_t}.$$

According to Table 6, the aggregate investment elasticity of 6.63 is consistent with the empirical findings in Zwick and Mahon (2017); the small-to-large elasticity ratio of 2.13 is also close to the empirical level. The response of investment is further decomposed into the extensive and intensive margins. Each margin accounts for an almost identical portion of the total response in the baseline model. However, the small-to-large elasticity ratios are greater in the extensive margin response than the intensive margin.

Table 6: Semi-elasticity of investment across the models and the decomposition

	Baseline	Fixed	Convex	Benchmark	Linear-Fixed	Quadratic-Fixed
Investment						
All	6.63	382.73	18.18	5.01	5.49	5.87
Small	9.85	313.76	14.8	4.32	5.41	7.06
Large	4.62	481.93	21.79	6.99	6.38	4.65
S/L ratio	2.13	0.65	0.68	0.62	0.85	1.52
Ext. margin						
All	3.34	84.63		2.58	2.94	3.37
Small	4.63	90.72		2.89	3.6	4.38
Large	1.99	74.28		2.27	2.29	2.35
S/L ratio	2.32	1.22		1.27	1.57	1.86
Int. margin						
All	3.28	152.6	18.18	2.43	2.54	2.5
Small	5.21	95.89	14.8	1.43	1.81	2.67
Large	2.62	244.77	21.79	4.71	4.09	2.29
S/L ratio	1.99	0.39	0.68	0.3	0.44	1.17
Spike ratio						
All	1.3	25.61	1.97	1.04	1.27	1.33
Small	2.36	37.97	0.74	1.24	1.67	2.42
Large	0.98	16.39	1.35	1.14	0.91	1.06
S/L ratio	2.4	2.32	0.55	1.09	1.84	2.29

Notes: The semi-elasticities of investment variables are computed from contemporaneous response to an interest rate change in the partial equilibrium. To address the asymmetry between responses to the positive and negative interest rate shocks, I report the average responses to the positive 1% and negative 1% interest rate changes.

As can be seen from the columns other than the second and the third in Table 6, the aggregate investment elasticities are well-matched with the empirical level once we consider both convex and fixed adjustment costs. Especially, the inclusion of convex adjustment cost dramatically dampens the aggregate elasticity, as can be seen from the aggregate elasticity in the third column compared to that of the second column (Winberry, 2021; Koby and Wolf, 2020). Again, this is consistent with the theoretical prediction of Proposition 2.

The cross-sectional elasticity ratio between small and large in other models than the baseline cannot match the empirical estimate of 1.95 from Zwick and Mahon (2017). However, as the fixed cost becomes size-dependent and as the intra-firm interdependence across establishments rises, the cross-sectional elasticity ratio increases. From the middle and lower part of the table, the size-dependence and the intra-firm linkages increase not only the extensive margin S/L ratio but the intensive margin S/L ratio. This is due to the selection effect on those large firms that remain to adjust despite the higher fixed cost.

Finally, I compare the business cycle statistics implied in the baseline model with the aggregate-level data. The aggregate-level data at the annual frequency is from NIPA data, and the sample period starts from 1955. All the variables are logged and linearly detrended. Figure 7 reports the business cycle statistics from the data and the model. Among the statistics, the time-series volatility of the logged output is the targeted moment.

The correlations across the aggregate variables in the baseline model are well-matched with the observed level in the data. Especially, the autocorrelation of aggregate investment and the cross-correlation between the aggregate investment and output are sharply matched even if they are not the targeted moment. For the relative volatilities of consumption and investment, the model's moments are slightly lower than the observed level.

Table 7: Business cycle statistics

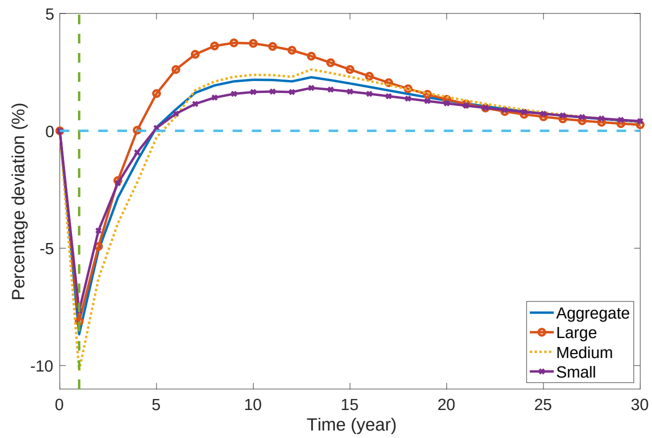
	Data	Model
$corr(Y_t, Y_{t-1})$	0.941	0.843
$corr(I_t, I_{t-1})$	0.742	0.742
$corr(C_t, C_{t-1})$	0.954	0.903
$corr(I_t, Y_t)$	0.795	0.796
$corr(L_t, Y_t)$	0.898	0.771
$corr(C_t, Y_t)$	0.978	0.980
$sd(Y_t)$	0.060	0.065
$sd(I_t)/sd(Y_t)$	1.976	1.809
$sd(C_t)/sd(Y_t)$	0.945	0.823

Notes: The business cycle statistics are obtained from the simulated data using the dynamic stochastic general equilibrium allocations. 5,000 firms are simulated for 1,000 periods (years). All the variables are logged and linearly detrended. The data counterpart is from NIPA data.

5.2 Synchronization

In this section, I analyze how the large and small firms respond to the same productivity shock using the impulse response analysis. Figure 4 plots the impulse responses of the spike ratios of all, large, medium, and small firms to the one-standard deviation negative aggregate TFP shock.[30] The medium firms are those in the 30th to 70th percentile range in the capital distribution. The impulse response is obtained from the method that computes the transition path to the stationary allocation after an unexpected negative one-standard deviation TFP shock, as described in Boppart et al. (2018). All the responses are expressed in percentage deviation from the steady-state level.

Figure 4: Impulse response of spike ratio



Notes: The impulse response of spike ratios are obtained from the transition dynamics to the stationary equilibrium allocations after an unexpected negative one-standard-deviation TFP shock.

Upon the arrival of the negative aggregate TFP shock, all the firms' extensive margin investment timings are synchronized. It is because firms realize it is not a good idea to install new large-scale capital as the business prospect is not promising in the near future. So, firms that are ready for the extensive margin investment tend to delay the plan, leading to synchronized timings of large-scale investments. The dynamics of investment timings after this initial synchronization are starkly different across the different firm sizes.

For large firms, initial synchronization leads to a surge in spiky investments. This is because the large firms are interest-inelastic in the model and thus strictly less affected by the general equilibrium effect.[31] An aggregate shock that contemporaneously incurs a 7% drop in the fraction of firms making large-scale investments leads to a 4% surge of the fraction in the subsequent period. The surge of large firms' lumpy investment is a consistent phenomenon with the empirical patterns observed in post-recession periods as in Figure 1.[32]

On the other hand, the synchronized investment timings of small and large firms are

[30]The shock is assumed to be as persistent as the calibrated aggregate TFP shock.

[31]The synchronization of large firms' investment timings is not the unique feature of the baseline model as it is observed in the benchmark model. This is because the extensive margin elasticity is still greater for large firms than small firms in the benchmark, as can be seen in Table 6. However, the magnitude of the surge is around 80% greater in the baseline model than in the benchmark model.

[32]It is worth noting that after the peak of the synchronized investments, the spike ratio moves in the opposite direction to the TFP dynamics.

spread out over the post-shock period. This is because the general equilibrium effect makes the small and large firms deviate from the concentrated period for large-scale investment. In other words, the general equilibrium effect strongly smooths their investment timings.

In the following section, I study how the nonlinearity in the large firms' spiky investments affects the business cycle under aggregate uncertainty.

5.3 Fragility after a surge of lumpy investments

In this section, I study how the synchronized investment timings of large firms affect the aggregate investment dynamics over the business cycle. First, I define a fragility index that captures how large fraction of large firms have just finished large-scale investments as follows:

$$Fragility_t := \frac{\sum \mathbb{I}\{s_{it} \leq \overline{s}\}\mathbb{I}\{k_{it} > \overline{k}\}}{\sum \mathbb{I}\{k_{it} > \overline{k}\}}$$

where s_{it} is the time from the last lumpy investment; $\overline{s}$ is the threshold where any firm i with s_{it} below the level has recently adjusted its capital in the extensive margin; $\overline{k}$ is the size threshold of large firms. If a great fraction of large firms have just finished a large-scale investment, a relatively small fraction of large firms are willing to make a large-scale investment due to the presence of the fixed adjustment cost. Over the business cycle, the fluctuations in this index interplay with the exogenous TFP fluctuations, as the following analyses will conclude.

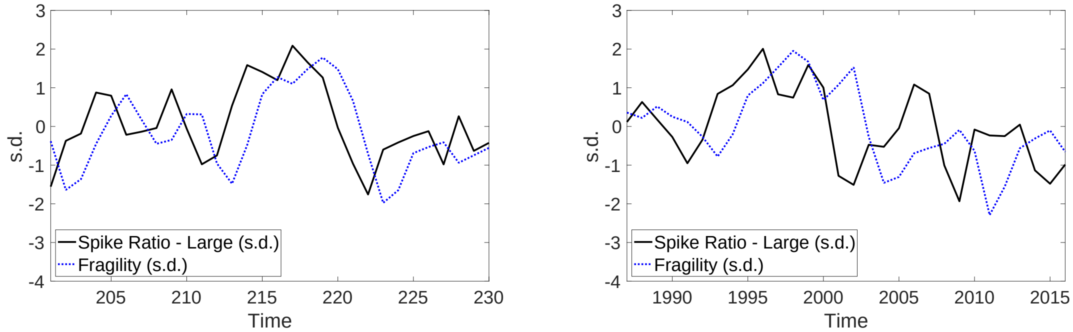
In both the model and the data, the median duration between two lumpy investments is 6 years. I set $\overline{s}$ as the half of this period, three years. In the regression that includes the fragility index, reported in Table 9, I found $\overline{s} = 3$ maximizes the fitness of the regression. The size cutoff $\overline{k}$ is set at the top 30th percentile of capital distribution in the simulated data and at the top 60th percentile in the Compustat data.[33]

It is worth noting that the fragility index is constructed from the readily observable micro-level variables. Especially, the measure is based on the past investment history of large firms, which are mostly listed and subject to financial reporting regulations. Therefore, the index can be measured in a timely manner and can contribute to predicting the near future of aggregate investment. This is a stark contribution to the existing responsiveness indices defined in the literature (Bachmann et al., 2013; Caballero and Engel, 1993).

Figure 5 shows the time series of fragility index and spike ratio in the simulation (panel (a))and the data (panel (b)), where each series is normalized by the standard deviation. In both panels, the time-series of spike ratio leads the fragility index by two to three years. As the average inaction takes around six years, around three years after a surge of lumpy investment (spike ratio), a trough is expected to arrive. By the definition of the fragility index, during this trough of lumpy investment, the index will rise, indicating only a small

[33] From the note in Table B.1 of Zwick and Mahon (2017), I check that the top 60% Compustat firms correspond to the top 30% firms in Zwick and Mahon (2017). In the model of this paper, large firms are defined as top 30% firms in terms of capital.

Figure 5: Time-series of fragility indices in simulation and data



(a) Simulation (b) Data

Notes: Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. Panel (a) plots a part of the simulated allocations. The solid line plots the aggregate investment growth rate (%). The dotted line plots the fragility indices normalized by the standard deviation. The fragility indices are calculated based on the distribution of large firms.

fraction of firms are willing to make a lumpy investment. Therefore, the growth rate of spike ratio and the fragility index comove in the opposite direction. Figure 6 is the scatter plot of the simulated time-series where the horizontal axis is the fragility index normalized by the standard deviation, and the vertical axis is the growth rate of large firms' spike ratio.[34] By fitting the relationship between the fragility and the growth rate of spike ratio into linear regression, I find the following relationship:

$$\Delta log(SpikeRatio_t)(\%) = -\underset{(0.0274)}{1.8936} * Fragility_t\ (s.d.) + \epsilon_t, \quad R^2 = 0.828$$

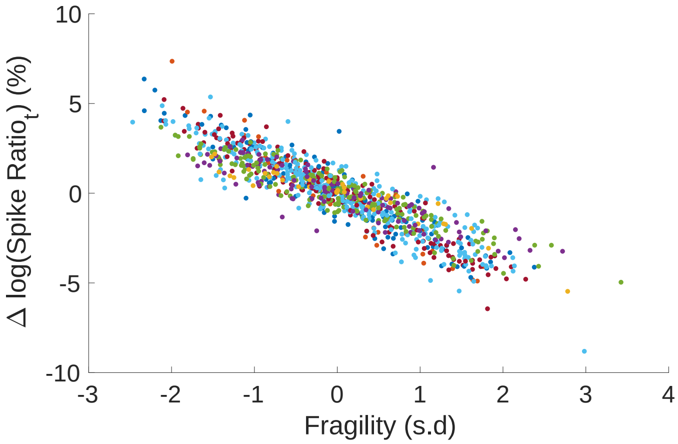
The relationship indicates that one standard deviation increase in fragility is associated with the growth rate of the large firms' spike ratio by 1.89%. As can be seen from the high R^2, these two variables are tightly related over the business cycle. While the growth rate of the large firms' spike ratio is not known before period t, the fragility index is known ahead of period t. This implies the growth rate of the large firms' spike ratio features a state-dependence; the fragility index has predictability for the one-period-ahead growth rate of the large firms' spike ratio.

Then, I study how the fluctuations in the fragility index affect the growth of aggregate investment combined with the TFP fluctuations. Table 8 reports the regression result of the following specification in both the model and the data:

$$\Delta log(I_t) = \alpha + \beta^{OutputShock} OutputShock_t + \beta^{Fragility} log(Fragility_t) + \epsilon_t$$

[34] The past aggregate shock A_{t-1} and the contemporaneous shock A_t are controlled by taking out fixed effects. The different colors of the dots are for different combinations of A_{t-1} and A_t.

Figure 6: Fragility index and the growth rate of the large firms' spike ratio



Notes: The vertical axis of the scatter plot is the spike ratio in percentage deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

where $\Delta log(I_t)$ is the aggregate investment. $OutputShock_t$ is a shock in the logged output, obtained from the residuals in the AR(1) fitting of the logged output process. The aggregate investment and output data are from NIPA data. For the simulated result, I proxy the output shock by the TFP shock as this is the only exogenous source of the output variation and can be explicitly measured. In this specification, $OutputShock_t$ exogenously arrives at t, while the $Fragility_t$ is determined at $t-1$. Therefore two variables are independent of each other.

As reported in Table 8, the coefficient estimates from the model and data are statistically indifferent. When the fragility index increases by 1%, aggregate investment decreases by 0.175% and 0.140% in the model and the data. The negative effect of the fragility index on the aggregate investment growth is due to the lack of lumpy investments from large firms. In Table D.6, I report the full regression results under different fragility indices and different specifications. In the fourth column of the full table, when the output shock is the only independent variable in the regression, around 51% of the investment growth rate variation is explained. Once the fragility fluctuation is considered, R^2 increases to 63%.

Using the estimate from the data in 8, I quantify the portion of the investment growth rate that is accounted for by the fluctuations in the fragility index. From the total standard deviation comparison, around 36% of aggregate investment volatility can be explained by the fragility fluctuations ($0.36 \approxeq 0.022/0.060$). Table 9 compares the investment growth rate and the fragility-adjusted investment growth rate in the recent three recessions of the sample period. The fragility is adjusted by deducting the predicted variation by the fragility index from the investment growth rate. In stark contrast to the other recessions, the recession in 2001 was greatly explained by the fragility fluctuations. Without the fragility fluctuations, the investment growth rate is mitigated to -4.340% instead of 7.627%, which is around a 43%

Table 8: Regressions of investment growth rate on fragility indices

	Dependent variable: $\Delta log(I_t)$	
	Model	Data
$OutputShock_t$	2.868	3.231
	(0.025)	(0.477)
$log(Fragility_t)$	-0.175	-0.140
	(0.005)	(0.047)
Constant	Yes	Yes
Observations	1,001	32
R^2	0.936	0.628
Adjusted R^2	0.935	0.602

Notes: The dependent variable is the growth rate of aggregate investment. The independent variables are TFP shocks obtained from fitting TFP series into AR(1) process, log of lagged fragility indices, and the growth rates of the fragility indices. The first column reports the regression coefficients from the simulated data. The fragility index is based on the years from the last lumpy investment of large firms. The second column reports the regression coefficients using a measure based on the years from the last lumpy investment of large firms in Compustat data. Output shock process is from NIPA. The numbers in the brackets are standard errors.

deduction in terms of magnitude. This is consistent with the well-known facts around the dot-com bubble crash, which caused the recession in 2001. Just before the crash, a great fraction of firms had jumped into a large-scale investment. This surge of lumpy investments increased the U.S. economy's fragility in the subsequent period.

Table 9: Investment growth rates during the recessions

	Investment growth rate (%):$\Delta log(I_t)$		
	Raw data (NIPA)	Fragility-adjusted	Adjusted portion (%)
Recession-1991	-2.140	-1.889	11.729
Recession-2001	-7.627	-4.340	43.097
Recession-2009	-16.359	-16.551	-1.174

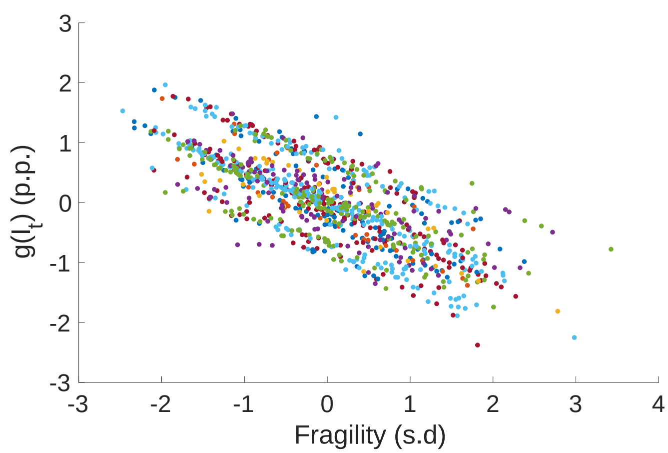
Notes: The first column reports the investment growth rate (%) at recession years of 1991, 2001, and 2009. The second column reports the adjusted investment growth rate after removing the predicted component from the fragility indices using the coefficients of the third column in Table 8. The third column reports the adjusted portion (%).

The recession in 1991 was the mildest recession among the recent recessions, and the component explained by fragility is not large (11.7%). The recession in 2009 is different from the others as the fragility index does not even predict the drop in investment growth. Despite the sharp rise in the fraction of lumpy investments in the prior years to the event, the fragility index did not rise much because the rise was concentrated into only a short period.

In contrast, the drop in the investment rate was the largest among the recent three recessions. There are two possible explanations for this inconsistency. The first is that the nature of the aggregate shock in 2009 was different from the ones in the prior recessions. The shock in 2009 originated from the financial sector, and the first-hand effect was likely through the financial constraint. Therefore, the firms most affected were small firms (Fort et al., 2013). As fragility fluctuations affect the aggregate investment through large firms' lumpy investments, it cannot sufficiently explain financial crisis. Second, the financial crisis was the largest recession after the Great Depression before the recent Covid recessions. Therefore, the magnitude of the exogenous shock might have been greater than the ones in the prior recessions.

To quantify the extra-response of aggregate investment coming from the fragility fluctuations at each time on the business cycle, I hit the economy with an unexpected one-standard-deviation TFP shock and compute the contemporaneous response under the general equilibrium. Figure 7 shows the state-dependent contemporaneous responses of aggregate investment.[35] The horizontal axis is the fragility index normalized by the standard deviation. The vertical axis is the deviation of aggregate investment response from the response at the steady-state in a percentage point. The prior aggregate shock A_{t-1} is controlled by teasing out the fixed effect, and the different colors of dots represent the different fixed-effect groups. Here the fragility index at t is a state of the economy at t as the index is determined before the beginning of period t.

Figure 7: State-dependent instantaneous responses to a negative aggregate TFP shock



Notes: The vertical axis of the scatter plot is the instantaneous response of aggregate investment to a negative one-standard-deviation TFP shock in percentage deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. In each responses, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

[35]Figure 2 can be understood as a data counterpart of this figure, as the residualized investment variation increases in the average of the recent spike ratio of large firms.

As can be seen from the figure, there is a significant negative relationship between the contemporaneous response of aggregate investment and the fragility index. By fitting the relationship into linear regression, I obtain the following result:

$$g(I_t)\ (p.p.) = \underset{(0.0151)}{-\,0.5605} * Fragility_t\ (s.d.) + \epsilon_t, \quad R^2 = 0.580$$

When the fragility index increases by one standard deviation, the contemporaneous response of the aggregate investment to the negative one-standard-deviation shock decreases by 0.56 percentage points. This result shows that the fragility fluctuations amplify the productivity-driven aggregate fluctuations.

5.4 Policy implication: State-dependent interest elasticity of aggregate investment

In this section, I discuss the policy implications of the fluctuations of the fragility index over the business cycle. In the economy captured in the baseline model, the aggregate investment features a strong history-dependence. This history-dependence not only affects the aggregate investment's response to the TFP shock but affects its elasticity to the interest rate change.

To study how the aggregate investment responds differently to the same interest shock depending on the fragility state, I hit the economy with an unexpected interest rate shock and compute the contemporaneous response under the partial equilibrium.[36] In particular, I compare the contemporaneous average change in the investment when the interest rate unexpectedly changes and returns immediately in the subsequent period to the level where the interest is supposed to be without the exogenous shock. The benchmark investment level is the contemporaneous investment when the interest rate is assumed to be staying at the same level. I calculate the average between the elasticity measured when the interest rate increases by 1% and the one measured when the interest rate drops by 1% to address the asymmetry in the responses to the positive and negative interest rate shocks.[37]

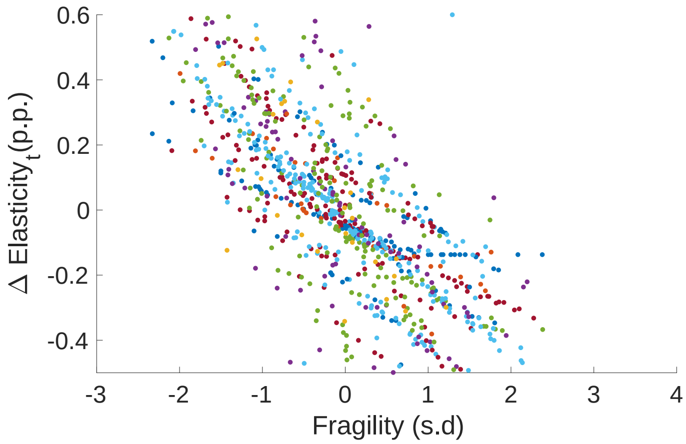
Figure 8 is the scatter plot of the interest elasticities of the aggregate investment in relation to the fragility state. The horizontal axis is the fragility index normalized by the standard deviation; the vertical axis is the interest elasticity in percentage point deviation from the steady-state.[38] According to the figure, there is a significant negative relationship between the fragility and the interest elasticity of aggregate investment. By fitting the relationship

[36] Therefore, the analysis is measuring the semi-elasticity of investment at each timing on the business cycle.

[37] For example, if the interest is 0.03 at period t, I first compute the firm-level investment in three cases: i) when the interest rate jumps up to 0.04 only in period t and then stays in 0.03; ii) when the interest rate drops down to 0.02 only in period t and then stays in 0.03; iii) when the interest rate stays at 0.03 forever. Then, I obtain the mean of the investment difference between case iii) and case i) and the investment difference between case iii) and case ii).

[38] The prior aggregate shock A_{t-1} is controlled by teasing out the fixed effect, and the different colors of dots represent the different fixed-effect groups.

Figure 8: State-dependent semi-elasticities of aggregate investment



Notes: The vertical axis of the scatter plot is the semi-elasticity of aggregate investment in percentage point deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. For each elasticity, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

into linear regression, I obtain the following result:

$$\underset{}{\Delta Elasticity_t \ (p.p) = } \underset{(0.0086)}{-\,0.2689} * Fragility_t \ (s.d) + \epsilon_t, \quad R^2 = 0.497$$

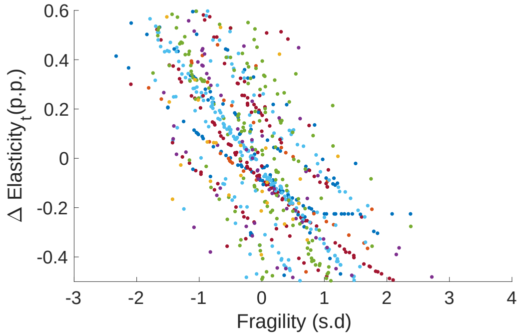
One standard deviation increase in the fragility index decreases the interest elasticity of aggregate investment by around 0.27 percentage points. The intuitive explanation for the result is that when the fragility index is high, there are not many large firms that can flexibly participate in and out of the large-scale investment. This decreases the interest elasticity of aggregate investment in a high fragility state.

To verify the interest elasticity fluctuations in the aggregate investment are driven by large firms, I separately compute the interest elasticities of large and small firms' investments. Figure 9 is the scatter plots of interest elasticities along with the fragility variation for large (panel (a)) and small firms (panel (b)). The negative relationship the fragility index and the elasticity is significantly stronger in large firms. When two different elasticities are fitted into linear regression, the following relationship is obtained:

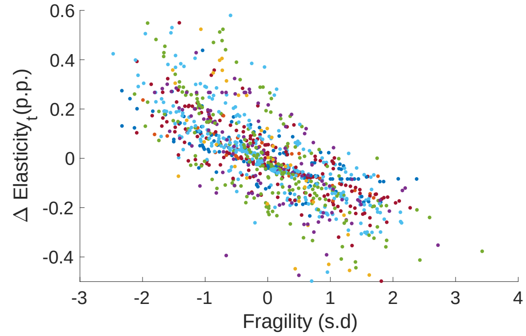
$$\Delta Elasticity_t^{Large} \ (p.p) = \underset{(0.0130)}{-\,0.3992} * Fragility_t \ (s.d) + \epsilon_t, \quad R^2 = 0.484$$

$$\Delta Elasticity_t^{Small} \ (p.p) = \underset{(0.0039)}{-\,0.1403} * Fragility_t \ (s.d) + \epsilon_t, \quad R^2 = 0.569$$

Figure 9: State-dependent semi-elasticities of investments: Decomposition



(a) Large firms



(b) Small firms

Notes: The vertical axis of the scatter plots is the semi-elasticity of large (panel (a)) and small (panel (b)) firms' investment in percentage point deviation from the average, and the horizontal axis is the fragility index in the standard deviation from the average. For each elasticity, contemporaneous and one-period-prior aggregate TFP fixed effects are controlled. Using the histogram method in Young (2010), firms are simulated for 1,000 periods (years) based on the dynamic stochastic general equilibrium allocations. The fragility indices are calculated based on the distribution of large firms.

When the fragility index increases by one standard deviation, large firms' investment elasticity decreases by around 0.40 percentage points. On the other hand, the same variation in the fragility index decreases small firms' elasticity by 0.14 percentage points, and the difference is statistically significant. The time-series correlation between the elasticities of the aggregate investment and the large firms' elasticities is 0.99. This result shows that large firms dominantly drive the stark negative relationship between the average interest elasticities (of all firms) and the fragility index. Although large firms are interest-inelastic, the time-series variation in their interest elasticities is greater than those of small firms. This is because large firms' responses are highly state-dependent, while small firms are flexible to adjust at all times, possibly due to their small fixed adjustment cost.

An important policy implication of the simulated result is that if the fragility index is high, the monetary policy would not effectively operate through the firm-level investment channel. Given there were recessions in the recent periods that happened in the time of high fragility, the policy implication echoes Tenreyro and Thwaites (2016) that monetary policies are less powerful during recessions. However, this paper adds to the findings by suggesting an endogenous mechanism of state dependence in monetary policy effectiveness. And importantly, the fragility index is a forward-looking variable and can be measured in a timely manner using readily observable large firms' data. Therefore, the fragility index can contribute to the optimal monetary policy design in the practical margin.

6 Conclusion

This paper develops a heterogeneous-firm real business cycle model in which the cross-section of the semi-elasticities of firm-level investment is matched with the empirical estimates. I theoretically and quantitatively point out that the cross-sectional ranking of the interest elasticities of investment between large and small firms is counterfactually flipped in existing model frameworks. Then, I incorporate a size-dependent fixed adjustment cost along with the convex adjustment cost into the model, which is conceptually based on the aggregated fixed adjustment cost at the production line level. These two adjustment costs help the model capture the cross-section of the elasticities consistent with the empirical estimates.

In the calibrated model, I show that the timings of large firms' lumpy investments are significantly synchronized following a negative TFP shock due to their low elasticity to the general equilibrium effect. This pattern is consistent with the firm-level data, as there have been synchronized lumpy investments of large firms in the post-shock periods. Then, from the state-dependent variation in the contemporaneous responses to the same negative aggregate TFP shock, I conclude that TFP-induced recessions are especially severe after a surge of large firms' lumpy investments. Also, the model features significant state-dependence in the interest elasticities driven by fragility index fluctuations. This implies that after a synchronized lumpy investment of large firms, the effectiveness of monetary policy falls due to the inelastic aggregate investment.

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A Appendix: Tables and figures

A.1 Conditional heteroskedasticity: Regression result

Table A.1: Residual volatility of aggregate investment and spike ratios

	Dependent variable: $log(\hat{\sigma}_t)$	
	Large	Non-large
$\overline{spike}_{t-1}$ (%)	0.337	0.077
	(0.138)	(0.074)
Constant	−4.131	−2.317
	(1.290)	(1.270)
Observations	35	35
R^2	0.154	0.032
Adjusted R^2	0.128	0.002

Notes: The dependent variable is the logged absolute value of the residuals from fitting the aggregate investment to capital ratio into AR(4) process. The independent variables are the past average spike ratio, $\overline{spike}_{t-1}$, and the intercept.

$\overline{spike}_{t-1}$ is defined as follows:

$$\overline{spike}_{t-1} := \frac{1}{J}\sum_{j=0}^{J-1} SpikeRatio_{t-1-j}$$

$$SpikeRatio_t := \frac{\#\text{Extensive margin adjustment}_t}{\#\text{Firms}_t}$$

where J is the number of past years to be included in the average. In the reported result, I use $J = 3$. The result is robust over $J = 1, 2, 4$.

B A theory of the interest elasticity and the firm size: The cross-section of interest elasticity

B.1 A model with convex adjustment cost : Propositions

Proposition 1 (Size-monotonicity in the interest elasticity).
Given $\mu > 0$, the following inequalities holds:

$$\begin{aligned}
(i) \quad & \frac{\partial}{\partial k}\left(\frac{\partial k^*}{\partial q}\right) > 0 \; for \; \forall k > 0 \\
(ii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial log k^*}{\partial q}\right) > 0 \; for \; \forall k > 0 \\
(iii) \quad & \frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right) > 0 \; for \; \forall k > 0 \\
(iv) \quad & \frac{\partial}{\partial k}\left(\frac{\partial log I^*}{\partial q}\right) > 0 \; if \; I^* > 0.
\end{aligned}$$

Proof.

$$log\left(1 + \mu\left(\frac{I^*}{k}\right)\right) = log\left(q\mathbb{E}z'\alpha\right) + (\alpha - 1)log\left((1-\delta)k + I^*\right)$$

The equation above holds for all possible k and q. I take a partial derivative with respect to q for both sides of the equation.

$$\left(\frac{\mu}{k + \mu I^*}\right)\frac{\partial I^*}{\partial q} = \frac{1}{q} + (\alpha - 1)\frac{1}{(1-\delta)k + I^*}\frac{\partial I^*}{\partial q}$$

Rearranging the terms, I get the following equations:

$$\begin{aligned}
\left(\frac{\mu}{k + \mu I^*} + \frac{1-\alpha}{(1-\delta)k + I^*}\right)\frac{\partial I^*}{\partial q} &= \frac{1}{q} \\
\left(\frac{\mu}{k + \mu I^*} + \frac{1-\alpha}{k^*}\right)\frac{\partial I^*}{\partial q} &= \frac{1}{q} \qquad (6)
\end{aligned}$$

where $k^* = (1-\delta)k + I^*$. Then, I take a log for both sides.

$$log\left(\frac{\mu}{k + \mu I^*} + \frac{1-\alpha}{k^*}\right) + log\left(\frac{\partial I^*}{\partial q}\right) = -log(q)$$

The equation above holds for all possible k and q. I take a partial derivative with respect to

k for both sides of the equation.

$$\frac{1}{\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}}\left(-\frac{\mu}{(k+\mu I^*)^2}\left(1+\mu\frac{\partial I^*}{\partial k}\right)-\frac{1-\alpha}{k^{*2}}\frac{\partial k^*}{\partial k}\right)+\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)=0$$

Therefore,

$$\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)=\frac{1}{\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}}\left(\frac{\mu}{(k+\mu I^*)^2}\left(1+\mu\frac{\partial I^*}{\partial k}\right)+\frac{1-\alpha}{k^{*2}}\frac{\partial k^*}{\partial k}\right).$$

Due to Lemma 1, all the terms on the right-hand side are positive except for $\left(1+\mu\frac{\partial I^*}{\partial k}\right)$. Thus, the following statement holds:

$$\left(1+\mu\frac{\partial I^*}{\partial k}\right)>0 \implies \frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)>0.$$

Going back to the inter-temporal optimality condition, I multiply k in the both sides to have

$$k+\mu I^*=q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k.$$

Then, I take a log and a partial derivative with respective to k. It leads to

$$\begin{aligned}\frac{1+\mu\frac{\partial I^*}{\partial k}}{k+\mu I^*}&=\frac{(\alpha-1)}{k^*}\frac{\partial k^*}{\partial k}+\frac{1}{k}\\&=\frac{1}{k}\left((\alpha-1)\frac{k}{k^*}\frac{\partial k^*}{\partial k}+1\right)\\&=\frac{1}{k}\left(1-(1-\alpha)\frac{\partial logk^*}{\partial logk}\right).\end{aligned}$$

From Lemma 2, $\frac{\partial logk^*}{\partial logk}<1$. Also I assume $\alpha<1$. Therefore, the right-hand side is positive. The denominator on the left-hand side is also positive because $k+\mu I^*=q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k>0$. Therefore, $\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)>0$. Then,

$$(iii)\quad \frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right)=\left(\frac{\partial I^*}{\partial q}\right)\frac{\partial}{\partial k}log\left(\frac{\partial I^*}{\partial q}\right)>0.$$

The right-hand side is positive because $\frac{\partial I^*}{\partial q}>0$ from equation (6). This result is formally stated in Lemma 3. As $\frac{\partial I^*}{\partial q}=\frac{\partial k^*}{\partial q}$, I conclude (i) $\frac{\partial}{\partial k}\left(\frac{\partial k^*}{\partial q}\right)>0$ for $\forall k>0$.

Now I will prove (ii) $\frac{\partial}{\partial k\partial q}log(k^*)>0$ and (iv) $\frac{\partial}{\partial k\partial q}log(I^*)>0$.

From Equation (6), the following is true:

$$\left(\frac{\mu}{k+\mu I^*}k^*+\frac{1-\alpha}{k^*}k^*\right)\frac{1}{k^*}\frac{\partial k^*}{\partial q}=\frac{1}{q}$$

As $\frac{\partial}{\partial k}\left(\frac{\partial log(k^*)}{\partial q}\right) = \frac{\partial}{\partial k}\frac{1}{k^*}\left(\frac{\partial k^*}{\partial q}\right)$,

$$\left(\frac{\mu}{\frac{k}{k^*} + \mu\frac{I^*}{k^*}} + 1 - \alpha\right)\frac{\partial log(k^*)}{\partial q} = \frac{1}{q}.$$

From $I^* = k^* - (1-\delta)k$,

$$\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*} + \mu} + 1 - \alpha\right)\frac{\partial log(k^*)}{\partial q} = \frac{1}{q}.$$

I take the partial derivatives with respect to k on both sides.

$$\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*} + \mu} + 1 - \alpha\right)\frac{\partial log(k^*)}{\partial q} + \left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*} + \mu} + 1 - \alpha\right)\frac{\partial}{\partial k}\frac{\partial log(k^*)}{\partial q} = 0.$$

By rearranging the terms, I obtain

$$\underbrace{\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*} + \mu} + 1 - \alpha\right)}_{>0}\frac{\partial}{\partial k}\frac{\partial log(k^*)}{\partial q} = -\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*} + \mu} + 1 - \alpha\right)\underbrace{\frac{\partial log(k^*)}{\partial q}}_{>0}.$$

From Lemma 3, $\frac{\partial log(k^*)}{\partial q} = \frac{1}{k^*}\frac{\partial k^*}{\partial q} = \frac{1}{k^*}\frac{\partial I^*}{\partial q} > 0$. Also, $\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu} + 1 - \alpha\right) > 0$. Therefore, the sign of $\frac{\partial}{\partial k}\frac{\partial log(k^*)}{\partial q}$ is equal to the sign of $-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu} + 1 - \alpha\right)$. Then, I investigate the sign of $-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu} + 1 - \alpha\right)$ as follows:

$$-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k-\mu(1-\delta)}{k^*}+\mu}+1-\alpha\right) = \left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)\left(\frac{1}{k^*}-\frac{(k-\mu(1-\delta))\frac{\partial k^*}{\partial k}}{(k^*)^2}\right)$$

$$= \left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)\frac{1}{k^*}\left(1-\frac{(k-\mu(1-\delta))\frac{\partial k^*}{\partial k}}{k^*}\right)$$

$$= \left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)\frac{1}{k^*}\left(1-\left(1-\mu(1-\delta)\frac{1}{k}\right)\frac{k}{k^*}\frac{\partial k^*}{\partial k}\right)$$

$$= \underbrace{\left(\frac{\mu}{\left(\frac{k-\mu(1-\delta)}{k^*}+\mu\right)^2}\right)\frac{1}{k^*}}_{>0}\underbrace{\left(1-\overbrace{\left(1-\mu(1-\delta)\frac{1}{k}\right)}^{<1}\overbrace{\frac{\partial logk^*}{\partial logk}}^{>0,<1}\right)}_{>0} > 0.$$

From Lemma 1 and Lemma 2, $0 < \frac{\partial logk^*}{\partial logk} < 1$. Thus,

$$(ii) \quad \frac{\partial}{\partial k}\frac{\partial log(k^*)}{\partial q} > 0.$$

Similarly, we can derive the following equation from Equation (6),

$$\underbrace{\left(\frac{\mu}{\frac{k}{I^*}+\mu}+(1-\alpha)\frac{I^*}{k^*}\right)}_{>0}\frac{\partial}{\partial k}\frac{\partial log(I^*)}{\partial q} = -\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k}{I^*}+\mu}+(1-\alpha)\frac{I^*}{k^*}\right)\underbrace{\frac{\partial log(I^*)}{\partial q}}_{>0}.$$

As $I^* > 0$, $\frac{\partial log(I^*)}{\partial q} = \frac{1}{I^*}\frac{\partial I^*}{\partial q} > 0$ from Lemma 3. And $\left(\frac{\mu}{\frac{k}{I^*}+\mu}+(1-\alpha)\frac{I^*}{k^*}\right) > 0$, as $I^* > 0$.

Thus, the sign of $\frac{\partial}{\partial k}\frac{\partial log(I^*)}{\partial q}$ is equal to the sign of $-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k}{I^*}+\mu}+(1-\alpha)\frac{I^*}{k^*}\right)$.

$$
\begin{aligned}
-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k}{I^*}+\mu}+(1-\alpha)\frac{I^*}{k^*}\right) &= -\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k}{I^*}+\mu}+\frac{(1-\alpha)}{1+\frac{(1-\delta)k}{I^*}}\right) \\
&= \left(\frac{\mu}{\left(\frac{k}{I^*}+\mu\right)^2}\right)\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right)+\frac{1-\alpha}{\left(1+\frac{(1-\delta)k}{I^*}\right)^2}(1-\delta)\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right) \\
&= \underbrace{\left(\left(\frac{\mu}{\left(\frac{k}{I^*}+\mu\right)^2}\right)+\frac{1-\alpha}{\left(1+\frac{(1-\delta)k}{I^*}\right)^2}(1-\delta)\right)}_{>0}\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right)
\end{aligned}
$$

And we can drive the sign of $\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right)$ as follows:

$$
\begin{aligned}
\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right) &= \frac{1}{I^*}\left(1-\frac{k}{I^*}\frac{\partial I^*}{\partial k}\right) \\
&= \frac{1}{I^*}\left(1-\frac{k}{I^*}\left(\frac{\partial k^*}{\partial k}-(1-\delta)\right)\right) \\
&> \frac{1}{I^*}\left(1-\frac{k}{I^*}\left(\frac{k^*}{k}-(1-\delta)\right)\right) \quad \left(\because \frac{\partial logk^*}{\partial logk}<1,\ \text{Lemma } 2\right) \\
&= \frac{1}{I^*}\left(1-\frac{k}{I^*}\left(\frac{I^*}{k}\right)\right) = 0
\end{aligned}
$$

Thus, $\left(\frac{\partial}{\partial k}\frac{k}{I^*}\right)>0$, so $-\frac{\partial}{\partial k}\left(\frac{\mu}{\frac{k}{I^*}+\mu}+(1-\alpha)\frac{I^*}{k^*}\right)>0$. Therefore,

$$
(iv) \quad \frac{\partial}{\partial k}\left(\frac{\partial logI^*}{\partial q}\right)>0 \text{ if } I^*>0.
$$

■

Proposition 2 (Elasticity dampening effect)**.**
Given $\mu>0$, if $I^*>0$, the following statements hold:

$$
\begin{aligned}
(i) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial k^*}{\partial q}\right)<0 \\
(ii) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial logk^*}{\partial q}\right)<0 \\
(iii) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial I^*}{\partial q}\right)<0 \\
(iv) &\quad \frac{\partial}{\partial \mu}\left(\frac{\partial logI^*}{\partial q}\right)\begin{cases}\leq 0 & \text{if } \frac{1}{1-\delta}\geq\mu \\ >0 & \text{if } \frac{1}{1-\delta}<\mu\end{cases}.
\end{aligned}
$$

Proof.
Taking partial derivative with respect to μ on Equation (6), I obtain

$$\underbrace{\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)}_{>0}\frac{\partial}{\partial\mu}\frac{\partial I^*}{\partial q}=-\frac{\partial}{\partial\mu}\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)\underbrace{\frac{\partial I^*}{\partial q}}_{>0}.$$

From Lemma 3, $\frac{\partial I^*}{\partial q}>0$. And $\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)>0$, as $k+\mu I^*=q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k>0$. Thus, the sign of $\frac{\partial}{\partial\mu}\frac{\partial I^*}{\partial q}$ is equal to the sign of $-\frac{\partial}{\partial\mu}\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)$.

$$\begin{aligned}-\frac{\partial}{\partial\mu}\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)&=-\left(\frac{k+\mu I^*-\mu\left(I^*+\mu\frac{\partial I^*}{\partial\mu}\right)}{(k+\mu I^*)^2}+(1-\alpha)\frac{-\frac{\partial k^*}{\partial\mu}}{(k^*)^2}\right)\\&=-\underbrace{\frac{k-\mu\frac{\partial I^*}{\partial\mu}}{(k+\mu I^*)^2}}_{>0}+\underbrace{\frac{(1-\alpha)}{(k^*)^2}\frac{\partial k^*}{\partial\mu}}_{<0}<0\end{aligned}$$

From Lemma 4, $\frac{\partial I^*}{\partial\mu}=\frac{\partial k^*}{\partial\mu}<0$. Thus the first term is positive and the second term is negative. Thus, the sign of the left-hand side is negative. Therefore, (i) and (iii) are proved.

$$\begin{aligned}&(i)\quad\frac{\partial}{\partial\mu}\left(\frac{\partial k^*}{\partial q}\right)<0\\&(iii)\quad\frac{\partial}{\partial\mu}\left(\frac{\partial I^*}{\partial q}\right)<0\end{aligned}$$

From the similar logic, the sign of $\frac{\partial}{\partial \mu}\left(\frac{\partial logk^*}{\partial q}\right)$ is equivalent to the sign of $-\frac{\partial}{\partial \mu}\left(\frac{\mu k^*}{k+\mu I^*}\right)$.

$$
\begin{aligned}
-\frac{\partial}{\partial \mu}\left(\frac{\mu k^*}{k+\mu I^*}\right) &= -\left(\frac{(\mu\frac{\partial k^*}{\partial \mu}+k^*)(k+\mu I^*)-\mu k^*\left(\mu\frac{\partial k^*}{\partial \mu}+I^*\right)}{(k+\mu I^*)^2}\right) \\
&= -\left(\frac{k^*k+k\mu\frac{\partial k^*}{\partial \mu}+\mu^2\frac{\partial k^*}{\partial \mu}(I^*-k^*)}{(k+\mu I^*)^2}\right) \\
&= -\left(\frac{k^2\left((1-\delta)+\frac{I^*}{k}+\frac{\mu}{k}\frac{\partial I^*}{\partial \mu}\right)-\mu^2\overbrace{\frac{\partial k^*}{\partial \mu}}^{<0\ (\because \text{Lemma } 4)}(1-\delta)k}{(k+\mu I^*)^2}\right) \\
&< -\left(\frac{k^2\left(\frac{I^*}{k}+\frac{\mu}{k}\frac{\partial I^*}{\partial \mu}\right)}{(k+\mu I^*)^2}\right) < 0
\end{aligned}
$$

The last inequality holds because $\frac{I^*}{k}+\frac{\mu}{k}\frac{\partial I^*}{\partial \mu} = \alpha(\alpha-1)q\mathbb{E}z'(k^*)^{\alpha-2}\frac{\partial k^*}{\partial \mu} > 0$, which is obtained from taking a partial derivative with respect to μ on the first-order optimality condition. Therefore, (ii) is proved.

$$
(ii) \quad \frac{\partial}{\partial \mu}\left(\frac{\partial logk^*}{\partial q}\right) < 0
$$

From the similar logic, the sign of $\frac{\partial}{\partial \mu}\left(\frac{\partial logI^*}{\partial q}\right)$ is equal to the sign of $-\frac{\partial}{\partial \mu}\left(\frac{\mu I^*}{k+\mu I^*}+(1-\alpha)\frac{I^*}{k^*}\right)$.

$$
\begin{aligned}
&-\frac{\partial}{\partial \mu}\left(\frac{\mu I^*}{k+\mu I^*}+(1-\alpha)\frac{I^*}{k^*}\right) \\
&= -\left(\frac{(\mu\frac{\partial I^*}{\partial \mu}+I^*)(k+\mu I^*)-\mu I^*\left(\mu\frac{\partial k^*}{\partial \mu}+I^*\right)}{(k+\mu I^*)^2}+\frac{1-\alpha}{(k^*)^2}\left(k^*\frac{\partial I^*}{\partial \mu}-I^*\frac{\partial k^*}{\partial \mu}\right)\right) \\
&= -\left(\frac{(\frac{\mu}{k}\frac{\partial I^*}{\partial \mu}+\frac{I^*}{k})k^2}{(k+\mu I^*)^2}+\frac{1-\alpha}{(k^*)^2}\left(\frac{\partial k^*}{\partial \mu}\right)(1-\delta)k\right) \\
&= -\left(\frac{(\alpha(\alpha-1)q\mathbb{E}z'(k^*)^{\alpha-2}\frac{\partial k^*}{\partial \mu})k^2}{(k+\mu I^*)^2}+\frac{1-\alpha}{(k^*)^2}\left(\frac{\partial k^*}{\partial \mu}\right)(1-\delta)k\right) \\
&= -\frac{1-\alpha}{(k^*)^2}\left(-\frac{(\alpha q\mathbb{E}z'(k^*)^{\alpha})k^2}{(k+\mu I^*)^2}+(1-\delta)k\right)\left(\frac{\partial k^*}{\partial \mu}\right)
\end{aligned}
$$

From the first-order condition $\alpha q\mathbb{E}z'(k^*)^{\alpha-1} = 1+\mu\left(\frac{I^*}{k}\right)$. Substituting this into the equation above, I obtain

$$
\begin{aligned}
-\frac{\partial}{\partial \mu}\left(\frac{\mu I^*}{k+\mu I^*}+(1-\alpha)\frac{I^*}{k^*}\right) &= -\frac{1-\alpha}{(k^*)^2}\left(-\frac{(1+\mu\left(\frac{I^*}{k}\right))k^*k^2}{(k+\mu I^*)^2}+(1-\delta)k\right)\left(\frac{\partial k^*}{\partial \mu}\right) \\
&= -\frac{1-\alpha}{(k^*)^2}k\left(-\frac{(k+\mu I^*)k^*}{(k+\mu I^*)^2}+(1-\delta)\right)\left(\frac{\partial k^*}{\partial \mu}\right) \\
&= \frac{1-\alpha}{(k^*)^2}k\left(\frac{k^*}{k+\mu I^*}-(1-\delta)\right)\left(\frac{\partial k^*}{\partial \mu}\right) \\
&= \frac{1-\alpha}{(k^*)^2}k(k+\mu I^*)\left(k^*-(1-\delta)(k+\mu I^*)\right)\left(\frac{\partial k^*}{\partial \mu}\right) \\
&= \frac{1-\alpha}{(k^*)^2}k(k+\mu I^*)I^*\underbrace{(1-(1-\delta)\mu)}_{(*)}\underbrace{\left(\frac{\partial k^*}{\partial \mu}\right)}_{<0}.
\end{aligned}
$$

Therefore, depending on the sign of the term $(*)$ above, the sign of $\frac{\partial}{\partial \mu}\left(\frac{\partial logI^*}{\partial q}\right)$ is determined.

$$
(iv) \quad \frac{\partial}{\partial \mu}\left(\frac{\partial logI^*}{\partial q}\right)\begin{cases}\leq 0 & \text{if } \frac{1}{1-\delta}\geq \mu \\ >0 & \text{if } \frac{1}{1-\delta}<\mu\end{cases}
$$

■

B.2 A model with convex adjustment cost: Lemmas

Lemma 1 (Size-monotonicity in future capital stock)**.**

$$For\ \forall k > 0,\ \frac{\partial k^*}{\partial k} > 0$$

Proof.
From the inter-temporal optimality condition,

$$1 + \mu\left(\frac{k^*}{k} - (1-\delta)\right) = q\mathbb{E}z'\alpha(k^*)^{\alpha-1}.$$

I take a partial derivative with respect to k:

$$\mu\frac{1}{k}\frac{\partial k^*}{\partial k} - \mu\frac{k^*}{k} = q\mathbb{E}z'\alpha(\alpha-1)((1-\delta)k + I^*)^{\alpha-2}\frac{\partial k^*}{\partial k}.$$

By rearranging the terms,

$$\frac{\partial k^*}{\partial k} = \frac{\mu\frac{k^*}{k}}{\left(\mu\frac{1}{k} - q\mathbb{E}z'\alpha(\alpha-1)((1-\delta)k + I^*)^{\alpha-2}\right)} > 0.$$

The last line is from $q\mathbb{E}z'\alpha(\alpha-1)((1-\delta)k + I^*)^{\alpha-2} < 0$, as $\alpha - 1 < 0$. ■

Lemma 2 (Size-elasticity of future capital stock)**.**

$$For\ \forall k > 0,\ \frac{\partial log(k^*)}{\partial log(k)} < 1$$

Proof.
By taking log in the both sides of the inter-temporal optimality condition,

$$log\left(1 + \mu\left(\frac{k^*}{k} - (1-\delta)\right)\right) = log(q\mathbb{E}z'\alpha(k^*)^{\alpha-1}).$$

Then, I take a partial derivative with respect to $log(k)$ to obtain

$$\frac{\mu\frac{\partial}{\partial logk}\left(\frac{k^*}{k}\right)}{1 + \mu\left(\frac{k^*}{k} - (1-\delta)\right)} = (\alpha-1)\frac{\partial logk^*}{\partial logk}.$$

Thus,

$$\mu\frac{\frac{\partial logk^*}{\partial logk}\frac{k^*}{k} - \frac{k^*}{k}}{1 + \mu\left(\frac{k^*}{k} - (1-\delta)\right)} = (\alpha-1)\frac{\partial logk^*}{\partial logk}.$$

By rearranging terms, I get

$$\left(\frac{\mu}{1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)}\frac{k^*}{k}-(\alpha-1)\right)\frac{\partial logk^*}{\partial logk}=\frac{\mu\frac{k^*}{k}}{1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)}.$$

By multiplying $1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)$, I get

$$\left(\mu\frac{k^*}{k}-(\alpha-1)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right)\right)\frac{\partial logk^*}{\partial logk}=\mu\frac{k^*}{k}.$$

The, it leads to

$$\frac{\partial logk^*}{\partial logk}=\frac{\mu\frac{k^*}{k}}{\mu\frac{k^*}{k}-(\alpha-1)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right)}.$$

From Lemma 1, $\frac{\partial logk^*}{\partial logk}>0$ and $\frac{k^*}{k}>0$. Thus the denominator on the right-hand side is also positive. Therefore, I have the following equivalence:

$$\begin{aligned}\frac{\partial logk^*}{\partial logk}<1 &\iff \mu\frac{k^*}{k}<\mu\frac{k^*}{k}-(\alpha-1)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right)\\ &\iff 0<(1-\alpha)\left(1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\right)\\ &\iff 0<1+\mu\left(\frac{k^*}{k}-(1-\delta)\right)\\ &\iff 0<q\mathbb{E}z'\alpha(k^*)^{\alpha-1}.\end{aligned}$$

Because the last inequality is true, I conclude $\frac{\partial logk^*}{\partial logk}<1$. ■

Lemma 3 (Investment monotonicity in discount factor)**.**

$$\frac{\partial I^*}{\partial q}>0$$

Proof. From Equation (6), I have

$$\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)\frac{\partial I^*}{\partial q}=\frac{1}{q}$$

By rearranging terms, I get

$$\frac{\partial I^*}{\partial q}=\frac{\frac{1}{q}}{\left(\frac{\mu}{k+\mu I^*}+\frac{1-\alpha}{k^*}\right)}.$$

Therefore, the following statement holds:

$$k + \mu I^* > 0 \implies \frac{\partial I^*}{\partial q} > 0.$$

Going back to the inter-temporal optimality condition, I multiply k in the both sides to have

$$k + \mu I^* = q\mathbb{E}z'\alpha(k^*)^{\alpha-1}k > 0.$$

Therefore, $\frac{\partial I^*}{\partial q} > 0$. ■

Lemma 4 (Investment and convex adjustment parameter).
For $\mu > 0$,

$$\frac{\partial I^*}{\partial \mu} = \frac{\partial k^*}{\partial \mu} < 0 \text{ if } I^* > 0.$$

Proof. From the first-order condition,

$$1 + \mu\left(\frac{I^*}{k}\right) = \alpha q\mathbb{E}z'(k^*)^{\alpha-1}$$

Taking a partial derivative w.r.t μ, I obtain

$$\frac{I^*}{k} + \frac{\mu}{k}\frac{\partial I^*}{\partial \mu} = \alpha(1-\alpha)q\mathbb{E}z'(k^*)^{\alpha-2}\frac{\partial k^*}{\partial \mu}.$$

From $I^* = k^* - (1-\delta)k$, $\frac{\partial I^*}{\partial \mu} = \frac{\partial k^*}{\partial \mu}$. Then, by rearranging terms, I get

$$\frac{\partial I^*}{\partial \mu} = \frac{\frac{I^*}{k}}{\left(\underbrace{\alpha(1-\alpha)q\mathbb{E}z'(k^*)^{\alpha-2}}_{<0} - \frac{\mu}{k}\right)} < 0.$$

■

B.3 A model with fixed adjustment cost: Proposition

Proposition 3 (The effect of the firm size and the price on the adjustment probability).
For $\forall k$ *s.t.* $\xi^*(k,q) < \overline{\xi}(q)$,

$$\frac{\partial \psi(k,q)}{\partial k}\frac{\partial \psi(k,q)}{\partial q} < 0 \text{ and } \frac{\partial}{\partial k}\frac{\partial}{\partial q}\psi(k,q) < 0.$$

Proof.
As $\xi^*(k,q) < \overline{\xi}$, $\psi(k,q) = \xi^*(k,q)/\xi$. By taking the cross-derivative with respect to q and k

on $\xi^*(k,q)$, I obtain

$$\frac{\partial^2 \xi^*(k,q)}{\partial q \partial k} = -\alpha \mathbb{E}_z z'((1-\delta)k)^{\alpha-1}(1-\delta) < 0.$$

Thus, $\frac{\partial}{\partial k}\frac{\partial}{\partial q}\psi(k,q) < 0$.

From Proposition 5, $\frac{\partial \xi^*(k,q)}{\partial k} < 0$ for $\forall k < \widehat{k}$, and $\frac{\partial \xi^*(k,q)}{\partial k} > 0$ for $\forall k > \widehat{k}$.

By taking a partial derivative with respect to q on F, I obtain

$$\frac{\partial \xi^*(k,q)}{\partial q} = \mathbb{E}z'(k^*)^{\alpha} - \mathbb{E}z'((1-\delta)k)^{\alpha}.$$

Thus, $\frac{\partial \xi^*(k,q)}{\partial q} > 0$ for $\forall k < \frac{k^*}{(1-\delta)} = \widehat{k}$, and $\frac{\partial \xi^*(k,q)}{\partial q} < 0$ for $\forall k > \frac{k^*}{(1-\delta)} = \widehat{k}$.

Therefore, $\frac{\partial \xi^*(k,q)}{\partial k}$ and $\frac{\partial \xi^*(k,q)}{\partial q}$ always take the opposite sign: $\frac{\partial \xi^*(k,q)}{\partial k}\frac{\partial \xi^*(k,q)}{\partial q} < 0$. And the equality holds when $k = \widehat{k}$. ■

B.4 A model with fixed adjustment cost: Lemmas

Lemma 5 (U-shaped probability of the extensive margin investment)**.**
Given $q > 0$, there uniquely exist $\widehat{k}$ and $\overline{k}$ such that

$$F(\overline{k}, q) = \overline{\xi},\ \ \xi^*(k, q) > \overline{\xi} \text{ for } \forall k > \overline{k},\ \text{ and}$$
$$\frac{\partial F}{\partial k}\Big|_{k=\widehat{k}} = 0$$

Proof.

$$\xi^*(k, q) := -I^* + q\mathbb{E}_z z'((1-\delta)k + I^*)^\alpha - q\mathbb{E}_z z'((1-\delta)k)^\alpha$$

After taking a partial derivative with respect to k, I get the following equation:[39]

$$\frac{\partial \xi^*(k, q)}{\partial k} = (1-\delta) - \alpha q\mathbb{E}_z z'((1-\delta)k)^{\alpha-1}(1-\delta).$$

Then, at $k = \widehat{k} := \frac{(\alpha q\mathbb{E}_z z')^{\frac{1}{1-\alpha}}}{1-\delta}$, $\frac{\partial \xi^*(k,q)}{\partial k}\Big|_{k=\widehat{k}} = 0$. From the first order condition, we can check $(\alpha q\mathbb{E}_z z')^{\frac{1}{1-\alpha}} = k^*$. Therefore, $\widehat{k} = \frac{k^*}{1-\delta}$.

Taking another partial derivative with respect to k, I obtain

$$\frac{\partial^2 \xi^*(k, q)}{\partial k^2} = \alpha(1-\alpha)q\mathbb{E}_z z'((1-\delta)k)^{\alpha-2}(1-\delta)^2 > 0.$$

Thus, for $\forall k < \widehat{k}$, $\frac{\partial \xi^*(k,q)}{\partial k} < 0$, and for $\forall k > \widehat{k}$, $\frac{\partial \xi^*(k,q)}{\partial k} > 0$. Therefore, $\xi^*(k, q) > F(\widehat{k}, q)$, for $\forall k > \widehat{k}$.

Then, I consider a limit case where $k \to \infty$.

$$\begin{aligned}\lim_{k\to\infty} \xi^*(k, q) &= \lim_{k\to\infty} -k^* + (1-\delta)k + q\mathbb{E}_z z'(k^*)^\alpha - q\mathbb{E}_z z'((1-\delta)k)^\alpha \\ &= \lim_{k\to\infty} \left(\frac{1}{\alpha} - 1\right) k^* + \underbrace{(1-\delta)k - q\mathbb{E}_z z'((1-\delta)k)^\alpha}_{\to\infty} \to \infty.\end{aligned}$$

$\xi^*(k, q)$ is continuous. Thus, if $\overline{\xi} \geq F(\widehat{k}, q)$, from the intermediate value theorem, there exists $\overline{k}$ such that $F(\overline{k}) = \overline{\xi} \geq F(\widehat{k}, q)$. Then, $\xi^*(k, q) > \overline{\xi}$ for $\forall k > \overline{k}$. If $\overline{\xi} < F(\widehat{k}, q)$, then, $\xi^*(k, q) > \overline{\xi}$ for $\forall k > 0$. ∎

Lemma 6 (The extensive margin response to interest rate change)**.**
For $\forall k \in (0, \widehat{k}(q))$

$$\frac{\partial}{\partial q}\xi^*(k, q) > 0$$

[39]The first order condition is applied after taking the partial derivative.

Proof.
By taking a partial derivative with respect to q on F, I obtain

$$\frac{\partial \xi^*(k,q)}{\partial q} = \mathbb{E}z'(k^*)^\alpha - \mathbb{E}z'((1-\delta)k)^\alpha.$$

Thus, $\frac{\partial \xi^*(k,q)}{\partial q} > 0$ for $\forall k < \frac{k^*}{(1-\delta)} = \widehat{k}$. ■

Lemma 7 (Size-monotonicity of the interest elasticity in a fixed-cost model)**.**

$$\frac{\partial}{\partial k}\left(\frac{\partial I^*}{\partial q}\right) = 0 \textit{ for } \forall k > 0$$

If $I^* > 0$*, then*

$$\frac{\partial}{\partial k}\left(\frac{\partial log I^*}{\partial q}\right) > 0 \textit{ for } \forall k > 0$$

Proof.
From the first order condition,

$$1 = \alpha q \mathbb{E}_z z'(k^*)^{\alpha-1}.$$

This implies that the future capital stock does not depend on the current size of the firm.

$$\frac{\partial k^*}{\partial k} = 0$$

From $I^* = k^* - (1-\delta)k$, the following equations hold

$$\frac{\partial I^*}{\partial k} = -(1-\delta),$$
$$\frac{\partial log I^*}{\partial k} = -\frac{(1-\delta)}{I^*} \text{ for } I^* > 0.$$

Taking a partial derivative with respect to q,

$$\frac{\partial^2 I^*}{\partial q \partial k} = 0,$$
$$\frac{\partial^2 log I^*}{\partial q \partial k} = \frac{(1-\delta)}{I^{*2}}\frac{\partial I^*}{\partial q} \text{ for } I^* > 0.$$

Going back to the first order condition, the following equation holds after taking the partial derivative with respect to q.

$$0 = \alpha \mathbb{E}z'(k^*)^{\alpha-1} + \alpha(\alpha-1)q\mathbb{E}z'(k^*)^{\alpha-2}\frac{\partial I^*}{\partial q}$$

Thus,

$$\frac{\partial I^*}{\partial q} = \frac{\alpha \mathbb{E} z'(k^*)^{\alpha-1}}{\alpha(1-\alpha) q \mathbb{E} z'(k^*)^{\alpha-2}} > 0.$$

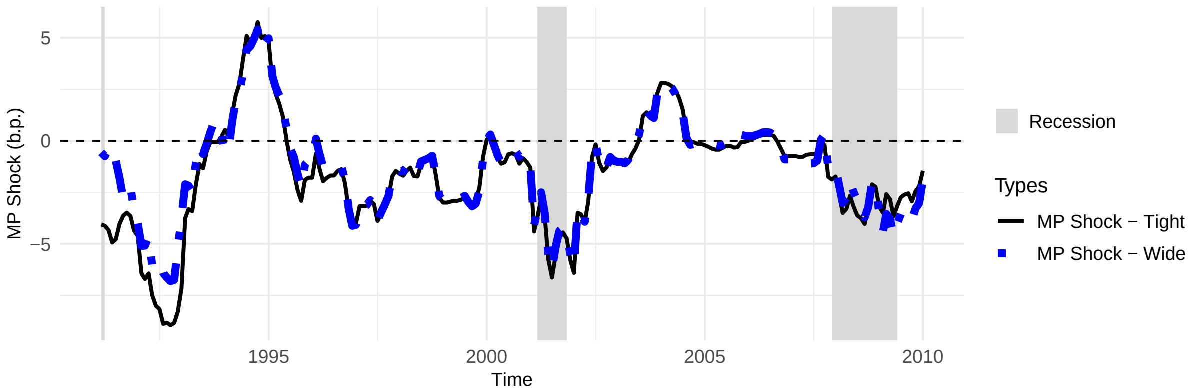
Therefore,

$$\frac{\partial^2 log I^*}{\partial q \partial k} = \frac{(1-\delta)}{I^{*2}} \frac{\partial I^*}{\partial q} > 0 \text{ for } I^* > 0.$$

■

B.5 Monetary policy shock

Figure B.1: One-year moving average monetary policy shock: March 1990 ∼ December 2009



Notes: The monetary policy shocks are obtained by time aggregating high-frequency monetary policy shocks identified from the unexpected jump (drop) in the federal funds rate during 30-minutes (Tight) and one-hour (Wide) windows around the FOMC announcement. To capture the unexpected component in the federal funds rate, I use the change in the rate implied by the current-month federal funds futures contract. All the data on the timings of the FOMC announcement and the high-frequency surprise are from Gurkaynak et al. (2005) and Gorodnichenko and Weber (2016).

B.6 Investment elasticities to the monetary policy shocks: Full tables

Table B.2: Investment sensitivity to the monetary policy shocks with the narrow window

	Dependent variables:									
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.1\}$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$		$log(I_{it}) \mid_{\frac{I_{it}}{k_{it}}>0.1}$		$log(I_{it}) \mid_{\frac{I_{it}}{k_{it}}>0.2}$	
	L	S	L	S	L	S	L	S	L	S
$MP_{Tight,t}$	-2.201	-7.025	-0.656	-2.993	-0.870	-2.072	-0.936	-2.317	-0.246	-3.512
	(0.606)	(2.41)	(0.363)	(0.688)	(0.366)	(0.676)	(0.676)	(1.668)	(0.912)	(2.187)
Obs.	29,400	7,903	29,400	7,903	29,400	7,903	19,524	5,039	11,181	3,643
R^2	0.929	0.791	0.596	0.562	0.603	0.558	0.954	0.865	0.96	0.895
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The dependent variable of the probit regression is the indicator of lumpy investment. The independent variables include monetary policy shocks, fixed effects (sector, year, and sector-year), and firm-level control variables (lagged total debt (DT), lagged current account (ACT), lagged size (AT), and sales (Sale) growth). The numbers in the bracket are the standard errors. The standard errors are clustered two-way by sector and year.

Table B.3: Investment sensitivity to the monetary policy shocks with the wide window

	Dependent variables:									
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.1\}$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$		$log(I_{it}) \mid_{\frac{I_{it}}{k_{it}}>0.1}$		$log(I_{it}) \mid_{\frac{I_{it}}{k_{it}}>0.2}$	
	L	S	L	S	L	S	L	S	L	S
$MP_{Wide,t}$	-2.178	-6.583	-0.643	-2.856	-0.762	-1.870	-0.850	-1.703	-0.333	-3.400
	(0.662)	(2.604)	(0.383)	(0.73)	(0.377)	(0.728)	(0.698)	(1.835)	(0.956)	(2.44)
Obs.	29,400	7,903	29,400	7,903	29,400	7,903	19,524	5,039	11,181	3,643
R^2	0.929	0.791	0.596	0.562	0.603	0.558	0.954	0.865	0.96	0.895
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The dependent variable of the probit regression is the indicator of lumpy investment. The independent variables include monetary policy shocks, fixed effects (sector, year, and sector-year), and firm-level control variables (lagged total debt (DT), lagged current account (ACT), lagged size (AT), and sales (Sale) growth). The numbers in the bracket are the standard errors. The standard errors are clustered two-way by sector and year.

B.7 Fixed parameters

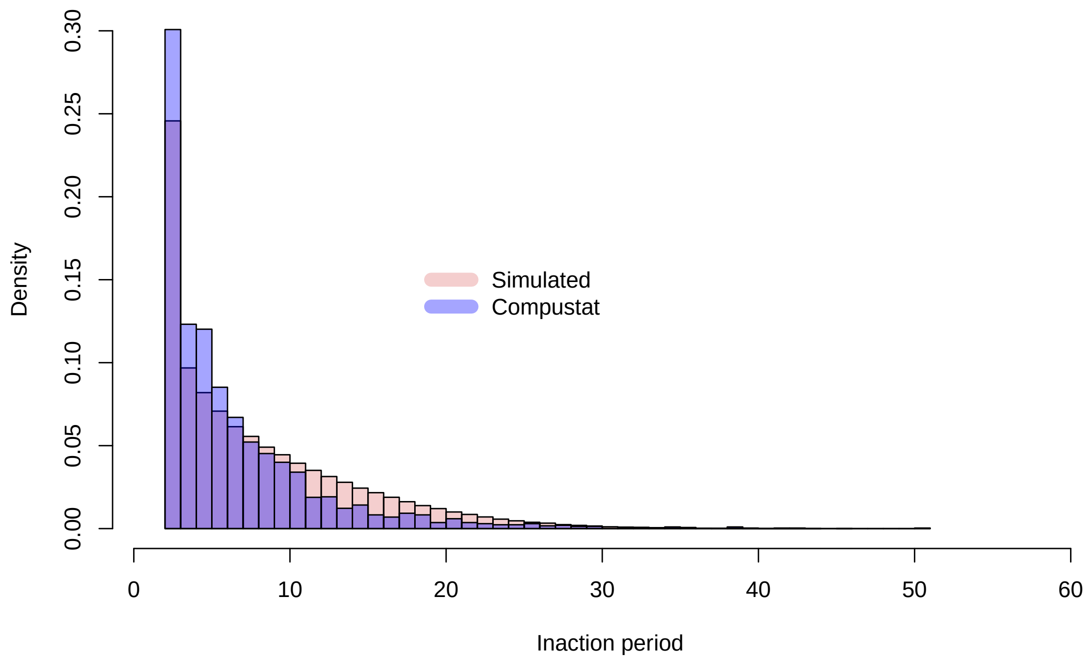
Table B.4: Fixed Parameters

Parameters	Description	Value
Firm-side Fundamentals		
α	Capital share	0.2800
γ	Labor share	0.6400
δ	Depreciation rate	0.0900
Household		
β	Discount factor	0.9770
η	Labor disutility parameter	2.4000
Aggregate TFP Process		
ρ_A	Persistence of aggregate TFP	0.8145

Notes: The fixed parameters are chosen at the level widely used in the relevant literature. The household labor disutility parameter is set at the level where the total labor supply becomes around one-third in the equilibrium. The persistence of aggregate TFP is fixed at 0.8145 following Bachmann et al. (2013).

C Additional model validation

Figure C.2: Distribution of inaction durations



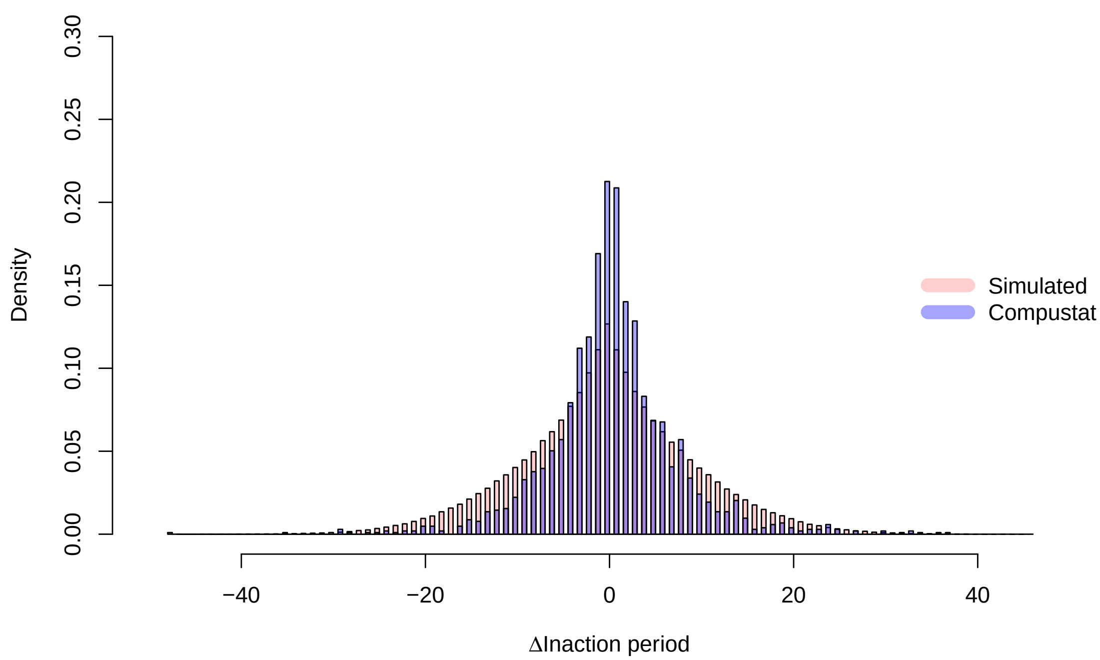
Notes: Based on the stationary equilibrium allocations, 5,000 firms are simulated for 1,000 periods (years). The inaction durations are obtained from the time gap between two neighboring firm-level lumpy investments.

Table C.5: Regression of inaction durations on the lagged terms

	Dependent variable: $log(t2Inv_{i,j})$					
	Compustat			Stationary equilibrium		
	All	Large	Non-large	All	Large	Non-large
$log(t2Inv_{i,j-1})$	0.900	0.908	0.877	0.846	0.864	0.852
$(s.e.)$	(0.012)	(0.014)	(0.023)	(0.001)	(0.002)	(0.001)
Observations	2,070	1,501	569	587,041	59,110	508,841

Notes: Based on the stationary equilibrium, 5,000 firms are simulated for 1,000 periods (years). The inaction durations are obtained from the time gap between two neighboring firm-level lumpy investments. The dependent variable is inaction duration, and the independent variable is the lagged inaction duration from the simulated data.

Figure C.3: Distribution of lag differences of inaction durations



Notes: Based on the stationary equilibrium allocations, 5,000 firms are simulated for 1,000 periods (years). The lag differences of inaction durations are obtained from the difference between two neighboring inaction durations.

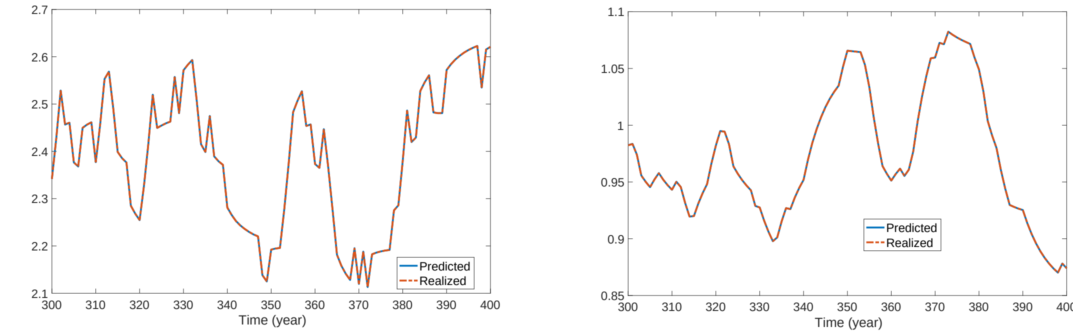
D Regressions of investment growth rate on fragility indices

Table D.6: Regressions of investment growth rate on fragility indices

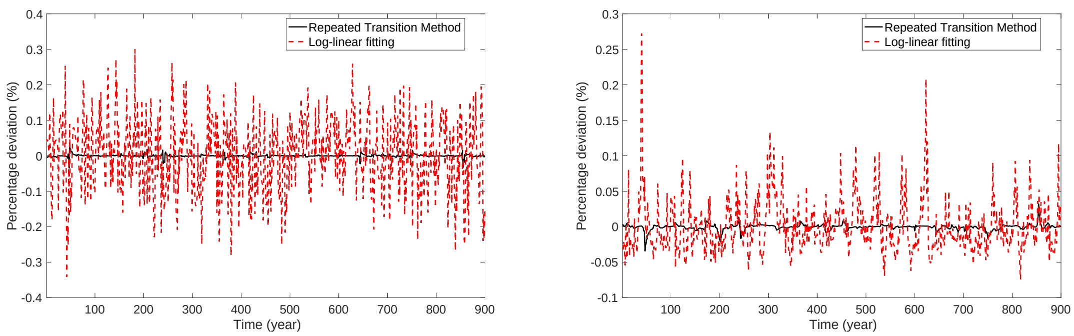
	Dependent variable: $\Delta log(I_t)$							
	Model			Data				
	Yshock	B30	T30	Yshock	T40	T50	T60	T70
$OutputShock_t$	2.873	2.860	2.868	2.846	3.223	3.211	3.231	3.236
	(0.036)	(0.032)	(0.025)	(0.518)	(0.473)	(0.475)	(0.477)	(0.479)
$log(Fragility_t)$		-0.097	-0.175		-0.133	-0.136	-0.140	-0.139
		(0.006)	(0.005)		(0.043)	(0.045)	(0.047)	(0.047)
Constant	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,000	1,001	1,001	31	32	32	32	32
R^2	0.866	0.892	0.936	0.510	0.633	0.629	0.628	0.626
Adjusted R^2	0.865	0.891	0.935	0.493	0.607	0.603	0.602	0.599

Notes: The dependent variable is the growth rate of aggregate investment. The independent variables are output shocks obtained from fitting output series into AR(1) process, log of lagged fragility indices, and the growth rates of the fragility indices. The first column reports the regression coefficients from the simulated data. The fragility index is based on the years from the last lumpy investment of large firms. The second column reports the regression coefficients using a measure based on the years from the last lumpy investment of large firms in Compustat data. TFP process is the output-based productivity from Bureau of Labor Statistics. The numbers in the brackets are standard errors.

Figure E.4: Aggregate fluctuations in the marginal utility and the aggregate capital stock



(a) Marginal utility, $p_t = \frac{1}{c_t}$ (b) Aggregate capital stock K_t



(c) Prediction error in marginal utility (d) Prediction error in aggregate capital stock

Notes: Panel (a) plots the rationally expected path and the simulated path of the marginal utility. Panel (b) plots the rationally expected path and the simulated path of the aggregate capital stock. Panel (c) plots the prediction errors in the marginal utility path from the repeated transition method and the log-linear fitting. Panel (d) plots the prediction errors in the aggregate capital stock path from the repeated transition method and the log-linear fitting.

Table E.7: The fitness comparison across the different law of motions: p_t

	Dependent variables: $log(p_t)$								
	R^2			$max(\|error\|)(\%)$			$mean(\|error\|)(\%)$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
A_1	0.9965	0.9995	0.9999	0.1960	0.0722	0.0393	0.0619	0.0225	0.0098
A_2	0.9951	0.9994	0.9999	0.2613	0.0936	0.0423	0.0756	0.0235	0.0117
A_3	0.9958	0.9993	0.9999	0.2793	0.1394	0.0676	0.0662	0.0263	0.0128
A_4	0.9945	0.9994	0.9999	0.3261	0.0900	0.0468	0.0657	0.0248	0.0115
A_5	0.9966	0.9992	0.9999	0.1954	0.1146	0.0669	0.0532	0.0266	0.0084

Notes: The table reports R^2, the maximum absolute prediction error, and the mean absolute prediction error by different law of motion (columns) and aggregate states (rows). Specification (1) includes a constant and log of contemporaneous capital stock as a independent variable; Specification (2) includes a constant, log of contemporaneous capital stocks, and log of fragility index as independent variables; Specification (3) includes constant and contemporaneous and lagged capital stocks up to three lags in a non-parametric form as independent variables.

Table E.8: The fitness comparison across the different law of motions: K_{t+1}

	Dependent variables: $log(K_{t+1})$								
	R^2			$max(\|error\|)(\%)$			$mean(\|error\|)(\%)$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
A_1	1.0000	1.0000	1.0000	0.0793	0.0785	0.0233	0.0150	0.0141	0.0057
A_2	0.9999	0.9999	1.0000	0.1253	0.1295	0.0402	0.0230	0.0237	0.0082
A_3	0.9999	0.9999	1.0000	0.2286	0.2248	0.0481	0.0210	0.0207	0.0090
A_4	0.9999	0.9999	1.0000	0.2503	0.2508	0.0784	0.0254	0.0244	0.0095
A_5	0.9998	0.9998	1.0000	0.1994	0.1886	0.0409	0.0259	0.0227	0.0076

Notes: The table reports R^2, the maximum absolute prediction error, and the mean absolute prediction error by different law of motion (columns) and aggregate states (rows). Specification (1) includes a constant and log of contemporaneous capital stock as a independent variable; Specification (2) includes a constant, log of contemporaneous capital stocks, and log of fragility index as independent variables; Specification (3) includes constant and contemporaneous and lagged capital stocks up to three lags in a non-parametric form as independent variables.