A Theory of Brand Positioning: Product-Portfolio View

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Abstract

Beyond real functional differences, brand positioning can have profound effects on the purchase decisions of consumers. Using a product-portfolio and consumer search framework, we provide a micro-foundation for why and how brand positioning can deliver credible information to consumers. Consumers form their perceptions of a brand from various interactions with all products under the same brand. We conceptualize brand positioning as the average location of a brand's products on a Hotelling-line. When consumers conduct their search for product matches, they are guided by how brands are positioned in the market. We show that niche brands naturally convey more information than mainstream brands. A firm with a mainstream brand has incentives to opportunistically dilute its brand by offering a wide range of products. A niche brand may arise as an equilibrium even in a monopolistic market because it serves as a commitment device for no dilution.

Keywords: brand positioning, product portfolio, mainstream and niche positioning, consumer search, hold-up problem

1 Introduction

She may not like what we've made that following season, but she'll give us the first look because she is a customer of ours, and it's our job to make sure we know what she'll like, and give her some of that.

– Stuart Weitzman, the founder of Stuart Weitzman

Brand positioning is one of the most critical and fundamental concepts in marketing and firm strategy. How a firm chooses to position itself in the market significantly affects its competitiveness and performance. A carefully crafted brand position can, for example, substantially increase the demand for a firm's products. While the advantages of honing a firm's unique position in the market may seem obvious, it is easy to take brand positioning for granted. Furthermore, despite the importance and practical relevance of brand positioning, the issue has rarely been the subject of rigorous analysis. What exactly is brand positioning? Why is it so valuable? How does it affect consumers' choices?

Marketing literature defines brand positioning as the way in which consumers perceive the brand (Kotler et al. 2014) and the overall view that consumers have of a brand—a view that is often formed by a unique bundle of associations in the minds of target customers (Avery and Gupta 2014). In other words, brand positioning is the particular place that a firm seeks to own. Consumers form their perception of the brand from various interactions with the brand's general line of products (that is, several different products under the same brand), which, taken together, identify and refine a brand's distinctiveness.

A consumer who owns Gucci products, for instance, may have in mind an overall image of Gucci as a brand that produces fashionable and sensual handbags and shoes. This perception is formed by the consumer's experiences with Gucci's products, and, critically, this image of the brand will affect the consumer's future apparel purchase decisions. When instead considering Hermes, which is another popular luxury brand, the consumer may have an entirely different brand image given the way the company has positioned itself: timeless elegance and classic luxury. Both Gucci and Hermes keep their brand positions and identities running consistently throughout their products. By doing so, Gucci and Hermes have effectively differentiated themselves from other luxury brands—consumers who are interested in luxury fashion will seek out Gucci for their next

spring collection without foreknowledge of the exact design or style of each individual product. Consumers who prefer classic styles may search Hermes first.

Brand positioning provides critical information about the characteristics of a firm's products, making it easier for consumers to locate their preferred products and choose where to purchase without blindly searching through several brands to find the right one for them. It is impossible to communicate to consumers what they can expect from a brand without a clear positioning. Brand positioning can thus serve as an efficient communication mechanism that invites consumers to search and patronage.

To formalize this idea, we begin by illuminating the fundamental concept of marketing by examining brand positioning under a product portfolio and consumer search framework. After providing a formal modeling structure on the concept of brand positioning, we explore several different equilibria and investigate the firm's optimal brand positioning strategy. Our equilibrium analysis sheds light on the optimal conditions and economic tradeoffs under which a firm should position its brand as either "mainstream" or "niche".

A key feature of our model is that consumers form their perception of a brand's position from various interactions with different products under the same brand. To capture this intricate nature of brand positioning, we conceptualize brand position as the average location of all products under the brand on a Hotelling-line. Consumers are uncertain about the location of each product but are aware of the average location of the brand's products—that is, its brand positioning. Based on this information, consumers can decide whether to search for a specific brand by visiting the store. Therefore, brand positioning conveys crucial information that guides the consumer's search decisions. If a consumer decides to search for a specific brand, she incurs a search cost.

We allow for two types of consumers in the market: regular consumers who incur search costs and shoppers who can search for a brand without incurring such costs. Once a consumer visits a brand, she can inspect multiple products under that brand and determine whether a product matches her needs before deciding whether to make a purchase. We construct a model where a consumer can find the right product with a probability that increases as the distance between her location and the product location on the Hotelling-line decreases.

To explore our idea in the simplest setting, we consider a monopolist of two products. Under this model setup, the brand is characterized by two parts: the average location of its two products, and the spread of its two products (that is, their distance apart). The firm first chooses its

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brand positioning—the average location of its two products. After observing the brand position-ing, consumers form rational expectations about the spread of two products and decide whether to visit the firm's store. When a consumer visits, she observes the locations of two products and can determine how well they match her preferences. She then makes a purchase decision. If neither of the two products matches her needs, she incurs only search costs—visiting a store but leaving empty-handed. Brand positioning, therefore, serves as a crucial means of communication for the firm to convey important information to its customers and reduce uncertainty, thereby spurring consumers to begin the search process. In the presence of search friction, however, positioning information alone may be insufficient to convince consumers to visit a brand.

When deciding whether to conduct a search for brands, consumers consider both their expected utility based on the brand positioning and their potential search costs. On the one hand, if the firm's brand positioning is close to the center of the Hotelling-line, the brand may appeal to the majority of consumers and, thus, spur a greater portion of consumers to visit the store. However, such a strategy of a central brand positioning may convey less *product* information, as various product positions or spread can support the same central location of brand positioning. For example, the same central brand positioning can have either two different products (i.e., a large spread case) or two similar products (i.e., a small spread case) around the center of the Hotelling-line. If two products are vastly different, then the probability of consumers located around the center of the Hotelling-line finding products that match their preferences is low. It may no longer be attractive for those consumers to incur search costs to visit the brand's store. On the other hand, if the brand positioning is close to one endpoint of the Hotelling-line, the brand would likely appeal to a smaller portion of consumers. However, these consumers may have a higher expected utility, since they may correctly infer that the locations of the two products must be close to that endpoint by definition of the brand positioning. For instance, if the average is exactly at one endpoint, the two products must also be exactly at the endpoint as well. Therefore, a brand with a narrow appeal (one, for instance, located at the end of the Hotelling-line) can convey more information about each product's location to consumers, which increases their expected utility. This is the main tradeoff of brand positioning—breadth of appeal versus clarity of the information communicated. In particular, the expectation of the spread can drastically change the consumer's search decision. Consumers can correctly anticipate this tradeoff of the brand and expect the right degree of spread in equilibrium.

Moreover, there exists the critical issue of the *hold-up problem*, caused by the presence of shoppers who do not incur a search cost. Suppose that there are only regular consumers who need to incur search costs to visit a brand. Based on the brand positioning and the expected level of spread (or locations of two products), regular consumers decide whether to search for a brand. When regular consumers visit the store, they have sunk the search costs. Thus, a consumer may still be locked in by the brand even if the brand deviates from the expected level of spread. However, importantly, such deviation increases neither the number of regular consumers who visit the store nor the probability of their matching. Thus, a brand has little incentive to deviate from the expected level of spread when there are only regular consumers in the market. The existence of shoppers who do not incur any search cost for visiting the store, however, can change the equilibrium dynamics dramatically. Brand may now have incentives to deviate, locating two products further away. The greater the spread, the more it can cater to shoppers who visit the store regardless of brand position since they do not incur any search cost. This is the classic hold-up problem that is created by the existence of shoppers.

As a result, anticipating the brand's opportunistic behavior, regular consumers update their beliefs about each product location; this, in turn, lowers their expected utility from finding a good product match. In particular, when there is a significantly large number of shoppers in the market, regular consumers may believe that a brand will spread two products so widely that their expected utility drops below their search costs. Hence, none of the regular consumers would choose to visit the brand, and there will be only shoppers in the market. Brand positioning information alone may then be insufficient to convince regular consumers to search for or visit the brand, which entails search costs. Given this, brand may sometimes need to credibly communicate its product spread, notably smaller spread so that it can increase the regular consumers' expected utility from visiting a brand.

Our focus in this article is on the credibility of such a communication role of brand positioning. We build a micro-model for how and why brand positioning can deliver credible information to consumers and provide a rationale for a firm's specific brand positioning strategy: "niche" or "mainstream" positioning. We first establish that any brand position in a Hotelling-line, whether central (mainstream brand positioning of appealing to the majority of consumers) or at the endpoints (niche brand positioning of appealing to a small portion of consumers), can be supported in equilibrium. Next, our main results show that the mainstream-positioned brand has greater incen-

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tive to spread its product locations, leading to brand dilution, while the niche-positioned brand has less incentive to do so. As brand positioning becomes more niche, the spread of individual products becomes smaller as the range of possible spread shrinks. This reduced spread of a specific brand's position can convey more product information. Hence, niche forms of brand positioning serve as the commitment mechanism that allows a brand to communicate credible information about its spread to consumers. Finally, we identify the conditions under which brands tend to adopt niche over mainstream positioning strategy. We find that search friction tends toward the proliferation of niche brands. In particular, unless there exist too many or too few shoppers, brands are better off with a niche brand position, one that appeals to a smaller range of consumers but has a higher chance of matching their preferences when search cost is high. In this way, the presence of shoppers can alter the equilibrium dynamics and dramatically change the firm's profit.

The article is organized as follows. The next section discusses the related literature. Section 3 develops the formal model, and Section 4 analyzes it. Section 5 extends the main model by endogenizing price, and Section 6 concludes.

2 Literature Review

The importance of brand positioning is well-recognized in academics and practice (Aaker 1996; Kotler et al. 2014), and our study contributes to several streams of research about brand positioning in marketing and economics. First, there is a large body of literature studying the concept of brand positioning from the psychological perspective focusing on the process of a consumer's mind. In the existing literature, brand positioning is defined as the way a firm wants consumers to perceive, think, and feel about its brand versus competitive entries (Trout and Ries 1986; Davis et al. 2000). Along this line of research, many papers point out that positioning is a process of emphasizing the brand's distinctive and motivating attributes in the light of competition (Jan Alsem and Kostelijk 2008) and establishing both the point of unique difference and the point of parity association with the category (Keller et al. 2011). We follow the definition of the traditional approach and provide a more formal economic modeling structure to understand the strategic use of brand positioning.

Several papers also study the issues on estimating consumer perceptions of the brand along attributes to inform marketing strategy (John et al. 2006; Lehmann et al. 2008). Most importantly, such estimates are used as a primary input for generating perceptual maps, which have been widely

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used by managers (Shocker and Srinivasan 1979; Green et al. 1989; Hauser and Koppelman 1979; Johnson and Hudson 1996; Steenkamp et al. 1994; Bijmolt and van de Velden 2012; Kaul and Rao 1995). While these papers provide practical means to understand how consumers perceive each brand's positioning, we focus on the economic rationale why those different positionings can affect consumers' behaviors.

Second, this research is related to the literature which investigates the issue of branding and brand positioning from an economic perspective.[1] Especially in the context of umbrella branding, researchers have examined whether firms can credibly convey vertical information about quality through branding (Sappington and Wernerfelt 1985; Wernerfelt 1988, 1991; Cabral 2000; Zhang 2015; Neeman et al. 2019; Klein et al. 2019; Yu 2020). In this paper, we study whether the firm can transmit horizontal information about its products through branding decisions. Kuksov (2007) considers a setting where consumers use brands as alternative communication means of their preference besides cheap talk in a matching market. Kuksov et al. (2013) posit that brand positioning or image is determined by the type of consumers who consume the brand's product. In our paper, a brand affects which type of consumers will buy its products by choosing the kind of products it will make available in the market. In that sense, the brand decides its *horizontal* positioning through its target customers, which are endogenously constructed through its product portfolio.

There is another strand of research on product positioning and its effects on the equilibrium outcomes. In particular, many researchers study static positioning in markets with strategic interactions with competition (Hauser and Shugan 2008; Moorthy 1988; Kuksov 2004; Lauga and Ofek 2011). Recently, more papers have analyzed the dynamic aspects of product positioning, namely repositioning of the product over time (Sweeting 2013; Jeziorski 2014). While most of the repositioning studies are empirical, there are a few analytical studies on this topic (Villas-Boas 2018; Cong and Zhou 2019). Villas-Boas (2018) analyzed a monopolist's optimal repositioning strategy under changing consumer preferences, and Cong and Zhou (2019) focused on the role of commitment for repositioning under competition. Our paper differs from the existing literature on product positioning by highlighting the difference between brand positioning and individual product positioning.

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[1]Also, see Bronnenberg et al. (2018) for an excellent review of the economics of brand and branding. Recent studies have focused on measuring the brand value in a static setting (Goldfarb et al. 2009), and dynamic environment (Borkovsky et al. 2017).

Finally, our paper is closely related to the stream of literature on consumer search theory. Zhou (2014) and Rhodes (2014) study consumer search over firms, each carrying multiple products. Consumers in our setting also search for multiple products at a firm's store. Still, unlike previous studies, in our setting, consumers' search decisions are guided by their prior observation of the firm's brand positioning. In this sense, our paper is related to the literature of ordered search (Armstrong 2017), where the search is non-random and some observable characteristics of the options guide consumers' search decisions. Armstrong et al. (2009) shows the value of being the first shopping destination or being "prominent" place when consumers engage in costly search. Several papers have examined how firms can become prominent by using various instruments to guide consumer search, for example, by charging a lower price than others (Chen and He 2011; Armstrong and Zhou 2011; Choi et al. 2018; Zhou 2011), through advertisement (Anderson and Renault 2006; Mayzlin and Shin 2011; Lu and Shin 2018), product design (Kuksov 2004; Bar-Isaac et al. 2012), service (Shin 2007; Janssen and Ke 2020), and targeted advertising (Shin and Yu 2019). This research examines the role of brand positioning through which the firm can influence consumer search decisions.

3   Model

Consider a market populated by consumers who are uniformly distributed on a Hotelling-line in $[-1, 1]$. A monopolistic firm sells two products to consumers and decides the locations of the two products, $x_1, x_2 \in [-1/2, 1/2]$. Therefore, we impose a restriction that the support for the firm's product positions is a subset of the consumer market. As shown below, this assumption greatly simplifies the equilibrium analysis by eliminating the cases of truncated demand at endpoints.

A consumer can find a product that satisfies her needs with a probability, which is a function of the distance from her location to the product location on the Hotelling-line. Specifically, a consumer located at $x \in [-1, 1]$ receives utility $u_i(x)$ from buying product $i \in \{1, 2\}$, where,

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u_i(x) = \begin{cases} 1, & \text{with probability } \theta - t|x - x_i|, \\ 0, & \text{otherwise.} \end{cases} \tag{1}
$$

Notice that the consumer's expected utility, $\mathrm{E}[u_i(x)] = \theta - t|x - x_i|$, which takes the same form as a standard Hotelling model. However, in our model, the consumer receives $u_i(x) = 1$ if

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the product i is a match. Otherwise, the product is not a match, and $u_i(x) = 0$. Thus, even a consumer who is located at exactly the same location as product i may only receive $u_i(x) = 1$ with probability θ. As the product locates further away from the consumer, a match is less likely. In our main model of consumers' utility specification in Equation (1), we have abstracted away the firm's pricing decision. In other words, we have assumed exogenous prices and normalized consumer utility of a matched product as one, and that of an unmatched product as zero. We endogenize the firm's price decision in Section 5, which our main findings are shown to be robust.

As an example of our probabilistic Hotelling model, consider a fashion-oriented consumer who has the same location (preference) as the fashion luxury brand Gucci on the Hotelling-line. However, Gucci's current collection may not necessarily satisfy her specific needs at the moment. For example, she may search for a long coat, but Gucci only offers short coats for this season. The parameter θ captures all unforeseen contingencies that may affect the match between consumers' specific needs and the product beyond the typical preference match.

To ensure that $\theta - t|x - x_i| \in [0, 1]$ for any $x \in [-1, 1]$ and $x_i \in [-1/2, 1/2]$, we impose the restriction that $0 < 3t/2 \le \theta \le 1$.[2] We also assume that $u_1(x)$ and $u_2(x)$ are independent. Hence, a consumer may find the match with a product located further from her location than the product that is closer to her. The stochastic nature of the matching utility, together with this independence assumption, ensures that the consumer prefers *both* products to be located close to her own location (or preference) so that she has a higher chance of finding the right match product. Even if the first product does not match, she has another chance to get matched from the other product. There is a deterministic outside option of zero. Consumers prefer a matched product to the outside option and prefer the outside option to an unmatched product.[3]

3.1 Brand Positioning

To capture the idea that consumers perceive a brand's positioning from various interactions with the brand's several different products, we define B, the brand positioning of the firm, as the average location of products under the brand. A brand that carries n products can choose the location of each of its individual products, which together determine the overall positioning of the brand. In

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[2]The farthest possible distance between a consumer and a product is $3/2$, so we need $\theta - 3t/2 \ge 0$. Also, the consumer can be located precisely at either product, so $\theta \le 1$.

[3]In a more general model setup, we can have consumers' utility of the outside option as $u_0 \in [0, 1)$. One can show that the general model is equivalent to our current model if we scale the search cost in the general model by $1/(1 - u_0)$.

our case, we assume that a brand carries two products ($n = 2$), with locations, x_1 and x_2. The brand positioning is

$$B = \frac{\sum_{k=1}^{n} x_k}{n} = \frac{x_1 + x_2}{2}. \tag{2}$$

Consumers do not observe each product's location under the brand but instead know the average location of the firm's products—brand positioning. This reflects our conviction that a brand image or positioning is an outcome of consumers' numerous interactions and experience with various products under the same brand. We capture the spirit of it parsimoniously through the average location of each individual product with equal weight to keep the model analytically tractable. In reality, some interactions may have a more substantial effect on consumer's perceptions about the brand than others. These interactions can also occur in many different forms, such as observing advertisements and not necessarily involving the purchase or use of products. For example, consumers have a brand image that Gucci is a fashion luxury through prior experience or advertisements without knowing the exact design or style of individual products for the coming season. Here, we focus on the firm's product decisions, abstracting how consumers can form those initial brand images through advertising messages or other communication decisions.

After observing the brand positioning B, consumers decide whether to visit the brand at a search cost and make a purchasing decision. There are two pieces of information relevant to consumer's decisions: the average location of the products B and the spread of two products Δ. From the definition of brand positioning in Equation (2), we can rewrite $x_1 = B - \Delta$ and $x_2 = B + \Delta$, where $\Delta \equiv (x_2 - x_1)/2$ represents the spread of the brand, defined by the distance between its two products. Without loss of generality, we assume that $\Delta \geq 0$, or equivalently, $x_1 \leq x_2$. Consumers consider both the brand positioning (which is observable) and their expectations about the spread (which is unobservable) to make their search decision.

3.2 Consumer Type: Regular Consumers vs Shoppers

There are two types of consumers in the market. An α fraction of consumers are regular consumers who have a positive search cost $s > 0$. The remaining $1 - \alpha$ fraction of consumers are shoppers with zero search costs. Regular consumers observe the firm's brand positioning B before visiting it. Still, they do not observe the exact locations of its two products. That is, they do not observe the spread Δ but have a rational expectation of Δ as $\widetilde{\Delta}$, which in equilibrium coincides with the

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firm's equilibrium choice, Δ^*. Let us denote the regular consumers' expectation of the locations of the two products as $\widetilde{x}_1 = B - \widetilde{\Delta}$ and $\widetilde{x}_2 = B + \widetilde{\Delta}$, respectively. Thus, regular consumers located at x do not observe the match values $u_1(x)$ and $u_2(x)$ a priori. Based on their expectation of $\widetilde{x}_1$ and $\widetilde{x}_2$, they form a rational expectation of the probability distribution of $u_1(x)$ and $u_2(x)$. By paying the search cost s to visit the store (or brand),[4] the regular consumers discover all the information relevant to their purchase decisions—the actual spread Δ as well as the match values $u_1(x)$ and $u_2(x)$. On the other hand, shoppers have zero search costs, so they observe everything freely.

To summarize, we construct a model of a monopolistic brand with two products. Consumers know their preferences, but some of them (i.e., regular consumers) are uncertain about each product location under the same brand before visiting a store because they only observe the average location of its products. In this setup, the game's sequence is as follows: First, the brand decides the locations of its two products, x_1 and x_2 to maximize profit. We first assume that the prices are exogenous. Therefore, the brand's *de facto* objective is to maximize demand. Second, regular consumers observe the brand $B = (x_1 + x_2)/2$ and decide whether to pay the search cost and visit the firm. Upon the visit, they discover the relevant information and decide which product to buy or take the outside option. Shoppers always visit and make purchase decisions after observing the exact matching values with both products.

3.3 Main Tradeoff: Breadth of Appeal vs. Clarity of the Information Communicated

Our model characterizes brand positioning in two parts: (1) the average location of its two products B, which is observed by consumer, and (2) the distance from this location to each product Δ, which is not observed by consumers. If B is close to 0, then the average location is close to the center of the Hotelling-line where a greater mass of consumers is concentrated. Therefore, we interpret a brand with B close to 0 as a *mainstream* brand, which can potentially appeal to the majority of consumers. On the other hand, if B is close to either end of $\pm 1/2$, the brand could only appeal to a smaller mass of consumers away from the center of the Hotelling-line. Therefore, we interpret a brand with B close to $\pm 1/2$ as a *niche* brand.[5]

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[4]We use the terms store and brand interchangeably. One can consider the situation where a consumer visits a brand shop to search for products.

[5]We discuss more carefully later on how close B should be to 0 to be a mainstream brand, and to $\pm 1/2$ to be a niche brand.

As consumers have heterogeneous preferences, the brand has an incentive to locate its products somewhat distant to capture more demand. Moreover, this would lead to a brand positioning closer to the center, $B = 0$, a mainstream brand positioning. On the other hand, from $B = 0$, consumers may not receive accurate information about the true locations of the brand's *products*. There can be many different combinations that can all generate $B = 0$ as long as $x_1 = -x_2 \in [0, 1/2]$. In this case, the brand position $B = 0$ does not necessarily provide accurate information about the exact style of any product that consumers can find in the store. Then, this lack of informational value can hurt the brand by discouraging consumer search.

By comparison, a niche brand positioning can convey more product information to consumers than a mainstream brand positioning. Particularly, in an extreme case, by observing $B = 1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = 1/2$; similarly, by observing $B = -1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = -1/2$. More generally any brand position B can be supported by $x_1 = B - \Delta$ and $x_2 = B + \Delta$ for $\Delta \in [0, \min\{1/2 + B, 1/2 - B\}]$. As B goes from 0 to $\pm 1/2$, the range of possible Δ gets smaller. A niche positioning is the strongest brand that can provide perfect product information with $\Delta = 0$. By claiming a niche brand positioning, the brand can communicate credibly that all of its products are indeed consistent with the overall brand image. Then, consumers who are close to the brand positioning will knowingly visit the brand. Thus, in the presence of consumer search costs, the brand can convince consumers to incur search costs due to the increased information value from positioning.

Choosing a mainstream average location (i.e., the center of the line) can appeal to a broader set of *shoppers*. On the other hand, a tighter expected spread between the two products can appeal to the local *regular* consumers. This tradeoff indicates that the brand may wish to position itself at the center (catering larger set of consumers) with smaller spreads of the products to induce consumer search from regular consumers. However, if the brand's position is close to the center, it can face temptations to opportunistically dilute its brand by locating two products in opposite directions further than expected. Because regular consumers only observe B, and thus, it won't change their search decisions but only serve more shoppers who would visit the brand anyway. Therefore, such a product-line decision (i.e., the brand is positioned close to the center with small spread) may not be credible to regular consumers. Thus, it can discourage regular consumers from incurring search costs. Therefore, in the presence of search friction, positioning information alone may be

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insufficient to convince consumers to visit a brand. On the other hand, a brand with a narrow appeal (i.e., the brand located at the end of the Hotelling-line) can convey more information about each product's location to consumers, which increases their expected utility. This tradeoff between breadth of appeal and clarity of the information communicated is central to brand positioning.

We explore this tradeoff in the subsequent equilibrium analysis. We identify conditions (essentially depending on the degree of search frictions in the market) for the brand to choose a niche versus mainstream brand positioning.

4   Equilibrium Analysis

We start our analysis by characterizing consumer's search decisions and demand and formulating the equilibrium condition. After that, we analyze two benchmark cases, one with only shoppers and the other with only regular consumers. These cases can be obtained by setting $\alpha = 0$ and $\alpha = 1$, respectively, in the general model. These benchmark models help us isolate the effect of search frictions on the optimal brand positioning, which we subsequently explore.

We first impose some restrictions on the search cost to simplify the equilibrium analysis and focus on the most interesting case. First, if the search cost is too high, no regular consumers will visit the brand even if the brand provides the highest possible benefits by locating its two products at the same location. To avoid this trivial case, we make the following assumption that ensures that regular consumers are willing to visit the brand if she is at the same location as the two products.

Assumption 1 $s < 1 - (1 - \theta)^2$.

Moreover, we want to simplify the analysis by requiring "no truncation" of demand for regular consumers. If demand truncation happens, it leads to unnecessary complications of corner cases and non-smooth demand, which makes the analysis rather tedious without adding insight. Particularly, we require the consumer located at 1 (or at -1) to not visit the brand even if both products are co-located as closely as possible at $1/2$ (or at $-1/2$). This translates into the following assumption.

Assumption 2 $s \geq 1 - (1 - \theta + t/2)^2$.

All the subsequent equilibrium analysis is performed under Assumptions 1 and 2.

4.1 Consumer's Search Decision, Demand and Equilibrium Concept

Given the brand's choice of brand positioning B, we calculate the mass of regular consumers who search (or visit the store). Shoppers have zero search cost, so they will always search.

By searching or visiting the brand, a regular consumer at location x pays the search cost s, and discovers the match utility with both products, $u_1(x)$ and $u_2(x)$. Her expected utility from the brand is,

$$\mathrm{E}[u(x)] = \mathrm{E}\left[\max\{0, u_1(x), u_2(x)\}\right] = 1 - (1 - \theta + t|x - \widetilde{x}_1|)(1 - \theta + t|x - \widetilde{x}_2|) \tag{3}$$
$$= 1 - \left(1 - \theta + t|x - B + \widetilde{\Delta}|\right)\left(1 - \theta + t|x - B - \widetilde{\Delta}|\right).$$

The regular consumer will visit the brand if and only if the benefit of search is greater than the cost of search:

$$\mathrm{E}[u(x)] \geq s.$$

So, we can now calculate the total number of visitors (or those who search the brand) among regular consumers. We denote the set of visitors on the Hotelling-line as $\mathcal{V}(B, \widetilde{\Delta})$, where,

$$\mathcal{V}(B, \widetilde{\Delta}) \equiv \begin{cases} \left[B - \Gamma_1(\widetilde{\Delta}), B + \Gamma_1(\widetilde{\Delta})\right] & \text{if } \widetilde{\Delta} \leq \Delta_1, \\ \left[B - \Gamma_1(\widetilde{\Delta}), B - \Gamma_2(\widetilde{\Delta})\right] \cup \left[B + \Gamma_2(\widetilde{\Delta}), B + \Gamma_1(\widetilde{\Delta})\right] & \text{if } \Delta_1 < \widetilde{\Delta} < \Delta_2, \\ \emptyset, & \text{otherwise,} \end{cases} \tag{4}$$

where $\Gamma_1(\widetilde{\Delta}) \equiv \sqrt{\widetilde{\Delta}^2 + (1-s)/t^2} - (1-\theta)/t$, $\Gamma_2(\widetilde{\Delta}) \equiv \sqrt{\left(\widetilde{\Delta} + (1-\theta)/t\right)^2 - (1-s)/t^2}$, $\Delta_1 \equiv \sqrt{1-s}/t - (1-\theta)/t$, and $\Delta_2 \equiv \left[(1-s) - (1-\theta)^2\right]/\left[2t(1-\theta)\right]$.[6]

Equation (4) shows that the total number of regular consumers who visit the brand can be characterized by three different cases, depending on how large the expected spread $\widetilde{\Delta}$ is. First, if $\widetilde{\Delta} \leq \Delta_1$, every consumer in between the two products is willing to visit the brand by incurring the search cost of s. We have a *strong brand* in this case, as shown by Figure 1-(a), where the gray line represents the set of regular consumers who visit the brand, $\mathcal{V}(B, \widetilde{\Delta}) = \left[B - \Gamma_1(\widetilde{\Delta}), B + \Gamma_1(\widetilde{\Delta})\right]$. Second, $\Delta_1 < \widetilde{\Delta} < \Delta_2$, some consumers located in between the two products find it not worthwhile to visit the brand because neither product is expected to provide a good match. We have a

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[6] For any given brand positioning $B \in [-1/2, 1/2]$, we have $\mathcal{V}(B, \widetilde{\Delta}) \in (-1, 1)$ under Assumption 2, and therefore the demand truncation never happens.

(a) A strong brand ($\widetilde{\Delta} \leq \Delta_1$)   (b) A medium brand ($\Delta_1 < \widetilde{\Delta} < \Delta_2$)   (c) A weak brand ($\widetilde{\Delta} \geq \Delta_2$)
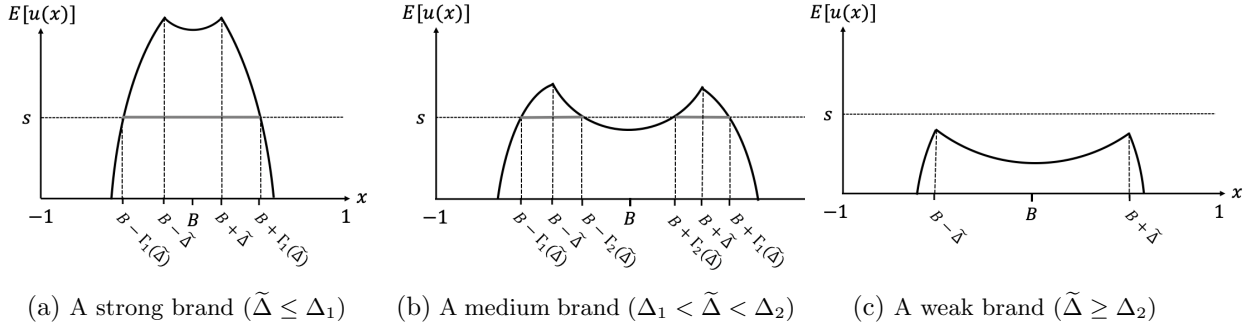
Figure 1: The set of regular consumers who visit the brand.

medium brand in this case, as shown by Figure 1-(b), where regular consumers who visit the brand come from two disjoint intervals that are, roughly speaking, located around $B - \widetilde{\Delta}$ (i.e., $x \in \left[B - \Gamma_1(\widetilde{\Delta}), B - \Gamma_2(\widetilde{\Delta})\right]$) and $B + \widetilde{\Delta}$ (i.e., $x \in \left[B + \Gamma_2(\widetilde{\Delta}), B + \Gamma_1(\widetilde{\Delta})\right]$), respectively. Finally, when $\widetilde{\Delta} \geq \Delta_2$, as illustrated by Figure 1-(c), the two products are so distant from each other that no regular consumer will visit the brand. We have a *weak brand* that is unattractive to regular consumers in this case, $\mathcal{V}(B, \widetilde{\Delta}) = \varnothing$. These definitions for a strong, medium, and weak brands are used throughout the paper.

Given the total number of regular consumers who visit, $\mathcal{V}(B, \widetilde{\Delta})$, we can derive the total consumer demand from both regular consumers and shoppers as following. The total consumer demand $D(B, \widetilde{\Delta}, \Delta)$ is a function of brand positioning B, consumer's expected spread $\widetilde{\Delta}$ (which affects regular consumers' visit decisions), and the actual spread Δ (which affects all consumers' purchase decisions):

$$D(B, \widetilde{\Delta}, \Delta) = \frac{\alpha}{2} \int_{x \in \mathcal{V}(B, \widetilde{\Delta})} \left[1 - (1 - \theta + t|B + \Delta - x|)(1 - \theta + t|B - \Delta - x|)\right] dx$$
$$+ \frac{1 - \alpha}{2} \int_{-1}^{1} \left[1 - (1 - \theta + t|B + \Delta - x|)(1 - \theta + t|B - \Delta - x|)\right] dx. \quad (5)$$

Using $D(B, \widetilde{\Delta}, \Delta)$ above, we formulate the firm's problem next. The firm chooses B and Δ based on the consumer's expected spread $\widetilde{\Delta}$ to maximize demand, where the equilibrium brand positioning, B^* and equilibrium brand spread, Δ^* are given by,

$$(B^*, \Delta^*) = \underset{(B, \Delta) \in \left\{0 \leq \Delta \leq \frac{1}{2} - |B|\right\}}{\arg \max} D(B, \Delta^*, \Delta). \quad (6)$$

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The right-hand side in Equation (6) involves a constrained optimization problem, where the constraint, $0 \leq \Delta \leq 1/2 - |B|$ is due to limited product space on the Hotelling-line for product positioning. From the perspective of regular consumers who do not observe Δ, we have their expected spread, $\widetilde{\Delta}$ must satisfy $0 \leq \widetilde{\Delta} \leq 1/2 - |B|$. This signifies that the firm can use brand positioning B to *directly* influence consumers' belief $\widetilde{\Delta}$, which is a unique feature to our model. In equilibrium, we require consumers' expectation $\widetilde{\Delta}$ coincides with the firm's optimal choice, Δ^*.

We are interested in the sequential equilibrium of the game (Kreps and Wilson 1982). Compared with perfect Bayesian equilibrium, sequential equilibrium uses trembling-hand perturbations to reach nodes that are off the equilibrium path, and require the players' strategies and beliefs to be sequentially rational and consistent on these nodes. Hence, even if the game reaches off the equilibrium path, the players choose actions optimally from then on, consistent with the equilibrium strategy. In our setting, when a firm deviates on B, sequential equilibrium requires that regular consumers' belief $\widetilde{\Delta}$ is consistent with the firm's optimal choice, Δ^* given the deviated position choice of B.

To solve for the sequential equilibria of the game, we proceed as the following two steps. First, we take the brand B as given and consider the brand's decision on the spread Δ. There are three cases to consider, according to Equation (4). The set of regular consumers who visit the brand, $\mathcal{V}(B, \Delta^*)$, is one interval when $\Delta^* \leq \Delta_1$ (which is the strong brand case), two disjoint intervals when $\Delta_1 < \Delta^* < \Delta_2$ (which is the medium brand case), or an empty-set when $\Delta^* \geq \Delta_2$ (which is the weak brand case). For each of the three cases, we calculate the firm's demand $D(B, \Delta^*, \Delta)$ based on Equation (5), and determine the equilibrium $\Delta^*(B)$ by taking derivatives of $D(B, \Delta^*, \Delta)$ with respect to Δ and solving the corresponding optimality conditions taking into account of the constraint of $0 \leq \Delta \leq 1/2 - |B|$. Second, we maximize $D(B, \Delta^*(B), \Delta^*(B))$ with respect to B to solve for the equilibrium brand positioning, B^*.

In the next section, we examine two benchmark cases where only one type of consumers exist in the market (shoppers only and regular consumers only). This helps us to identify the underlying forces for the firm's optimal brand positioning decision.

4.2 Benchmarks: Shoppers Only ($\alpha = 0$) vs. Regular Consumers Only ($\alpha = 1$)

As the first benchmark, we consider the case with only shoppers, which is by setting $\alpha = 0$ in the main model. In this case, all consumers in the market do not incur search costs to visit the brand,

and thus, both the brand positioning B and the spread of products Δ are observable to consumers since all consumers can immediately inspect both products without incurring any cost. Therefore, the problem reverts to a simple product positioning problem, and there is no special role of brand positioning beyond the product differentiation. The later comparison between the main model and this benchmark can highlight the unique role of brand positioning.

It is easy to solve the complete-information game, the equilibrium of which is presented by the following proposition.

Proposition 1 (Shoppers Only) *If there are only shoppers in the market ($\alpha = 0$), there exists a unique equilibrium with $(B^*, \Delta^*) = (0, \max\{\Delta^*_{\alpha=0}, 0\})$, where $\Delta^*_{\alpha=0} \equiv [1 - (1-\theta)/t]/2$. Moreover, the optimal spread Δ^* decreases in θ and t: $\partial \Delta^*/\partial \theta \leq 0$ and $\partial \Delta^*/\partial t \leq 0$.*

Proof. See the Appendix. ∎

Proposition 1 implies that in the presence of only shoppers in the market, the firm will position the brand in the center with $B^* = 0$, which is the standard result in product positioning problem. Also, as θ increases, consumers have higher match probabilities, and as t increases, consumers' preferences become more heterogeneous. Both results in a larger spread, Δ^*, which is quite intuitive.

Next, we study the alternative benchmark case with only regular consumers in the market (that is, $\alpha = 1$). This case is crucially different from the previous benchmark with only the shoppers because the regular consumers only observe B and not Δ. Therefore, in equilibrium, the unobservable Δ must be credibly communicated through the firm's choice of B. The following proposition summarizes the findings in equilibrium.

Proposition 2 (Regular Consumers Only) *If there are only regular consumers in the market ($\alpha = 1$), there exist a unique set of equilibria such that $\Delta^* = \max\{\Delta^*_{\alpha=1}, 0\}$ and B^* takes any value in $[-(1/2 - \Delta^*), 1/2 - \Delta^*]$, where $\Delta^*_{\alpha=1} \equiv [\sqrt{3(1-s) + 4(1-\theta)^2} - 4(1-\theta)]/(3t) \leq \Delta_1$. Therefore, some regular consumers will visit the brand in equilibrium, who come from one interval on the Hotelling-line. Moreover, the optimal spread Δ^* decreases in search cost s: $\partial \Delta^*/\partial s \leq 0$.*

Proof. See the Appendix. ∎

We find that, in the presence of only regular consumers, the unobserved product information Δ can be communicated credibly, which is a stark contrast to the main model later. Moreover, in equilibrium, the firm chooses a strong brand with a small spread of $\Delta^* \leq \Delta_1$. This is intuitive

because the spread has to be sufficiently small for regular consumers to pay a visit to the store. Based on the observed B and the expected small spread Δ^*, the regular consumers close to B will visit the store. Therefore the firm will indeed choose a small spread to serve these regular consumers.

It is immediate to see that a higher search cost s leads to a smaller Δ^* and, thus, a stronger brand.[7] The brand needs to compensate the consumer's high search costs by placing two products closer, which increases their expected utility. Also, we notice that $B^* = 0 \in [-(1/2-\Delta^*), 1/2-\Delta^*]$, and therefore, the mainstream brand positioning of $B^* = 0$ is always an equilibrium. We can now compare the equilibria across the two benchmark cases when there are only shoppers ($\alpha = 0$), and only regular consumers ($\alpha = 1$) by comparing the equilibrium spread Δ^*.

Proposition 3 *The equilibrium spread, Δ^* in the case with only regular consumers ($\alpha = 1$) is less than or equal to that in the case with only shoppers ($\alpha = 0$): $\max\{\Delta^*_{\alpha=1}, 0\} \leq \max\{\Delta^*_{\alpha=0}, 0\}$.*

Proof. See the Appendix. ■

For the regular consumers who must pay a search cost to visit the brand, the brand needs to provide enough benefits by locating two products sufficiently close. In contrast, for shoppers who can visit freely, the brand wants to spread out its products sufficiently to maximize market coverage. If both shoppers and regular consumers co-exist in the market, the brand faces a trade-off in choosing the spread of the brand between serving the two types of consumers.

4.3 Equilibrium Brand Positioning

In this section, we analyze the main model with $\alpha \in (0, 1)$. In order to encourage some regular consumers to visit its store, the brand would want to locate their products close to each other by choosing a small spread. However, once regular consumers (who cannot observe the spread of the product directly, but only anticipate the spread) visit the brand, the brand is tempted to deviate by increasing the spread to better serve the shoppers. This is the classic hold-up problem. The regular consumers are rational and can anticipate the brand's opportunistic behavior. This can be costly for the brand because regular consumers can be discouraged from visiting the brand. Therefore, we will show that the brand may discipline itself by choosing its brand location B close to either

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[7]However, the threshold for a strong brand Δ_1 decreases in s at a faster rate. Therefore, as we state in the results in Section 4.3, as s increases, the brand is harder to maintain a strong brand in equilibrium.

end of the Hotelling-line. In other words, the brand may endogenously choose a niche brand as a commitment device to preserve a strong brand (or, the small spread), and thereby serve both the shoppers and the regular consumers.

Importantly, this hold-up problem never happens under the two benchmark cases. When there are only shoppers, the brand cannot hold them up once they visit the store. When there are only regular consumers, there is no incentive for the brand to deviate by spreading the products further apart. The regular consumers make a visit decision based on their own expectations. Therefore, such deviation does not increase the number of regular consumers who visit the store. Moreover, such deviation (i.e., increasing the ex-post spread) will reduce the match probability of those who visit the store, further decreasing the demand eventually. Thus, this hold-up problems uniquely arises in the main model where the market consists of both segments of the shoppers and the regular consumers.

We now solve the sequential equilibrium under the main model. Given any $B \in [-1/2, 1/2]$, there are three possible cases of $\Delta^* \leq \Delta_1$ (a strong brand), $\Delta_1 < \Delta^* < \Delta_2$ (a medium brand) and $\Delta^* \geq \Delta_2$ (a weak brand). For each case, we first relax the constraint $\Delta \leq 1/2 - |B|$ and solve the resulting unconstrained optimization problem in Equation (6), and then consider when the constraint $\Delta \leq 1/2 - |B|$ will be binding. In order to highlight the tradeoff in the firm's branding decision, our analysis focuses on the two more interesting cases of a strong brand and a weak brand.[8]

Let's analyze the first case of a strong brand. Given $\Delta^* \leq \Delta_1$, all visitors come from one interval on the Hotelling-line. Based on Equations (4) and (5), we have the first- and second-order optimality conditions imply $\Delta^* = \max\{\Delta_\alpha^*, 0\}$, where, the unconstrained optimal spread Δ_α^* is

$$\Delta_\alpha^* \equiv \frac{\alpha\sqrt{[(1+\alpha)(1-\theta) - (1-\alpha)t]^2 + (4-\alpha^2)(1-s)} - 2[(1+\alpha)(1-\theta) - (1-\alpha)t]}{(4-\alpha^2)t},$$

which does not depend on B.

Lemma 1 *The unconstrained optimal spread Δ_α^* decreases with α: $\partial\Delta_\alpha^*/\partial\alpha \leq 0$.*

Proof. See the Appendix. ∎

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[8]In the Appendix, we analyze all three cases including a medium brand and identify conditions under which a medium brand cannot be an equilibrium. See Lemma A-1 in the Proof of Proposition 5.

The constraint is not binding if $\max\{\Delta_\alpha^*, 0\} \leq 1/2 - |B|$; otherwise, we have the constraint binding and $\Delta^* = 1/2 - |B|$. To summarize,

$$\Delta^*(B) = \begin{cases} \max\{\Delta_\alpha^*, 0\}, & \text{if } |B| \leq 1/2 - \max\{\Delta_\alpha^*, 0\}, \\ 1/2 - |B|, & \text{otherwise.} \end{cases} \tag{7}$$

This spread can be part of an equilibrium only if it satisfies the assumed condition for a strong brand that $\Delta^*(B) \leq \Delta_1$ so that the firm's optimal choice of the spread is consistent with the consumers' expected spread. Whether a strong brand can be maintained is important because a strong brand can serve both shoppers and regular consumers. On the other hand, if only a weak brand can be sustained, then the firm is unable to serve the regular consumers completely, thus potentially hurting the firm's profit.

The following proposition identifies conditions under which a strong brand can be sustained in equilibrium.

Proposition 4 *1. If $s \leq 1 - (1 - \theta + t)^2/4$ or $\alpha \geq \widehat{\alpha}$, then $\Delta^*(B) \leq \Delta_1$ for all $B \in [-1/2, 1/2]$. Therefore, any positioning (regardless of whether it is a mainstream or a niche positioning) can sustain a strong brand.*

2. Otherwise (i.e., if $s > 1 - (1 - \theta + t)^2/4$ and $\alpha < \widehat{\alpha}$), then $\Delta^*(B) \leq \Delta_1$ if and only if $|B| \geq 1/2 - \Delta_1$. Therefore, only a niche brand positioning (i.e., $|B| \geq 1/2 - \Delta_1$) can sustain a strong brand.

Proof. See the Appendix. ∎

This proposition shows that a strong brand with a small spread $\Delta^* \leq \Delta_1$ can always be achieved by a niche positioning, i.e., $|B| \geq 1/2 - \Delta_1$. However, a mainstream positioning (i.e., around the center location of the Hotelling-line such that $|B| < 1/2 - \Delta_1$) can only sustain a strong brand when s is sufficiently small, or α is sufficiently large.

More precisely, when s is relatively low, or α is relatively high, there is enough number of regular consumers who are willing to visit the brand. Then, the firm can maintain a strong brand with a relatively small spread ($\Delta^* \leq \Delta_1$) regardless of its brand positioning $B \in [-1/2, 1/2]$. On the other hand, if s is sufficiently large and α is sufficiently small, then the firm has to position its brand close to an end ($|B| > 1/2 - \Delta_1$) in order to sustain a small spread. This is because in

the presence of a large segment of shoppers (i.e., α is sufficiently small), a brand positioned in the middle is tempted to increase the spread to better serve the shoppers, which is evident from the fact that $\partial \Delta_\alpha^* / \partial \alpha \leq 0$ (from Lemma 1). Moreover, a high search cost requires an extremely narrow spread to provide sufficient benefit from consumer search. Putting these two effects together, a mainstream brand's unconstrained optimal choice of spread cannot coincide with the consumers' expected spread that can justify their search costs. This can cost the firm significantly because regular consumers will no longer trust the firm to serve products close to their tastes and therefore decide not to visit. Again, this is the aforementioned hold-up problem.

Therefore, the proposition implies that the firm may overcome the hold-up problem by positioning its brand as a niche positioning. The benefit of choosing a niche positioning is that the firm can attract both the shoppers and regular consumers. However, by choosing a brand's position far away from the center of the Hotelling-line, the firm is unable to serve the shoppers optimally (as shown in Proposition 1). Given these tradeoffs, the firm will choose the brand positioning B^* optimally to maximize $D(B, \Delta^*(B), \Delta^*(B))$.[9] Note that, in equilibrium, the firm chooses not only whether to be a mainstream or niche brand, but also how close to the central location a mainstream brand will be, as well as how close to an end a niche brand will be.

We show that two types of equilibria can arise under different parameter regions: mainstream and niche. First, as the next result shows, a mainstream brand equilibrium is always positioned at the center of the Hotelling-line, $B^* = 0$. However, we may sometimes have a strong or weak mainstream brand.

Proposition 5 (Mainstream Positioning) *Mainstream positioning can be an equilibrium either as a strong brand or a weak brand.*

1. If $s \leq 1 - (t+1-\theta)^2/4$ or $\alpha \geq \widehat{\alpha}$, then a strong mainstream brand $(B^*, \Delta^*) = (0, \max\{\Delta_\alpha^*, 0\})$ is an equilibrium. The brand serves both regular consumers and shoppers.[10]

2. If $s > 1 - t(1-\theta)$ and $\alpha < \overline{\alpha}_{main}(< \widehat{\alpha})$, then a weak mainstream brand $(B^*, \Delta^*) = (0, \Delta_{\alpha=0}^*)$ is an equilibrium. In this case, the brand only serves the shoppers.

Proof. See the Appendix. ∎

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[9]In the Appendix (proof to Proposition 5), we follow a similar procedure to analyze the other two cases with $\Delta_1 < \Delta^* < \Delta_2$ and $\Delta^* \geq \Delta_2$.

[10]We can further prove the uniqueness of this equilibrium under $s \leq 1 - (t+1-\theta)^2/4$.

The composition of consumers (i.e., the size of α) and consumer's search cost s affect the brand's optimal brand positioning decision critically. As implied by Proposition 4, when s is relatively small, or α is relatively high, there are enough number of regular consumers who are willing to visit the brand. Then, the firm does not suffer from a hold-up problem, and, therefore, even a mainstream positioning can attain a strong brand with a small spread $\Delta^* \leq \Delta_1$. It is then an equilibrium for the firm to set $\Delta^* = \max\{\Delta_\alpha^*, 0\}$ that attracts both regular consumers and shoppers to visit.

However, if consumer search cost is sufficiently high ($s > 1 - t(1-\theta)$) and there exist a sufficiently large number of shoppers ($\alpha < \overline{\alpha}_{main}$), then only a weak brand is feasible for a mainstream positioning, serving only the shoppers.[11] In this case where the mainstream brand is weak, the firm may potentially find it more profitable to choose a niche brand positioning, where it can maintain a strong brand, serving both shoppers and regular consumers. While the brand would likely appeal to a smaller portion of the entire population, but it can attract both segments. This benefit of monetizing on the regular consumers is not so large if the size of the regular consumers (α) is small. Therefore, appealing to a small group of regular consumers by choosing a niche positioning cannot justify forsaking the central brand positioning where the firm can serve the large group of shoppers efficiently. This is the main tradeoffs between choosing the mainstream vs. niche brand positioning. It is further noted that, given mainstream positioning, it is optimal for the brand to position as the brand at the center of the Hotelling-line, $B^* = 0$, in order to best serve the shoppers.

Next, we identify the conditions under which a niche positioning can be the firm's equilibrium choice in the following proposition. The firm chooses a niche brand positioning with $|B^*| = 1/2 - \Delta_1 > 0$ and one product exactly at one endpoint with $|B^*| + \Delta^* = 1/2$.

Proposition 6 (Niche Positioning) *If $s > 1 - t(1 - \theta)$ and $\underline{\alpha}_{niche} < \alpha < \overline{\alpha}_{niche}(< \widehat{\alpha})$, then the unique set of equilibria are strong niche brands: $(B^*, \Delta^*) = (\pm(1/2 - \Delta_1), \Delta_1)$.[12]*

Proof. See the Appendix. ∎

Following the discussions for Proposition 5, the firm finds niche positioning more profitable if (1) a mainstream positioning suffers from a hold-up problem, and (2) the benefit of attracting regular consumers as a niche positioning is large enough. For condition (1), consumer's search cost

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[11]The condition $s > 1 - t(1 - \theta)$ implies $\Delta_{\alpha=0}^* > \Delta_2$ such that the spread $\Delta_{\alpha=0}^*$ is part of an equilibrium for a mainstream brand where no regular consumers visit the brand.

[12]We have $\underline{\alpha}_{niche} \geq \overline{\alpha}_{main}$. Therefore, the interval of α, $(\underline{\alpha}_{niche}, \overline{\alpha}_{niche})$, for which a niche brand positioning is optimal in Proposition 6 is greater than the interval $[0, \overline{\alpha}_{main})$ for which a weak mainstream brand is optimal in Part 2 of Proposition 5.

has to be sufficiently large ($s > 1 - t(1 - \theta)$) and there cannot be too many regular consumers ($\alpha < \overline{\alpha}_{niche}$), as shown in Proposition 4 and Proposition 5. In this case, the regular consumers will not visit the brand if it is positioned in the center of the Hotelling-line ($B = 0$), which is illustrated in Figure 2-(a). This is because of the hold-up problem, where the existence of shoppers induces the firm to spread the two products apart. Still, regular consumers demand a lower brand spread to justify their search costs. In this case, this proposition suggests that a niche brand positioning can serve as a commitment not to stretch its products excessively. In particular, condition (2) is satisfied if $\alpha > \underline{\alpha}_{niche}$ such that there are enough regular consumers in the market. Then, the brand finds it optimal to sacrifice the central location to position its brand toward an endpoint. The niche brand positioning provides the brand with a commitment to locate its products close to one another. In turn, regular consumers close to the expected locations of the brand's products have higher expected utility and choose to visit. Therefore, unlike mainstream brand positioning depicted in Figure 2-(a), the brand can serve some regular consumers by positioning it as a niche brand, which is illustrated in Figure 2-(b).

Finally, we find that it is optimal for the firm to choose B such that the niche brand is placed as close as possible to the center of the Hotelling-line. This is because the firm can benefit from moving closer to the center position and thereby better serve the shoppers. Based on these two forces coming from incentives to serve regular consumers vs. shoppers, each pulling the brand positioning in opposite directions, the firm chooses an optimal location $(|B^*|, \Delta^*) = (1/2 - \Delta_1, \Delta_1)$. This is precisely the equilibrium location B^* where it must be close enough to the endpoints so that regular consumers are willing to visit, while at the same time as close as possible to the center to maximize the demand from the shoppers.

Mainstream vs. Niche Brand Positioning

We have characterized two different types of equilibria—mainstream positioning and niche positioning. Proposition 5 and 6 together demonstrate the opposing forces induced by two segments of consumers—regular consumers and shoppers–and how their interplay fundamentally influences the brand's positioning choice between a mainstream and a niche brand. For regular consumers, the brand has an incentive to establish a strong brand positioning by placing its products close to each other. However, the presence of shoppers tempts the brand to spread out its product locations to accommodate the shopper's heterogeneous preferences more efficiently. The more shoppers there

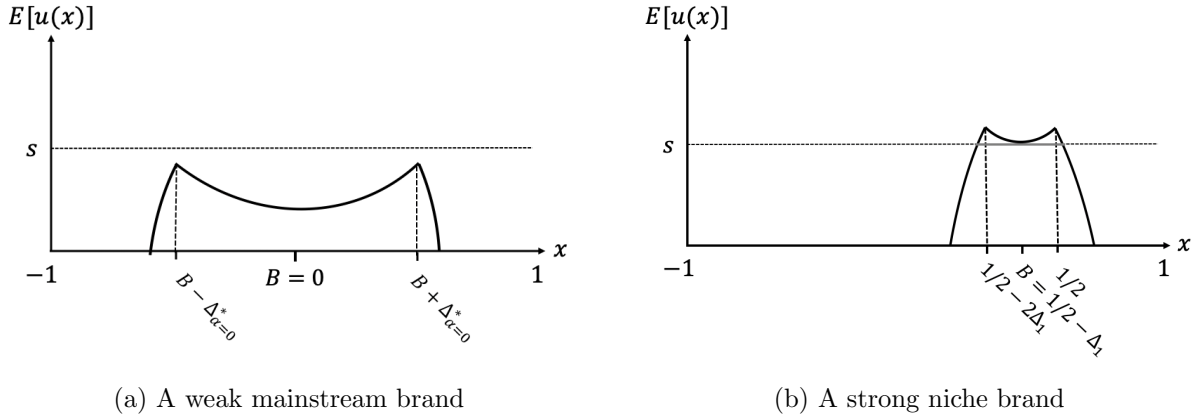(a) A weak mainstream brand   (b) A strong niche brand

Figure 2: Regular consumers' search decision when facing a niche vs. a mainstream brand under $s > 1 - t(1 - \theta)$ and $\underline{\alpha}_{niche} < \alpha < \overline{\alpha}_{niche}$.

are, the stronger this temptation becomes. This will discourage regular consumers from visiting the brand. Therefore, the brand needs a mechanism to overcome the temptation if it aims to serve both shoppers and regular consumers. We show that niche brand positioning can do this role.

A niche brand can be more profitable than a mainstream brand under a specific condition, i.e., if α is in an intermediate interval $(\underline{\alpha}_{niche}, \overline{\alpha}_{niche})$, and the consumer search cost is sufficiently large, i.e. $s > 1 - t(1 - \theta)$. This condition is represented by the red dotted area in Figure 3. Otherwise, mainstream positioning is optimal. This implies that even if niche brand can provide more information about the brand's products, under a wide range of parameter space, the brand prefers a mainstream brand to a niche brand. More specifically, whenever the strong mainstream brand (illustrated with horizontal lines in Figure 3) exists, it is an equilibrium. On the other hand, a weak mainstream brand (depicted by vertical lines in Figure 3) can only be an equilibrium if α is either sufficiently small or sufficiently large. In an intermediate region, a weak mainstream brand it is dominated by a niche branding. Lastly, if $s < 1 - (t + 1 - \theta)^2/4$ or $\alpha \geq \widehat{\alpha}$, a strong mainstream brand is the unique equilibrium. In summary, the figure shows that unless there exist too many or too few shoppers, brands are better off with a niche brand position when search costs is sufficiently high. Thus, search friction tends toward the proliferation of niche brands.

Notice that Proposition 5 and 6 together do not cover the entire parameter space. In particular, one intermediate region for s and two intermediate intervals of α are missing. This is because we have identified a sufficient condition in order to rule out the existence of a medium brand equilibrium where regular consumers visit the brand from two disjoint intervals. Therefore, in these missing
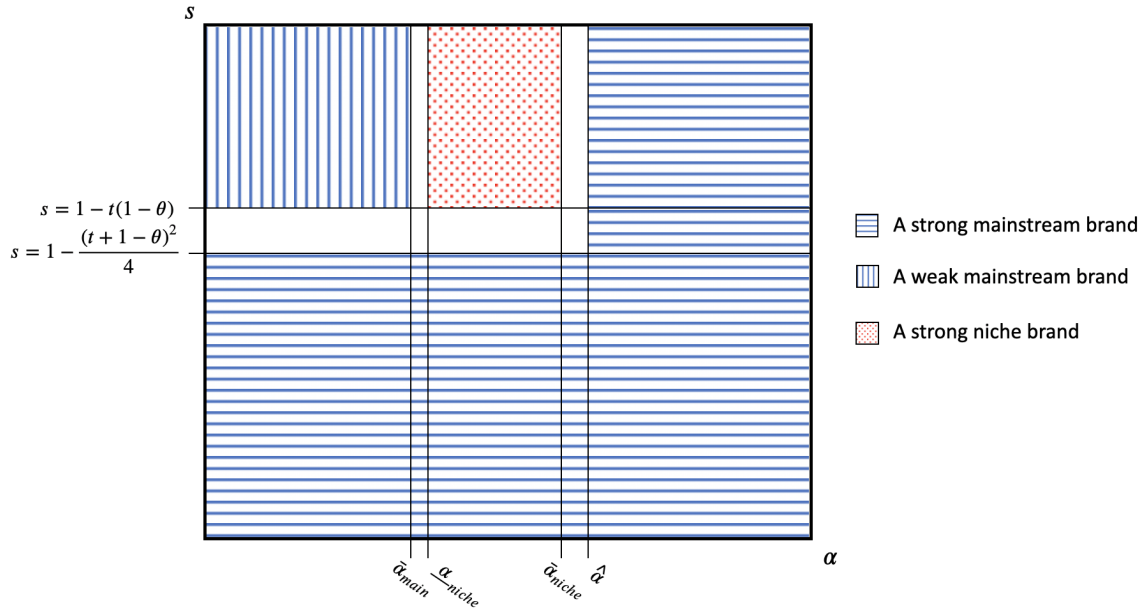
23

Figure 3: The equilibrium brand positioning.

regions, a medium brand equilibrium may exist. However, an analysis of a medium brand would add significant technical complexities and little additional insight, and thus, we focus on a strong brand and weak brand. We believe that this decision help us clearly understand the important tradeoffs in the firm's brand positioning decision and how it affects the role of branding as a communication mechanism.

5 Endogenous Prices

In our main analysis above, we have assumed that the prices of the two products are exogenously given, and the normalized consumer utility of a matched product is one. In this section, we will assess the robustness of those results under exogenous pricing scenarios by constructing a new model with endogenous prices. Here, we will show that the new model under endogenous equilibrium prices is exactly equivalent to the original model. Therefore, our findings do not depend on the exogenous price assumption.

In fact, we can endogenize the prices by modifying consumers' match utility distribution in

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Equation (1) as the following,

$$u_i(x) = \begin{cases} v_i - p_i, & \text{with probability } \theta - t|x - x_i|, \\ -p_i, & \text{otherwise.} \end{cases}$$

where p_i is the price of product i, and v_i is consumer-specific consumption value, which follows a binary distribution with $v_i = v_0$ with probability φ and $v_i = v_0 + 1$ with probability $1 - \varphi$. It is assumed that all consumers observe their own v_i value at the beginning of the game. In other words, v_i represents the ex-ante heterogeneity among consumers. The firm sets the prices and the positions of the two products simultaneously, and those prices are not observable to regular consumers before they pay the search cost and visit the store.

It is straightforward to argue that the firm will set the equilibrium price for each product as either $v_0 + 1$ or v_0. (To be more precise, the prices are set slightly below $v_0 + 1$ or v_0 so that consumers prefer to make a purchase than the outside option.) If the equilibrium price is set at $v_0 + 1$, only the regular consumers with $v_i = v_0 + 1$ will make a purchase if they find a match, in which case they derive zero utility. Consequently, it is not worthwhile for them to pay the search cost and visit the store in the first place. In this case, the hold-up problem is so severe that no regular consumer is willing to pay the search cost and visit. It becomes a trivial case, which is not very interesting. A more interesting case happens when v_0 is sufficiently high and/or φ is sufficiently high, in which cases, the firm will optimally set the equilibrium price for both products at v_0. We focus on this latter case.

Under the equilibrium price, the regular consumers with $v_i = v_0 + 1$ has the following utility distribution:

$$u_i(x) = \begin{cases} 1, & \text{with probability } \theta - t|x - x_i|, \\ -v_0, & \text{otherwise.} \end{cases}$$

which is equivalent to the utility distribution in Equation (1) under the exogenous pricing assumption. In fact, the realized value of $u_i(x) = -v_0$ given mismatch does not matter, because in this case, the consumer will always take the outside option. On the other hand, the regular consumers with $v_i = v_0$ expect zero utility upon visiting the store and thus has no incentive to visit. They leave the market without any purchase.

Similarly, for shoppers, there are two different kinds of shoppers under the new setting: φ fraction of shoppers who have $v = v_0$, and the rest who have $v = v_0 + 1$. Nevertheless, these two

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kinds of shoppers' purchase decisions are exactly the same irrespective of their types. Therefore, we can just treat them as a homogeneous group of shoppers, as in the original model. This implies that if we redefine $\widetilde{\alpha} \equiv \frac{\alpha(1-\varphi)}{\alpha(1-\varphi)+(1-\alpha)}$, which denote the fraction of "effective" regular consumers, our new model with endogenous prices are equivalent to the original model in the main analysis, which demonstrates the robustness of our results to the exogenous pricing assumption.

6   Conclusion

A brand's position is one of a firm's most valuable assets: it can have a significant impact on consumers' purchase decisions; it helps firms create market differentiation by providing or articulating critical information about product characteristics; it facilitates consumer search and aids them in determining where to purchase products without having to search through multiple brands. However, the important issue of brand positioning remains an under-researched topic in economics and marketing. There is a lack of a framework for thinking about the design of brand positioning in a unified and consistent way.

In this paper, we conceptualize the role of brand positioning through a product portfolio and consumer search framework. First, we provide a micro-foundation for how and why brand positioning can deliver credible information to consumers and we provide rationale for a firm's specific brand positioning strategy: "niche" or "mainstream". Consumers are uncertain about each product location of the brand but are aware of the average location of the brand's products—in other words, its brand positioning. Based on this information, consumers can make decisions about whether to visit a brand (or firm) first, while brands can simultaneously determine their individual product locations. We identify the conditions under which brands tend to adopt a niche positioning strategy over a mainstream strategy and find that niche brands prevail when the fraction of regular consumers in the population is within the intermediate range and search cost is sufficiently high. We also find that a mainstream-positioned brand has a greater incentive to spread its individual product locations, which leads to brand dilution, while the niche-positioned brand has less incentive to do so. The niche-positioned brand can thus serve as a commitment tool for a brand not to spread its individual product location and thereby invite more consumers to visit the store. Therefore, our results shed light on the coexistence of different brand positioning and provide practical guidance for firms to form an optimal brand positioning strategy.

The current research focuses on the mechanism of how brand positioning can affect consumer choice and market outcomes. A natural extension of this work would be to understand how initial perceptions of brand positioning are formed. One can consider endogenizing initial brand perception by capturing the dynamics of firm and consumer interactions through a dynamic game framework or by explicitly incorporating other forms of communications. Brand positioning can be established not only by prior product experience, which is the focus of the current paper, but also by other communications, such as advertisements. It would be interesting to investigate, for example, the brand's optimal advertising strategy and the credibility of its advertising messages. We leave these issues for future research.

Finally, we only examine the monopolistic firm's brand positioning choice. Future research can extend the current framework to a competitive situation. How strategic considerations can change the optimal brand positioning under competition can be a fruitful and important venue for further investigation, which will would broaden the implications of our study. We hope that our modeling framework of brand position will serve as a workhorse model for addressing these topics for further research.

Appendix

Proof of Proposition 1

Proof. Under $\alpha = 0$, by Equation (5), $D(B, \Delta^*, \Delta)$ does not depend on Δ^*. For simplicity we use the shorthand notation of D for $D(B, \Delta^*, \Delta)$ when there is no confusion. We have

$$
\left.\frac{\partial D}{\partial B}\right|_{B=B^*} = -2t(1 - \theta + t)B^* = 0, \text{ and } \frac{\partial^2 D}{\partial B^2} = -2t(1 - \theta + t) < 0 \Rightarrow B^* = 0.
$$

$$
\left.\frac{\partial D}{\partial \Delta}\right|_{\Delta=\Delta^*} = -4t^2 \Delta^* \left[\Delta^* - \frac{1}{2}\left(1 - \frac{1-\theta}{t}\right)\right] = 0, \text{ and}
$$

$$
\left.\frac{\partial^2 D}{\partial \Delta^2}\right|_{\Delta=\Delta^*} = -8t^2 \left[\Delta^* - \frac{1}{4}\left(1 - \frac{1-\theta}{t}\right)\right] \le 0 \Rightarrow \Delta^* = \max\left\{\frac{1}{2}\left(1 - \frac{1-\theta}{t}\right), 0\right\}.
$$

Notice that we always have $\Delta^* \le 1/2$. The comparative statics of Δ^* with respect to t and θ is straightforward to obtain. ∎

Proof of Proposition 2

Proof. Given any $B \in [-1/2, 1/2]$, let's consider three cases below depending on Δ^* according to Equation (4). For each case, we will first relax the constraint $\Delta \le 1/2 - |B|$ and solve the resulting unconstrained optimization problem in Equation (6), and then consider when will the constraint $\Delta \le 1/2 - |B|$ be binding. For simplicity we use the shorthand notation of D for $D(B, \Delta^*, \Delta)$ when there is no confusion.

First, $\Delta^* \le \Delta_1$, in which case, all visitors come from one interval on the Hotelling-line. Based on equations (4) and (5), we have the first- and second-order optimality conditions:

$$
\left.\frac{\partial D}{\partial \Delta}\right|_{\Delta=\Delta^*} = 2t^2 \Delta^* \left[\sqrt{\frac{1-s}{t^2} + (\Delta^*)^2} - 2\Delta^* - \frac{2(1-\theta)}{t}\right] = 0,
$$

$$
\left.\frac{\partial^2 D}{\partial \Delta^2}\right|_{\Delta=\Delta^*} = 2t^2 \left[\sqrt{\frac{1-s}{t^2} + (\Delta^*)^2} - 4\Delta^* - \frac{2(1-\theta)}{t}\right] \le 0.
$$

This implies that $\Delta^* = \max\left\{[\sqrt{3(1-s) + 4(1-\theta)^2} - 4(1-\theta)]/(3t), 0\right\} = \max\{\Delta^*_{\alpha=1}, 0\}$, which does not depend on B. This implies that,

$$
\Delta^*(B) = \begin{cases} \max\{\Delta^*_{\alpha=1}, 0\}, & \text{if } |B| \le 1/2 - \max\{\Delta^*_{\alpha=1}, 0\}, \\ 1/2 - |B|, & \text{otherwise.} \end{cases} \tag{8}
$$

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Lastly, We can easily verify that under Assumption 1, $\Delta^*_{\alpha=1} \leq \Delta_1$ and thus $\Delta^*(B) \leq \Delta_1$.

Second, $\Delta_1 < \Delta^* < \Delta^*_2$, in which case, visitors come from two disjoint intervals on the Hotelling-line. Based on equations (4) and (5), we find that,

$$\frac{\partial D}{\partial \Delta}\Big|_{\Delta=\Delta^*} = 2t^2 \left[\Delta^* \sqrt{\Delta^{*2} + \frac{1-s}{t^2}} - \left(\Delta^* + \frac{1-\theta}{t} \right) \left(2\Delta^* - \sqrt{\left(\Delta^* + \frac{1-\theta}{t} \right)^2 - \frac{1-s}{t^2}} \right) \right].$$

Next, we will show that $\Delta^* < \Delta_2$ implies that $\partial_\Delta D|_{\Delta=\Delta^*} < 0$. In fact, $\Delta^* < \Delta_2$ is equivalent to,

$$\Delta^{*2} + \frac{1-s}{t^2} - \left(\Delta^* + \frac{1-\theta}{t} \right)^2 > 0. \tag{9}$$

$\partial_\Delta D|_{\Delta=\Delta^*} < 0$ is equivalent to,

$$\Delta^* \left[\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} - \left(\Delta^* + \frac{1-\theta}{t} \right) \right] < \left(\Delta^* + \frac{1-\theta}{t} \right) \left[\Delta^* - \sqrt{\left(\Delta^* + \frac{1-\theta}{t} \right)^2 - \frac{1-s}{t^2}} \right]. \tag{10}$$

Further notice that,

$$\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} - \left(\Delta^* + \frac{1-\theta}{t} \right) = \frac{\Delta^{*2} + \frac{1-s}{t^2} - \left(\Delta^* + \frac{1-\theta}{t} \right)^2}{\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} + \left(\Delta^* + \frac{1-\theta}{t} \right)},$$

$$\Delta^* - \sqrt{\left(\Delta^* + \frac{1-\theta}{t} \right)^2 - \frac{1-s}{t^2}} = \frac{\Delta^{*2} + \frac{1-s}{t^2} - \left(\Delta^* + \frac{1-\theta}{t} \right)^2}{\Delta^* + \sqrt{\left(\Delta^* + \frac{1-\theta}{t} \right)^2 - \frac{1-s}{t^2}}}.$$

Based on these two equations above, Equation (10) can be equivalently rewritten as,

$$\Delta^* \left[\Delta^* + \sqrt{\left(\Delta^* + \frac{1-\theta}{t} \right)^2 - \frac{1-s}{t^2}} \right] < \left(\Delta^* + \frac{1-\theta}{t} \right) \left[\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} + \left(\Delta^* + \frac{1-\theta}{t} \right) \right], \tag{11}$$

By multiplying both sides of Equation (10) with $(\Delta^* + (1-\theta)/t)$ and multiplying both sides of Equation (11) with Δ^* and summing them up, we have that $\partial_\Delta D|_{\Delta=\Delta^*} < 0$ is equivalent to,

$$\sqrt{\left(\Delta^* + \frac{1-\theta}{t} \right)^2 - \frac{1-s}{t^2}} \left[\Delta^{*2} + \left(\Delta^* + \frac{1-\theta}{t} \right)^2 \right] < \Delta^* \left[3 \left(\Delta^* + \frac{1-\theta}{t} \right)^2 - \Delta^{*2} \right],$$

which holds because $\sqrt{(\Delta^* + (1-\theta)/t)^2 - (1-s)/t^2} < \Delta^*$ by Equation (9) and $\Delta^{*2} + (\Delta^* + (1-$

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$\theta)/t)^2 < 3(\Delta^* + (1-\theta)/t)^2 - \Delta^{*2}$. Therefore, we have proved that under $\Delta_1 < \Delta^* < \Delta_2$, we have $\partial_\Delta D|_{\Delta=\Delta^*} < 0$. This further implies that there does not exist an equilibrium with $\Delta_1 < \Delta^* < \Delta_2$.

Thirdly, $\Delta^* \geq \Delta_2$, which results in zero demand and cannot be in equilibrium.

So far, we have identified an equilibrium candidate for a given B as in Equation (8). Next, we solve for $B^* = \arg\max_B D(B, \Delta^*(B), \Delta^*(B))$.

Consider $\Delta^*(B)$ given by Equation (8). By symmetry, it is without loss of generality to consider $B \geq 0$. We have the following four observations: (1) $\partial_B D(B, \Delta^*, \Delta) = 0$. (2) From Equation (8), it is obvious that $\Delta^{*\prime}(B) \leq 0$ for $B \geq 0$. (3) By the first-order optimality condition, we have that $\partial_\Delta D(B, \Delta^*(B), \Delta^*(B)) \geq 0$, where the equality holds when $|B| + \Delta^*(B) < 1/2$, and the inequality holds when $|B| + \Delta^*(B) = 1/2$. (4) $\partial_{\Delta^*} D(B, \Delta^*(B), \Delta^*(B)) = s\Delta^*(B)/\sqrt{\Delta^*(B)^2 + (1-s)/t^2} > 0$. These four observations together imply that,

$$\frac{dD(B, \Delta^*(B), \Delta^*(B))}{dB} = \partial_B D(B, \Delta^*(B), \Delta^*(B)) +$$
$$[\partial_{\Delta^*} D(B, \Delta^*(B), \Delta^*(B)) + \partial_\Delta D(B, \Delta^*(B), \Delta^*(B))]\Delta^{*\prime}(B) \leq 0. \quad (12)$$

This implies that the equilibrium brand positioning, B^* takes any value such that $|B^*| \leq 1/2 - \max\{\Delta^*_{\alpha=1}, 0\}$, and correspondingly, the equilibrium spread, $\Delta^* = \max\{\Delta^*_{\alpha=1}, 0\}$. Moreover, later in Proposition 3, we show that $\max\{\Delta^*_{\alpha=1}, 0\} \leq \max\{\Delta^*_{\alpha=0}, 0\} \leq 1/2$. Notice that this equilibrium exists for the entire parameter space under Assumptions 1 and 2. ∎

Proof of Proposition 3

Proof. We prove that $\max\{\Delta^*_{\alpha=0}, 0\} \geq \max\{\Delta^*_{\alpha=1}, 0\}$. Notice that $\Delta^*_{\alpha=1}$ decreases with s, and $s \geq 1 - (1-\theta+t/2)^2$ under Assumption 2. This implies that $\Delta^*_{\alpha=1}$ takes the maximum value of $[\sqrt{3(1-\theta+t/2)^2 + 4(1-\theta)^2} - 4(1-\theta)]/(3t)$ at $s = 1 - (1-\theta+t/2)^2$. Therefore, we only need to prove that

$$\max\{\Delta^*_{\alpha=0}, 0\} \geq \max\left\{\frac{\sqrt{3(1-\theta+t/2)^2 + 4(1-\theta)^2} - 4(1-\theta)}{3t}, 0\right\}.$$

To prove this, we only need to prove that

$$\frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t} > 0 \Rightarrow$$

$$\Delta^*_{\alpha=0} \geq \frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t}.$$

Further notice that

$$\frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t} > 0 \Leftrightarrow t > 2(1-\theta), \text{ and}$$

$$\Delta^*_{\alpha=0} \geq \frac{\sqrt{3(1-\theta+t/2)^2+4(1-\theta)^2}-4(1-\theta)}{3t} \Leftrightarrow 6t^2+18(1-\theta)t-3(1-\theta)^2 \geq 0.$$

$t > 2(1-\theta)$ implies that $6t^2+18(1-\theta)t-3(1-\theta)^2 \geq 57(1-\theta)^2 \geq 0$. This completes the proof. ∎

Proof of Lemma 1

Proof. First differentiating D with respect to Δ results in

$$\frac{\partial D}{\partial \Delta} = 2t \cdot \Delta^*_\alpha \cdot \left(-(1+\alpha)(1-\theta) + t\left(1 - 2\Delta^*_\alpha - \alpha\left(1 - \sqrt{(\Delta^*_\alpha)^2 + \frac{1-s}{t^2}}\right)\right)\right) = 0.$$

Differentiating both sides with respect to α, we have

$$-(1-\theta) - t \cdot \left(1 - \sqrt{(\Delta^*_\alpha)^2 + \frac{1-s}{t^2}}\right) = \frac{\partial \Delta^*_\alpha}{\partial \alpha} \cdot t \cdot \underbrace{\left(2 - \alpha \cdot \frac{\Delta^*_\alpha}{\sqrt{(\Delta^*_\alpha)^2 + \frac{1-s}{t^2}}}\right)}_{>0}.$$

The left-hand side is negative if and only if $t^2(\Delta^*_\alpha)^2 + 1 - s < (1-\theta+t)^2$. Given $\Delta^*_\alpha \leq 1/2$, we only need to show $t^2/4 + 1 - s < (1-\theta+t)^2$, or equivalently, $s > 1 - (1-\theta+t/2) \cdot (1-\theta+3t/2)$. This always holds by Assumption 2. ∎

Proof of Proposition 4

Proof. Using $\Delta^*_{\alpha=1} \leq \Delta_1$ and $\partial\Delta^*_\alpha/\partial\alpha < 0$, it is straightforward to show that if $\Delta^*_{\alpha=0} > \Delta_1$, then there exists $\widehat{\alpha} \in (0,1)$ such that $\Delta^*_\alpha \leq \Delta_1$ if and only if $\alpha \geq \widehat{\alpha}$. Note that $\Delta^*_{\alpha=0} > \Delta_1$ if and only if $s > 1 - (t+1-\theta)^2/4$. Also, if $s \leq 1 - (t+1-\theta)^2/4$ such that $\Delta^*_{\alpha=0} \leq \Delta_1$, then $\Delta^*_\alpha \leq \Delta_1$ for all $\alpha \in [0,1]$. Putting all these conditions together, $\Delta^*_\alpha < \Delta_1$ if $s \leq 1 - (1-\theta+t)^2/4$ or $\alpha \geq \widehat{\alpha}$.

Otherwise, if $s > 1 - (1 - \theta + t)^2/4$ and $\alpha \leq \widehat{\alpha}$, then $\Delta_\alpha^* \geq \Delta_1$.

Then, the results in Part 1 and 2 follow immediately from Equation (7). ■

Proof of Proposition 5

Proof. Two propositions will be proven in the following steps. (1) We rule out equilibria involving a medium brand in which the equilibrium spread Δ^* satisfies $\Delta_1 < \Delta^* < \Delta_2$. (2) Equilibria for a mainstream positioning are analyzed for a strong brand ($\Delta^* \leq \Delta_1$) and weak brand ($\Delta^* \geq \Delta_2$) cases. (3) An equilibrium with a niche positioning is identified. (4) We compare the profits and characterize the equilibrium brand positioning decision.

Lemma A-1 *If $s > 1 - t(1 - \theta)$ and $\alpha < \overline{\alpha}_{niche}(\leq \widehat{\alpha})$, then a medium brand equilibrium does not exist for a mainstream positioning. Moreover, for a niche positioning, a medium brand cannot be an equilibrium if $s > 1 - t(1 - \theta)$ and $\alpha > \underline{\alpha}$. Therefore, a medium brand does not exist if $s > 1 - t(1 - \theta)$ and $\underline{\alpha} < \alpha < \overline{\alpha}_{niche}$.*

Proof. Suppose such an equilibrium existed such that an equilibrium spread Δ^* satisfies $\Delta_1 < \Delta^* < \Delta_2$ does not exist. Then, the regular consumers who visit the brand come from two disjoint intervals. Similar to the analysis of a strong brand in the main text, we first consider an unconstrained optimization problem. A medium brand implies that $|B|$ cannot be to close to an end, i.e., $|B| < 1/2 - \Delta_1$.

In the proof of Proposition 2, we have proved that $\partial_\Delta D|_{\Delta = \Delta^*, \alpha = 1} < 0$. In the proof of Proposition 1, we have proved that $\partial_\Delta D|_{\Delta = \Delta^*, \alpha = 0} > 0$ if $\Delta_{\alpha=0}^* \geq \Delta_2$, or equivalently, $s > 1 - t(1 - \theta)$. This implies that if $s > 1 - t(1 - \theta)$, we have $\partial_\Delta D|_{\Delta = \Delta^*} = \alpha \partial_\Delta D|_{\Delta = \Delta^*, \alpha = 1} + (1 - \alpha)\partial_\Delta D|_{\Delta = \Delta^*, \alpha = 0} > 0$ if α is not too large, i.e., $\alpha < \check{\alpha}$ for some $\check{\alpha} \in (0, 1)$. To summarize, we have shown that if $s > 1 - t(1 - \theta)$ and $\alpha < \overline{\alpha}_{niche} := \min\{\widehat{\alpha}, \check{\alpha}\}$, $\partial_\Delta D|_{\Delta = \Delta^*} > 0$ for $\Delta_1 < \Delta^* < \Delta_2$. This implies that the following two things. First, if $|B| < 1/2 - \Delta_2$, then there is no medium brand which meets the consumers' expected spread Δ^*. Second, if $1/2 - \Delta_2 \leq |B| < 1/2 - \Delta_1$, then the firm will want to further increase a spread Δ such that the only candidate for an equilibrium spread is $\Delta^* = 1/2 - B$. Therefore, a medium brand may be feasible with a niche positioning where $1/2 - \Delta_2 \leq |B| < 1/2 - \Delta_1$ and $\Delta^*(B) = 1/2 - B$.

It remains to characterize the optimal choice of B. For this, we compute $dD(B, \Delta^*(B), \Delta^*(B))/dB$. In particular, we rule out any equilibrium with a medium brand by finding a condition that

$dD(B, \Delta^*(B), \Delta^*(B))/dB > 0$, which would imply that the firm wants to position its brand close enough to an end until it reaches $|B| = 1/2 - \Delta_1$. When there is no confusion, $D(B, \Delta^*(B), \Delta^*(B))$ is abbreviated by $D^*(B)$. Equation (12) applies here, where $D^*(B) = \alpha \cdot D^*(B)|_{\alpha=1} + (1 - \alpha) \cdot D^*(B)|_{\alpha=0}$. Then, we identify a condition for $dD^*(B)/dB > 0$ by showing that $dD^*(B)|_{\alpha=1}/dB > 0$ and $dD^*(B)|_{\alpha=0}/dB < 0$, and hence by linearity of $D^*(B)$ in α, we need $\alpha > \underline{\alpha}$ for some $\underline{\alpha} \in (0, 1)$.

First, for $\alpha = 1$ case,

$$\frac{\partial D}{\partial \Delta^*}\Big|_{\Delta=\Delta^*,\alpha=1} = \frac{s\left[\Delta^*\sqrt{\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}} - \left(\Delta^* + \frac{1-\theta}{t}\right)\sqrt{\Delta^{*2} + \frac{1-s}{t^2}}\right]}{\sqrt{\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}}\sqrt{\Delta^{*2} + \frac{1-s}{t^2}}} < 0$$

$$\Leftrightarrow \Delta^*\sqrt{\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}} - \left(\Delta^* + \frac{1-\theta}{t}\right)\sqrt{\Delta^{*2} + \frac{1-s}{t^2}} < 0$$

$$\Leftrightarrow \Delta^{*2}\left[\left(\Delta^* + \frac{1-\theta}{t}\right)^2 - \frac{1-s}{t^2}\right] - \left(\Delta^* + \frac{1-\theta}{t}\right)^2\left(\Delta^{*2} + \frac{1-s}{t^2}\right) < 0$$

$$\Leftrightarrow -\frac{1-s}{t^2}\left[\left(\Delta^* + \frac{1-\theta}{t}\right)^2 + \Delta^{*2}\right] < 0.$$

This proves that $\partial_{\Delta^*} D^*(B)|_{\Delta=\Delta^*,\alpha=1} < 0$. Moreover, it is easy to show that $\partial_B D|_{\alpha=1} = 0$ and $\Delta^{*\prime}(B) = -1$. This implies that $dD^*(B)/dB|_{\alpha=1} = \partial_B D^*(B)|_{\alpha=1} - (\partial_\Delta D^*(B)|_{\Delta=\Delta^*,\alpha=1} + \partial_{\Delta^*} D^*(B)|_{\Delta=\Delta^*,\alpha=1}) > 0$.

Second, for $\alpha = 0$ case,

$$\frac{dD^*(B)}{dB}\Big|_{\alpha=0} = t\left[4t\Delta^*(B)^2 + 4(1-\theta)\Delta^*(B) - t - (1-\theta)\right] < 0$$

$$\Leftrightarrow \Delta^*(B) < \frac{\sqrt{t^2 + t(1-\theta) + (1-\theta)^2} - (1-\theta)}{2t}.$$

Notice that $\Delta^*(B) < \Delta_2$, so to show $dD^*(B)/dB|_{\alpha=0} < 0$, we only need to show that $\Delta_2 \leq [\sqrt{t^2 + t(1-\theta) + (1-\theta)^2} - (1-\theta)]/(2t)$, or, $s \geq 1 - (1-\theta)\sqrt{t^2 + t(1-\theta) + (1-\theta)^2}$, which holds if $s > 1 - t(1-\theta)$. This shows that $dD^*(B)/dB|_{\alpha=0} < 0$.

Putting together the two cases of $\alpha = 1$ and $\alpha = 0$, there exists $\underline{\alpha}$ such that $dD(B, \Delta^*(B), \Delta^*(B))/dB > 0$ if and only if $\alpha > \underline{\alpha}$. Therefore, if $s > 1 - t(1-\theta)$ and $\underline{\alpha} < \alpha < \overline{\alpha}_{niche}$, then a medium cannot be an equilibrium. ∎

Lemma A-2 *Given a mainstream positioning, both a strong brand and a weak brand can be part*

of an equilibrium.

Proof. First, suppose that consumers facing a brand positioned as mainstream expect a strong brand, i.e., $|B| < 1/2 - \Delta_1$ and $\Delta^* \leq \Delta_1$. From Proposition 4, this is true if and only if $s \leq 1 - (t + 1 - \theta)^2/4$, or $\alpha \geq \widehat{\alpha}$, and the optimal spread consistent with consumers' expected spread is $\Delta^*(B) = \max\{0, \Delta_\alpha^*\}$. Then, it is straightforward to show that it is optimal to set $B^* = 0$. This is because the demand $D(B, \Delta^*(B), \Delta^*(B))$ consists of the demand from the shoppers and the regular consumers. The demand from the regular consumers is independent of B for a mainstream positioning. On the other hand, for the demand from the shoppers, it is optimal to set $B^* = 0$. Therefore, the optimal branding decision for a mainstream and strong brand is $(B^*, \Delta^*) = (0, \max\{\Delta_\alpha^*, 0\})$.

Next, suppose that consumers expect a weak brand given a mainstream positioning, i.e., $|B| < 1/2 - \Delta_1$ and $\Delta^* \geq \Delta_2$. Then, no regular consumers will visit the firm, and accordingly the firm's optimal branding decision is equivalent to the firm's decision in the first benchmark with only the shoppers. Therefore, the firm sets $(B^*, \Delta^*) = (0, \Delta_{\alpha=0}^*)$. Then, this can be part of an equilibrium only if $\Delta_{\alpha=0}^* \geq \Delta_2$, which holds if $s > 1 - t(1 - \theta)$. ∎

Lemma A-3 *The optimal niche positioning is $(B^*, \Delta^*) = (\pm(1/2 - \Delta_1), \Delta_1)$.*

Proof. In Lemma A-1, we ruled out a medium brand case for a niche positioning. The only remaining possibility for an equilibrium with a niche positioning is a strong brand where $\Delta^* \leq \Delta_1$. If $s > 1 - t(1 - \theta) \geq 1 - (t + 1 - \theta)^2/4$ and $\alpha < \widehat{\alpha}$, then Proposition 4 implies that $\Delta^*(B) \leq \Delta_1$ if and only if $|B| \geq 1/2 - \Delta_1$. For $|B| \geq 1/2 - \Delta_1$, we have $\Delta^*(B) = 1/2 - |B|$ given by Equation (7). In this case, by the proofs of Propositions 1 and 2, we know both the shoppers' demand and the regular consumers' demand decreases with $|B|$, and therefore the total demand, $D(B, \Delta^*(B), \Delta^*(B))$ achieves the maximum at $|B^*| = 1/2 - \Delta_1$. ∎

Profit analysis.

To summarize the findings thus far, we have three different candidates in different parameter regions: (1) If $s \leq 1 - (t + 1 - \theta)^2/4$ or $\alpha \geq \widehat{\alpha}$, then $(B^*, \Delta^*) = (0, \max\{0, \Delta_\alpha^*\})$ is the optimal strong mainstream brand; (2) If $s > 1 - t(1 - \theta)(> 1 - (t + 1 - \theta)^2/4)$, then $(B^*, \Delta^*) = (0, \Delta_{\alpha=0}^*)$ is the optimal weak and mainstream brand; (3) If $s > 1 - t(1 - \theta)$ and $\underline{\alpha} < \alpha < \overline{\alpha}_{niche}$, then $(B^*, \Delta^*) = (\pm(1/2 - \Delta_1), \Delta_1)$ is the optimal strong and niche brand.

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It remains to compare the profits among the three branding decisions wherever appropriate. The demand under each of the three identified candidates for equilibrium is respectively denoted by $D_{main}^{strong}, D_{main}^{weak}, D_{niche}^{strong}$. There are the following three cases:

1. If either $s < 1 - (t + 1 - \theta)^2/4$ or $\alpha \geq \widehat{\alpha}$, then $(B, \Delta^*) = (0, \Delta_\alpha^*)$ (a strong mainstream brand) is the unique equilibrium.

2. If $s > 1 - t(1 - \theta)$ and $\alpha < \underline{\alpha}_{main}$, then $(B, \Delta^*) = (0, \Delta_{\alpha=0}^*)$ (a weak mainstream brand) is the unique equilibrium.

3. If $s > 1 - t(1 - \theta)$ and $\underline{\alpha} \leq \alpha < \overline{\alpha}_{niche}$, then a branding decision which gives greater profit between $(B, \Delta^*) = (0, \Delta_{\alpha=0}^*)$ (a weak mainstream brand) and $(B, \Delta^*) = (1/2 - \Delta_1, \Delta_1)$ (a strong niche brand) is the unique equilibrium. We have $D_{main}^{weak} = (1 - \alpha) \cdot D^*(0, \Delta_{\alpha=0}^*)|_{\alpha=0}$, and $D_{niche}^{strong} = \alpha \cdot D^*(1/2 - \Delta_1, \Delta_1)|_{\alpha=1} + (1 - \alpha) \cdot D^*(1/2 - \Delta_1, \Delta_1)|_{\alpha=0}$. Therefore, $D_{main}^{weak} > D_{niche}^{strong}$ if and only if α is not too large, i.e., $\alpha < \underline{\alpha}$. Hence, if $\underline{\alpha} = \underline{\alpha}_{main} \leq \alpha < \overline{\alpha}_{main} := \min\{\underline{\alpha}, \overline{\alpha}_{niche}\}$, then the weak mainstream brand is the unique equilibrium.

This completes the proof. ∎

Proof of Proposition 6

Proof. The proof follows immediately from the proof of Proposition 5 case (4) when $s > 1 - t(1 - \theta)$ and $\underline{\alpha} < \alpha < \overline{\alpha}_{niche}$. In particular, if $\max\{\underline{\alpha}, \underline{\underline{\alpha}}\} = \underline{\alpha}_{niche} < \alpha < \overline{\alpha}_{niche}$, then $D_{niche}^{strong} > D_{main}^{weak}$, and therefore the strong niche is the unique equilibrium.

Finally, for this result, the interval $\alpha \in (\max\{\underline{\alpha}, \underline{\underline{\alpha}}\}, \overline{\alpha}_{niche})$ must be nonempty. We are unable to show these properties for the general parameter space. Instead, we present a numeric analysis to show that these intervals are nonempty under some parameter setting. Take $s = 0.9, \theta = 0.8, t = 0.2$. Then, $\widehat{\alpha} = \check{\alpha} = \overline{\alpha}_{niche} = 0.475$, $\underline{\alpha} = \underline{\alpha}_{main} = 0.271$, and $\underline{\underline{\alpha}} = \overline{\alpha}_{main} = 0.047$. Therefore, $\underline{\alpha}_{niche} = 0.271$. Consequently, our main result holds for all parameter regions $\alpha \in (0.271, 0.475)$. ∎

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