Abstract

Sovereign debtors’ public capital influences defaults and debt restructurings. We compile a dataset on public expenditure composition around restructurings with private external creditors. We show that public investment (i) experiences a severe decline and slow recovery, (ii) differs from public consumption and transfers, (iii) reduces its share in public expenditure, and (iv) relates with restructuring delays. We develop a theoretical model of defaultable debt that explicitly embeds public expenditure composition, capital accumulation, and multi-round debt renegotiations. The model quantitatively shows that severe decline and slow recovery in public investment — “debt overhang” — delay debt settlement. Theoretical predictions are supported by data.

JEL Classification Codes: F34, F41, H63

Key words: Debt Overhang; Sovereign Debt; Sovereign Default; Debt Restructuring; Public Investment; Public Capital.

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1 Introduction

Sovereign debtors’ public capital influences defaults and debt restructurings. We compile a new dataset on public expenditure composition around restructurings with private external creditors. We show that public investment (i) experiences a severe decline and slow recovery, (ii) differs from public consumption and transfers, (iii) reduces its share in public expenditure, and (iv) relates with restructuring delays. To explain these stylized facts, we develop a theoretical model of defaultable debt that explicitly embeds public expenditure composition, capital accumulation, and multi-round debt renegotiations. Our model quantitatively replicates these stylized facts and shows both severe decline and slow recovery in public investment—“debt overhang”—delay debt settlement. Our theoretical predictions are supported by data: both a panel analysis using 111 post-default restructuring episodes and a calibration exercise for Argentine default and restructuring in 2001–05.

Our paper starts by presenting a comprehensive dataset on public expenditure composition at 179 privately-held external debt restructurings over 1978–2010. Following the detailed categories in the U.S. Bureau of Economic Analysis (BEA 2005), we classify public consumption, investment, transfers, and capital. Our dataset provides much wider coverage of restructuring episodes, time-series, and categories and is thus superior to existing databases (e.g., the IMF World Economic Outlook and the World Bank Global Development Finance). We then combine our new data with an existing dataset on the duration and strategies of restructurings from Asomuma and Trebesch (2016), which cover the same sample of restructuring episodes.

The consolidated datasets provide four new stylized facts on sovereign debt overhang and public expenditure composition around post-default restructurings—sovereigns default first and renegotiate their debt later. First, public investment experiences a severe decline and slow recovery during restructurings. Second, on the contrary, public consumption and transfers experience a short-lived decline and quick recovery. Third, public expenditure tilts remarkably towards consumption and transfers during debt crisis. Fourth, the severe decline in public investment is associated with lengthy restructuring delays, i.e., long restructuring duration. We confirm these findings through both panel and cross-sectional regressions for 111 episodes.

Our empirical findings unveil a new dimension of sovereign debt overhang and public expenditure composition, which the literature has not fully explored yet. More specifically, we confront the following two main questions: Why public investment experiences a severe decline and slow recovery during debt crisis, but public consumption and transfers do not? What is the role of public capital accumulation (investment) on restructuring delays and debt settlements? These questions challenge the current understanding in the literature that public capital accumulation is irrelevant to sovereign debtors’ choice of default and debt settlement, and expenditure composition. To our knowledge, we are the first to shed light on the role of public capital on sovereign debt overhang, public expenditure composition, and debt restructurings.

To address these questions, we construct a theoretical sovereign debt model that explicitly embeds public expenditure composition, capital accumulation, and multi-round debt renegotiations with a risk averse sovereign debtor and its risk-neutral foreign creditors. The model is built
on the classical setup of Eaton and Gersovitz (1981) as in the recent quantitative analysis of sovereign debt (e.g., Arellano 2008; Aguiar and Gopinath 2006). In particular, our small open economy with defaultable debt follows two conventional frameworks in the literature: (i) fiscal policy—distinct private and public sectors due to both distortionary consumption tax and two different consumption goods—(Cuadra et al. 2010; Arellano and Bai 2017) and (ii) multi-round debt renegotiations after default (Benjamin and Wright 2013; Bi 2008).

The important theoretical innovation is incorporating endogenous public investment, capital accumulation, and production with public capital and labor in the model with endogenous defaults and renegotiations. We explicitly depart from two standard modeling approaches: an exogenous income process (e.g., Arellano 2008; Aguiar and Gopinath 2006) and endogenous production with elastic labor supply (e.g., Mendoza and Yue 2012; Cuadra et al. 2010). At each period, the sovereign chooses its expenditure composition—public consumption, investment, and transfers—together with its choice of default, settlement, delay, and external borrowing. Its choice of public investment—net of depreciation and adjustment costs—interacting with its decision of default, settlement, delay and external borrowing, pins down public capital stock, which in turn, together with elastic labor determines production.

We emphasize two novel predictions in our model. First, our model provides main implications on the sovereign’s default, debt settlements and restructuring delays. After default, the sovereign delays renegotiations when public capital is low; it opts to invest in public capital and refrains using resources for recovered debt repayments given high marginal product of public capital (equivalent to high shadow value of public capital). On the contrary, the sovereign agrees on debt settlement when public capital is high; it hesitates to invest in public capital and chooses to use resources for recovered debt payments given low marginal product of public capital (corresponding to low shadow value of public capital). As a result, restructuring delays and debt settlements are driven by both marginal product of public capital and recovery of repayment capacity (Benjamin and Wright 2013; Bi 2008)—this corresponds to recovery of productivity and an associated response of elastic labor in our model.

Before default, i.e., ex-ante, there are two opposing effects of public capital on default similar to those of “total” capital in Gordon and Guerron-Quintana (2018). On the one hand, when public capital is high, repayment capacity, i.e., output is high—defined as “smoothing channel” in their paper—resulting in low likelihood of default. On the other hand, when public capital is high, costs of financial exclusion are low—defined as “autarky channel” in their paper—because public capital plays its role of smoothing the household’s utility. This leads to high likelihood of default. In addition, our model newly includes the third channel, “restructuring channel”—

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1See also the survey by Aguiar and Amador (2014) and Aguiar et al. (2016).
2See Ohanian et al. (2018) for the role of marginal product of “total” capital on international capital flows in Asia and Latin America.
3Gordon and Guerron-Quintana (2018) and Park (2017) focus on the role of “total capital” on sovereign defaults. In their framework, the government, i.e., social planner decides only “total consumption” and “total investment” with default and external borrowing choice. There exists no fiscal policy—public consumption, investment, transfers and taxation. The government can allocate resources across sectors without any fiscal constraint.
quick settlements—which reduces costs of default resulting in high likelihood of default. As a result, total net effects of public capital through these three channels on default are balanced in our model. This differentiates our paper from Gordon and Guerron-Quintana (2018) where there is no restructuring channel and the smoothing channel dominates the autarky channel.\(^4\)

Second, the model explains the well-known “sovereign debt overhang” effect on public investment: current high level of external debt negatively influences future investment. Our paper differs from the literature on sovereign debt overhang (e.g., Aguiar et al. 2009) in which cyclical debt overhang arises endogenously due to the limited ability of the government. Our model explains not only the conventional phase, a “run-up to default period” but also one new phase of sovereign debt overhang, a “restructuring (run-up to debt settlement) period” with different drivers of debt overhang. In the run-up to default, low productivity, high external debt payments, and the sovereign’s consumption smoothing motive and myopia interact and drive a sharp decline in investment, resulting in a default (Aguiar et al. 2009). On the one hand, the sovereign with a high consumption smoothing motive and myopia opts to smooth the household’s utility through public consumption and transfers when output is low. On the other hand, the sovereign disproportionally reduces public investment due to high burden of external debt payments.

During debt restructurings, a combination of slow recovery of productivity, no external borrowing, and the sovereign’s consumption-smoothing motive and myopia generates both deceleration of public capital accumulation and lengthy renegotiations, which in turn, interact and result in debt overhang. Public capital accumulation is slow due to both no external borrowing and the sovereign’s consumption-smoothing motive and myopia. Renegotiations are delayed because of both high marginal product of public capital (when public capital is low) and slow recovery of productivity. This cycle continues until the sovereign accumulates public capital to a high level and agrees on debt settlement.

Our theoretical predictions that both severe decline and slow recovery in public investment delay debt settlement are supported by the data. First, the panel analysis on debt settlement using 111 post-default episodes at an annual frequency confirms the theoretical predictions. Second, the quantitative analysis calibrated to the Argentine default and restructuring in 2001–05 replicates the four stylized facts: (i) a severe decline and slow recovery of public investment, (ii) a short-lived decline and quick recovery of public consumption and transfers, (iii) public expenditure tilting towards consumption and transfers, and (iv) an association between public investment decline and restructuring duration.

**Literature Review** Our paper contributes to both theoretical and empirical literature on public debt overhang. Based on theoretical models, Aguiar et al. (2009) explain that a government’s lack of commitment induces a negative correlation between investment and the stock of government debt (a debt overhang effect). Similarly, Ostry et al. (2015) show the distortionary impact of high public debt on investment and growth. With empirical analysis, both Reinhart et al. (2012) and Ostry et al. (2015) find that public debt indebtedness is highly associated

\(^4\)Hamann et al. (2018) also find similar two opposing effects of oil reserves on default.
with low public and private investment and growth in advanced economies. Moreover, our paper provides both new empirical and theoretical explanations on the mechanisms of sovereign debt overhang in two different phases: pre-default and restructuring periods.

In the theoretical literature, the paper is also related to Cuadra et al. (2010), Arellano and Bai (2017), D’Erasmo and Mendoza (2016, 2017), Pouzo and Presno (2016), Hatchondo et al. (2017), Bianchi et al. (2017), and Karantounias (2018), which all study interactions between fiscal policy and a sovereign’s default and external borrowing choice.⁵,⁶ These studies explicitly embed different fiscal policy instruments on expenditure—public consumption and transfers—and on revenue—labor tax, consumption tax, and lump-sum income tax—in the model with endogenous production with elastic labor. The current paper differs from the existing literature in that by introducing public investment, we newly find the role of public capital on a sovereign’s default, debt settlement, and restructuring delays.

Lastly, the theoretical work on sovereign debt restructurings models the outcomes of default and debt renegotiation as a bargaining game between a sovereign debtor and its creditors.⁷ With a multi-round bargaining game, both Benjamin and Wright (2013) and Bi (2008) explain that recovery of the debtor’s repayment capacity generates delays, and Asonuma and Joo (2017) show that both the debtor’s repayment capacity and its creditors’ consumption-smoothing motive interact and drive the delays. On the contrary, Bai and Zhang (2012) find that delays arise due to information asymmetry between the debtor and its creditors. We fill a gap in the literature by explaining an additional channel of delays driven by marginal product of public capital.

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⁵Mendoza et al. (2014) explore interactions between fiscal policy, i.e., different taxation methods and external borrowing choice in highly-integrated two-country set-up without the sovereigns’ default choice.

⁶For empirical analysis on sovereign debt and fiscal policy, see Kaminsky, Reinhart and Vegh (2005), Ilzetzki, et al. (2013) and Franekl et al. (2013).

2 Dataset and Stylized Facts

2.1 New Dataset on Public Expenditure Composition

Confronted with debt crises and restructurings, sovereigns often implement fiscal adjustment programs associated with financing by multilateral sources, e.g., an IMF-supported program. The fiscal adjustment programs feature substantially different treatment in public expenditure compositions, particularly large cuts in public investment in contrast with small cuts in public consumption. Representative episodes are Latin American debt crises in the early 1980s. Argentina, Brazil, Mexico, and Peru implemented partial adjustment programs accompanied by monetization of deficits (Calderon et al. 2003). The momentum toward achieving fiscal austerity under the programs resulted in severe reductions in public investment: 1.1–2.8 percent of GDP on average in 1983–87. The sharp reductions in public investment, which were not completely offset by increases in private investment, resulted in a significant drop in GDP growth.

To explore explicitly the role of public expenditure composition in both sovereign debt overhang and restructurings, it is crucial to identify precisely each category of public expenditure and understand its dynamics. For this purpose, we code new data on the public expenditure composition—consumption, investment, transfers, and capital—at 179 privately-held external debt restructurings over 1978–2010.

One main challenge for this coding exercise was a lack of high quality data on public expenditure composition satisfying criterion for (i) cross-country (in particular defaulting countries), (ii) times series, and (iii) category coverage simultaneously. The IMF World Economic Outlook (WEO) database provides annual data on government spending components, but the data meet only the third criteria and neither the first nor the second—available only for recent years (since 2000) and for advanced economies. The WB Global Development Finance (GDF) database provides annual consumption data, i.e., general government final consumption, while the data meet both the first and second criterion and not the third. This is because the indicator covers only one category of consumption and lacks the compensation of general government employees—one of the large components of public consumption—underestimating total public consumption.

To have high quality data on categories of public expenditure, we therefore combine the limited yearly data on public expenditure from the IMF Fiscal Affairs Department (FAD) and WEO, WB GDF, with rich information from a new broad range of sources. Important quan-
tative sources for us in particular are the IMF Staff Reports from the IMF archives (Article IV consultation, IMF-supported program requests and reviews, information annexes, etc.). For a detailed definition of public consumption and transfers, we follow BEA (2005). The coding decision is documented in detail for each of the 179 restructurings and backed by the exact sources used for coding. Appendix A shows coding examples and the underlying sources for a few exemplary cases.

Table 1 summarizes our public expenditure composition dataset demonstrating four main advantages compared to existing ones—IMF WEO or WB GDF. First of all and most importantly, it is the first comprehensive public expenditure composition data covering wide range of categories including transfers—little has been covered in existing data sources. Second, each expenditure category in our dataset covers at least 70 percent of all restructuring episodes (124 cases out of 179 episodes). Third, each expenditure category covers the time window around restructuring episodes: pre-restructuring, restructuring and post-restructuring periods. Fourth, each expenditure series is comprised of sub-categories; for instance, public consumption series include compensation of general government employees and pension.

Table 1: Public Consumption, Investment, Transfers and Capital for Restructurings in 1978–2010

<table>
<thead>
<tr>
<th>Restructuring Episodes</th>
<th>Observation</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev.</th>
<th>Percent of GDP</th>
<th>Restructuring periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Consumption, average</td>
<td>179</td>
<td>124</td>
<td>13.1</td>
<td>9.4</td>
<td>124</td>
<td>12.0</td>
</tr>
<tr>
<td>Public Investment, average</td>
<td>151</td>
<td>151</td>
<td>4.7</td>
<td>3.4</td>
<td>151</td>
<td>3.7</td>
</tr>
<tr>
<td>Public Transfers, average</td>
<td>124</td>
<td>124</td>
<td>5.3</td>
<td>3.1</td>
<td>124</td>
<td>3.9</td>
</tr>
<tr>
<td>Public Capital, average</td>
<td>151</td>
<td>151</td>
<td>75.0</td>
<td>49.3</td>
<td>151</td>
<td>74.2</td>
</tr>
</tbody>
</table>

1/ For all components of public expenditure, our dataset has both series in real and level (constant 2011 US dollars), and in percent of GDP.

2/ For each restructuring episode, we take an average of public expenditure series for corresponding periods: (i) pre-restructuring periods, i.e., over 3 years before the start of restructurings; (ii) restructuring periods, i.e., from the start to the end of restructurings; (iii) post-restructuring periods, i.e., over 3 years after the end of restructurings. Then, we take an average of the obtained statistics across restructuring observations.

BEA (2005) defines that expenditures consisting of compensation of general government employees, consumption of fixed capital (CFC), and intermediate purchases of goods and services less sales to other sectors and own-account production of structures and software. It excludes current transactions of government enterprises, interest paid or received by government, and subsidies.
2.2 Empirical Results: Four Stylized Facts

We merge our newly-constructed data with the existing dataset on the duration and strategies (preemptive or post-default) of restructurings from Asonuma and Trebesch (2016). Asonuma and Trebesch (2016) differentiate post-default episodes (111 cases covering 62 percent of all episodes)—the government defaults first and renegotiates its debt—from preemptive exchanges—negotiations take place prior to a payment default. Our findings for post-default debt restructurings in 1978–2010 can be summarized in four main stylized facts.\textsuperscript{12}

**Stylized fact 1:** Public investment experiences a severe decline and slow recovery around restructurings.

Figure 1: Public Investment around Restructurings

Panel (i) shows that public investment declines markedly at the onset of debt crisis (year 0) and stays below the pre-crisis level in the subsequent years. Public investment only recovers to the pre-crisis level in year 4, leading to the debt settlement in year 5. Average investment in the restructuring period shown by the red dashed line is significantly lower than that in the pre-restructuring period.

pre-restructuring period shown by the green dotted line. Panel (ii) shows that public investment increases steadily after the settlement (year 0). Average investment in the post-restructuring period shown by the purple dotted line is significantly higher than that in the restructuring period.

- Stylized fact 2: Public consumption and transfers experience a short-lived decline and quick recovery around restructurings.

Figure 2: Public Consumption and Transfers

(i) Around Start of Restructurings

(ii) Around End of Restructurings

Figure 2 shows the dynamics of public consumption and transfers around restructurings. We follow the same presentation approach as in Figure 1 in terms of both time horizon, timing of events (both start and end of debt crisis), scale (real and level), normalization of the series at levels at the two events, and average in the three periods. Panel (i) shows that public consumption and transfers drop at the onset of default (year 0). At the same time, public consumption and transfers start to quickly recover and reach the pre-crisis level in year 2. Owing to the quick recovery, average public consumption and transfers in the restructuring period shown by the red dashed line are slightly higher than that in the pre-restructuring period shown by the green dotted line. Panel (ii) shows that public consumption and transfers increase mildly after the settlement (year 0). Average consumption and transfers in the post restructuring period shown by the purple dotted line is significantly higher than those in the restructuring period.

A contrast between Figures 1 and 2 shows a substantial difference in the dynamics of public consumption and transfers, and investment. Public investment experiences a severe decline and slow recovery, while public consumption and transfers experience a short-lived decline and quick recovery.

To obtain more systemic and robust evidence—not contaminated by business cycle effects—we apply a standard panel fixed effects regression of public investment, consumption, and transfers (all measured as a deviation from the trend) for post-default restructurings reported in
Table 2. We use two measures of public investment: (i) public investment deviation from the trend, and (ii) growth rate of public capital. Main explanatory variables are dummy variables for restructuring and post-restructuring periods, and lagged public and publicly-guaranteed (PPG) external debt (in percent of GDP) from the World Bank World Development Indicators (WDI) database. GDP deviation from the trend is included to control the effects of the business cycle.

Table 2: Public Investment, Capital, Consumption, and Transfers around Restructurings

<table>
<thead>
<tr>
<th></th>
<th>Public Investment</th>
<th>Public Capital</th>
<th>Public Consumption</th>
<th>Public Transfers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>deviation from trend, current</td>
<td>percentage change, current</td>
<td>deviation from trend, current</td>
<td>percentage change, current</td>
</tr>
<tr>
<td>Restructuring period (current, dummy)</td>
<td>-0.14*** (-0.03)</td>
<td>-1.26*** (-0.23)</td>
<td>0.007 (0.011)</td>
<td>0.02 (0.03)</td>
</tr>
<tr>
<td>Post-restructing period (current, dummy)</td>
<td>-0.07** (0.03)</td>
<td>-0.87*** (0.24)</td>
<td>0.003 (0.012)</td>
<td>-0.04 (0.03)</td>
</tr>
<tr>
<td>PPG external debt (lagged, percent of GDP)</td>
<td>-0.0007*** (0.003)</td>
<td>-0.01*** (0.002)</td>
<td>0.00003 (0.0001)</td>
<td>0.000005 (0.0002)</td>
</tr>
<tr>
<td>GDP, deviation from trend (current, percent)</td>
<td>0.03*** (0.003)</td>
<td>0.07*** (0.03)</td>
<td>0.011*** (0.001)</td>
<td>0.012*** (0.003)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.10*** (0.03)</td>
<td>4.18*** (0.21)</td>
<td>-0.008* (0.01)</td>
<td>0.01 (0.03)</td>
</tr>
<tr>
<td>Episode-specific fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Number of restructuring episodes</td>
<td>97</td>
<td>96</td>
<td>92</td>
<td>81</td>
</tr>
<tr>
<td>Number of observations</td>
<td>1,043</td>
<td>996</td>
<td>949</td>
<td>747</td>
</tr>
<tr>
<td>F-statistics</td>
<td>37.94</td>
<td>21.39</td>
<td>19.59</td>
<td>4.02</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.139</td>
<td>0.087</td>
<td>0.084</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Notes: The table shows results from fixed effects OLS regressions. The dependent variables are public investment deviation from the trend in column (1), public capital percentage change in column (2), public consumption deviation from the trend in column (3), and public transfers deviation from the trend in column (4). The main explanatory variables are dummy variables for restructuring and post-restructuring periods, and lagged public and publicly-guaranteed (PPG) external debt (in percent of GDP). Significance levels are denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. All regressions include episode-specific fixed effects. Robust standard errors clustered on the episode level are in parentheses.

1/ A dummy variable for the restructuring period is set 1 prior to the start of restructurings and 0 during both restructuring and post-restructuring periods.

2/ A dummy variable for the post-restructuring period is set 1 after to the end of restructurings and 0 during both pre-restructuring and restructuring periods.

3/ A deviation from the trend is a percentage deviation from the trend obtained by applying a Hodrick-Prescott (HP) filter to annual series with filter of 6.25.

4/ Percentage change of public capital from the level in the previous year.

The main result reported in Table 2 is that public investment—measured both deviation from the trend and percentage change in public capital—is significantly lower during restructurings than that in the pre-restructuring period (columns 1 and 2). Quantitatively, on average, public capital growth rate is lower by 1.3 percent during restructurings than that in the pre-restructuring period. Public investment is significantly higher in post-restructuring period than that during restructurings. The second result concerns the effect of public external debt on public investment: public investment is negatively and significantly correlated with public external debt, i.e., “debt overhang effects”. Our innovation here is to find the evidence on emerging and
low-income countries (EM and LIC) during debt crisis. The empirical literature mostly focuses on the debt overhang effects in advanced economies (Reinhart et al. 2012; Ostry et al. 2015).

On the contrary, neither public consumption nor transfers in the restructuring period differs significantly from that in the pre- or post-restructuring periods (columns 3 and 4). Furthermore, both consumption and transfers are positively and insignificantly correlated with public external debt.

• **Stylized fact 3: Public expenditure tilts remarkably towards consumption and transfers during debt crisis.**

Figure 3: Public Expenditure Composition around Restructurings

Panels (i) and (ii) in Figure 3 show average changes in public consumption and transfers, and investment, respectively—measured as percent of GDP between the pre-restructuring and restructuring periods for our sample of post-default restructurings. To be comparable with average in pre-restructuring period, we take average over the first 3 years during debt restructurings. A difference in public consumption/transfers-to-GDP ratio between pre-restructuring and restructuring periods is small, i.e., 0.7 percent of GDP on average—equivalent to 4 percent of pre-restructuring consumption/transfers-to-GDP ratio. On the contrary, a substantial difference in public investment-to-GDP ratio, i.e., 0.9 percent of GDP on average—equivalent to 21 percent of pre-restructuring investment-to-GDP ratio—emerges between pre-restructuring and restructuring periods.

Panel (iii) in Figure 3 shows that share of public investment in public expenditure is reduced to 19 percent in the restructuring period—2 percent lower than that in the pre-restructuring period. Public expenditure tilts significantly towards consumption and transfers during debt restructuring.

Table 3 provides econometric support for this stylized fact. It reports results of panel fixed effects regression of public expenditure component (both in percent of GDP and percent of
Table 3: Public Expenditure Composition around Restructurings

<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
<th>Column (2)</th>
<th>Column (3)</th>
<th>Column (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Investment (percent of GDP)</td>
<td>-0.85***</td>
<td>-0.79*</td>
<td>0.38**</td>
<td>-2.23***</td>
</tr>
<tr>
<td>Restructuring period (current, dummy)</td>
<td>(0.18)</td>
<td>(0.44)</td>
<td>(0.18)</td>
<td>(0.80)</td>
</tr>
<tr>
<td>Restructuring period*PPG external debt</td>
<td>-0.003**</td>
<td>-0.005</td>
<td>-0.009</td>
<td>-0.084</td>
</tr>
<tr>
<td>(lagged, percent of GDP)</td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Post-restructuring period (current, dummy)**</td>
<td>-0.47*</td>
<td>-1.04</td>
<td>0.38</td>
<td>-0.58</td>
</tr>
<tr>
<td>(lagged, percent of GDP)</td>
<td>(0.27)</td>
<td>(0.65)</td>
<td>(0.26)</td>
<td>(1.19)</td>
</tr>
<tr>
<td>GDP, deviation from trend (current, percent)**</td>
<td>0.03**</td>
<td>0.02</td>
<td>0.03*</td>
<td>0.08</td>
</tr>
<tr>
<td>Constant</td>
<td>4.25***</td>
<td>13.08***</td>
<td>2.99***</td>
<td>22.58***</td>
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<tr>
<td>Episode-specific fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Number of restructuring episodes</td>
<td>95</td>
<td>93</td>
<td>93</td>
<td>91</td>
</tr>
<tr>
<td>Number of observations</td>
<td>1,028</td>
<td>882</td>
<td>882</td>
<td>863</td>
</tr>
<tr>
<td>F-statistics</td>
<td>13.47</td>
<td>7.46</td>
<td>1.89</td>
<td>3.79</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.068</td>
<td>0.045</td>
<td>0.012</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Notes: The table shows results from fixed effects OLS regressions. The dependent variables are public investment (percent of GDP) in column (1), public consumption (percent of GDP) in column (2), public transfers (percent of GDP) in column (3), and public investment (percent of public expenditure) in column (4). The main explanatory variables are dummy variables for restructuring and post-restructuring periods, and interactive terms of dummy variables for restructuring and post-restructuring periods and lagged PPG external debt (percent of GDP). Significance levels denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. All regressions include episode-specific fixed effects. Robust standard errors clustered on the episode level in parentheses.

1/ A dummy variable for the restructuring period is set 1 during restructuring periods and 0 during both pre-restructuring and post-restructuring periods.

2/ An interactive term of dummy variable for the restructuring period and lagged public and publicly guaranteed debt (percent of GDP).

3/ A dummy variable for the post-restructuring period is set 1 after to the end of restructurings and 0 during both pre-restructuring and restructuring periods.

4/ An interactive term of dummy variable for the post-restructuring period and lagged public and publicly guaranteed debt (percent of GDP).

5/ A deviation from the trend is a percentage deviation from the trend and annual change in the trend obtained by applying a Hodrick-Prescott (HP) filter to annual series with filter of 6.25.

Public investment on average is significantly reduced by 0.9 percent of GDP—equivalent to 20 percent of estimated average public investment—during restructurings (column 1). Moreover, the reduction in public investment is larger when PPG external debt is high. The sum of these two terms correspond to “debt overhang effects during debt restructurings”. This is a new finding in the empirical literature on sovereign debt overhang.

Public consumption on average is less remarkably reduced by 0.8 percent of GDP—equivalent to 6 percent of estimated average public consumption—in the restructuring period (column 2). The reduction in public consumption is not significantly associated with level of PPG external debt. In contrast, public transfers are significantly increased by 0.4 percent—equivalent to 13 percent of estimated average public transfers—during restructurings (column 3) and this increase...
is not significantly associated with level of PPG external debt. These are consistent with scatter plots on PPG external debt and public expenditure components in Figure B3 in Appendix B.2.

Most importantly, as a result of these changes in public expenditure components, Column 4 shows that the share of public investment in public expenditure is significantly lower (by 2.2 percent of public expenditure) during restructurings than that in the pre-restructuring period.

- **Stylized fact 4: The sharp decline in public investment is associated with long restructuring duration.**

Figure 4 shows a scatter plot of duration of restructurings and the decline in public investment at the start of restructurings—measured as percentage point change of public investment-to-GDP ratio over two years from the pre-restructuring year (year -1) to one year after the start of restructurings (year 1). Restructuring duration and the decline in public investment are negatively correlated shown by a downward-sloping fitted line. It shows that restructurings are protracted when sovereign debtors experience the severe decline in public investment.

Figure 4: Public Investment Decline and Duration of Restructurings

The correlation is also supported by Table B.1 in Appendix B.3 which reports results of a cross-sectional regression of restructuring duration for our sample of post-default restructurings. We use several measures of public investment decline: (i) cumulative differences in public investment-to-GDP ratio over the first two and one years, respectively; (ii) a cumulative percentage change in public investment (level) over the first two years. Column (1) shows results

---

13 The stylized facts presented in this subsection are not necessarily causal. We do not claim to identify an effect from the sovereign’s public expenditure choice to outcomes of restructurings i.e., duration.
for a bare-bones model with public investment decline. In columns (2)–(4) we add a con-
tentional set of controls for restructuring duration used in the empirical literature on sovereign
debt (Kohlscheen 2009; Trebesch 2018; Bai and Zhang 2012; Asonuma and Joo 2017). The
set includes (i) the debtors’ macroeconomic variables—GDP deviation from the trend obtained
by applying a Hodrick-Prescott (H-P) filter, external debt-to-GDP ratio, export-to-debt service
ratio (all at the end of restructurings), (ii) external financing such as a dummy variable for
an IMF-supported program and London Interbank Offered Rate (LIBOR), (iii) a restructuring
method variable (a dummy variable for bond exchanges), and (iv) pre-restructuring level of
public capital.

Regression results confirm robustness of the negative correlation between public investment
decline and duration of restructurings for different measures of public investment decline.
3 Theoretical Model

3.1 Summary of Theoretical Findings

Our theoretical model is built for the purpose of shedding light on the role of public capital and debt overhang during sovereign debt crisis. In particular, our model of sovereign debt embeds explicitly endogenous public capital accumulation and production, and post-default multi-round renegotiations with a risk averse sovereign and its risk-neutral creditors, and replicates the aforementioned four stylized facts. To account for different economic situations for sovereign debtors, we take a two-step approach. First, we use a conventional small open economy model with sovereign debt and fiscal policy—public and private sectors are separated by distortionary consumption tax and two different consumption goods in line with Cuadra et al. (2010) and Arellano and Bai (2017)—as benchmark and derive main qualitative results in Sections 3, 4, and 5. Then we incorporate each of the specific assumptions used in the previous studies (Gordon and Guerron-Quintana 2018; Arellano and Bai 2017; Cuadra et al. 2010; Benjamin and Wright 2013) in our framework and show robustness of our model in Appendix E.

First, the model provides main implications on the sovereign’s default, debt settlements and restructuring delays. After default, the sovereign delays the renegotiations when public capital is low; it opts to invest in public capital and refrains using resources for recovered debt repayments given high marginal product of public capital (equivalent to high shadow value of public capital). On the contrary, the sovereign agrees on debt settlement when public capital is high; it hesitates to invest in public capital and chooses to use resources for recovered debt payments given low marginal product of public capital (corresponding to low shadow value of public capital). As a result, restructuring delays and debt settlements are driven by both marginal product of public capital and recovery of repayment capacity (Benjamin and Wright 2013; Bi 2008)—this corresponds to recovery of productivity and an associated response of elastic labor in our model.

Before default, i.e., ex-ante, there are two opposing effects of public capital on default similar to those of “total” capital in Gordon and Guerron-Quintana (2018). On the one hand, when public capital is high, repayment capacity, i.e., output is high—defined as “smoothing channel” in their paper—resulting in low likelihood of default. On the other hand, when public capital is high, costs of financial exclusion are low—defined as “autarky channel” in their paper—because public capital plays its role of smoothing household’s utility. This leads to high likelihood of default. In addition, our model newly includes the third channel, “restructuring channel”—quick settlements—which reduces costs of default resulting in high likelihood of default. As a result, total net effects of public capital through these three channels on default are balanced in our model.

Second, our model explains mechanisms of sovereign debt overhang in two phases. Prior to default, low productivity, high external debt payments, and the sovereign’s consumption-smoothing motive and myopia interact and drive a sharp decline in investment leading to debt overhang (Aguirar et al. 2009). On the one hand, the sovereign with a high consumption-smoothing motive and myopia opts to smooth the household’s utility through public consumption
and transfers when output is low. On the other hand, the sovereign disproportionally reduces public investment due to high burden of external debt payments.

During debt restructurings, a combination of slow recovery of productivity, no external borrowing, and the sovereign’s consumption-smoothing motive and myopia generates both deceleration of public capital accumulation and lengthy renegotiations, which in turn, interact and result in debt overhang. Public capital accumulation is slow due to both no external borrowing and the sovereign’s consumption-smoothing motive and myopia. Renegotiations are delayed because of both high marginal product of public capital (when public capital is low) and slow recovery of productivity. This cycle continues until the sovereign accumulates public capital to a high level and agrees on debt settlement.

3.2 General Points

There are four agents in the model: a household, a firm, a sovereign government, and foreign creditors. The sovereign (equivalently household) is risk averse and cannot affect the global risk-free interest rate. Foreign creditors are risk neutral, and they can borrow or lend as much as needed at a constant risk-free interest rate in the international capital market.

In each period, an exogenous stochastic productivity shock $a_t$ materializes. It is stochastic, drawn from a compact set $A = [a_{\text{min}}, a_{\text{max}}] \subset R$. $\mu(a_{t+1}|a_t)$ is probability distribution of a shock $a_{t+1}$ conditional on previous realization $a_t$. In addition, the sovereign has a credit record $h_t \in [0, 1]$, which indicates whether the sovereign has maintained access to the market ($h_t = 0$) or whether it has lost market access due to defaults ($h_t = 1$).

After observing the productivity shock, the sovereign government receives consumption tax revenues and decides expenditure composition—public consumption, investment and transfers— together with its default and restructuring choice and external borrowing. Total consumption tax revenues are determined by the household’s optimal choice of private consumption given constant consumption tax rate.$^{14}$ Public consumption and transfers are provided to the household to smooth his utility directly or through private consumption, respectively. Public capital rented to the firm is accumulated through net investment and is also subject to depreciation and adjustment costs.

The household receives profits from the firm, and public consumption and transfers from the government, respectively. He then chooses consumption and labor supply and pays taxes to the government. The firm chooses labor demand, produces consumption goods using labor, public capital and private capital—assumed to be constant$^{15}$—, and pays profits to the household.

The sovereign bond market is incomplete. The sovereign can borrow and lend only via one-period, zero-coupon sovereign bonds.$^{16}$ $b_{t+1}$ denotes the amount of bonds to be repaid in the

---

$^{14}$We follow the convention in the theoretical literature (Arellano and Bai 2017; Alfaro and Konczuk 2016), which is supported by findings on value-added taxes in developing countries in Gunter et al. (2017).

$^{15}$The assumption follows the theoretical literature with production (e.g., Mendoza and Yue 2012; Azzimonti 2015).

$^{16}$Our model of debt restructuring with one-period bonds follows Benjamin and Wright (2013), Bi (2008), and Yue (2010). Relaxing the model to include long-duration bonds does not provide additional insights but
next period whose set is shown by $B = [b_{\text{min}}, b_{\text{max}}] \subset \mathbb{R}$ where $b_{\text{min}} \leq 0 \leq b_{\text{max}}$. We set the lower bound for the sovereign’s bond holding at $b_{\text{min}} = -y_{\text{max}}/r^*$, which is the largest debt that the sovereign could repay. The upper bound $b_{\text{max}}$ is the high level of assets that the sovereign may accumulate.\footnote{17} We assume $q(b_{t+1}, k_{t+1}^g, 0, a_t)$ to be price of sovereign bonds with sovereign’s asset position $b_{t+1}$, public capital $k_{t+1}^g$, good credit record ($h_t = 0$), and a productivity shock $a_t$. The bond price is determined in equilibrium.

We assume that the creditors always commit to repay their debt. However, the sovereign is free to decide whether to repay its debt or to default. If the sovereign chooses to repay its debt, it will preserve access to the international capital market in the next period. On the contrary, if it chooses not to pay its debt, it is then subject to both exclusion from the international capital market and direct productivity loss.\footnote{18,19} When a default occurs, the sovereign and the creditors negotiate a reduction of unpaid debt via a multi-round bargaining. At the renegotiation, one party, who is randomly selected with exogenous and constant probability, chooses whether to propose an offer with haircuts (recovery rates) or to pass its option. The other party decides whether to accept or reject the proposal. If the offer with haircuts is proposed and accepted, then the sovereign regains access to the international capital market in the next period ($h_{t+1} = 0$), and the creditors receive recovered debt payments. Otherwise, both parties continue the negotiation over debt in arrears in the next period.

In order to avoid permanent exclusion from the international capital market and direct productivity losses, the sovereign has an incentive to negotiate over haircuts. Similarly, the creditors are also willing to negotiate over the reduction of unpaid debt because they prefer to maximize the recovered debt payments.

### 3.3 Timing of the Model

Figure 5 summarizes the timing of decisions within each period.

1. The sovereign starts current period with initial assets/debt, and public capital. We are in increases technical difficulty to track the model. This is because old bonds are exchanged with new bonds with the same maturity and smaller outstanding (debt stock), i.e., no change in maturity structure of bonds due to an exchange (Hatchondo et al. 2014; Sunder-Plassmann 2018). See Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyingungor (2012) for long-duration bond models without debt restructurings.

\footnote{17} exists when the interest rates on the sovereign’s savings are sufficiently low compared to the discount factor, which is satisfied as $(1 + r^*)\beta < 1.$

\footnote{18} The direct productivity loss assumption in our production model is conceptually equivalent to "output costs" assumption in the conventional endowment model e.g., Arellano (2008), Aguiar and Gopinath (2006), and Yue (2010). In this regard, the direct production loss is widely accepted in the sovereign debt literature with endogenous production (Cuadra et al. 2010; Arellano and Bai 2017; Gordon and Guerron-Quintana 2018). Both assumptions are broadly in line with empirical estimates of output loss at the time of defaults and restructurings (Sturzenegger 2004; Tomz and Wright 2007; Borenstein and Panizza 2009; De Paoli et al. 2009; Levy-Yeyati and Panizza 2011; Asuomua and Trebesch 2016; Trebesch and Zabel 2017; Asuomua et al. 2017).

\footnote{19} Mendoza and Yue (2012) provide micro-foundation of this conventional assumption that exclusion from credit markets leads to losses in production efficiency, which result in output costs due to a lack of imported inputs and labor reallocation away from final goods production.
Figure 5: Timing of Model

2. A productivity shock \((a_t)\) realizes. The sovereign decides whether to repay its debt or to default.

3. (a) In node (B) (repayment node), if repayment is chosen, we move to the upper branch of a tree. The sovereign maintains market access \((h_{t+1} = 0)\) and chooses assets/debt, and public consumption, capital, and transfers. Default risk is determined and foreign creditors choose next-period sovereign bonds. Bond prices are determined in the market. The household chooses his consumption and labor supply, and the firm chooses labor demand. We proceed to node (A) next period.

(b) In node (C) (default node), if default is chosen, we move on to the lower branch of a tree. The sovereign loses access to the international capital market \((h_{t+1} = 1)\), suffers the direct productivity loss, and chooses public consumption, capital and transfers. The household chooses his consumption and labor supply, and the firm chooses labor demand.

4. A productivity shock \((a_{t+1})\) realizes.

5. In node (D) (default node), with constant probability, the sovereign has an opportunity to propose an offer to its creditors. Otherwise, the creditors do. The proposer decides whether to propose an offer to the other party.
6. (a) In node (E) (propose node), if the proposer chooses to propose, the other party decides whether to accept or reject the offer. If the other accepts the offer, the sovereign regains market access in the next period \((h_{t+2} = 0)\). We move back to node (A) next period. On the contrary, if the other rejects the offer, the sovereign remains in autarky \((h_{t+2} = 1)\). We again move to node (D).

(b) In node (F) (pass node) if the proposer opts to pass, the sovereign remains in autarky \((h_{t+2} = 1)\). We again move to node (D).
4 Recursive Equilibrium

4.1 Household’s Problem

This section defines the stationary recursive equilibrium of our model. A representative household maximizes a standard time-separable utility function:

\[ E^0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t, g_t) \]

where \( U(c_t, l_t, g_t) = (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) \)

where \( 0 < \beta < 1 \) is the discount factor and \( c_t, l_t, g_t \) denote private consumption, labor supply and public consumption in period \( t \), respectively. \( U(\cdot) \) is the period utility function, which is separable between a multiple of private consumption and labor supply, and public consumption. Both \( u(\cdot) \) and \( v(\cdot) \) are continuous, strictly increasing, strictly concave, and satisfy the Inada conditions. \( \lambda \) denotes the weight on public consumption in the household’s utility function.

The household takes as given the wage rate \( w_t \), profits paid by a firm \( \pi^F_t \), public transfers \( T_t \), public consumption \( g_t \) and taxation \( \tau \), and chooses consumption and labor supply. He does not borrow directly from abroad, but the government borrows, provides public consumption and transfers, and makes default decisions internalizing the household’s utility.\(^{20}\) The household’s optimization problem is written as:

\[
\max E^0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) \right] \tag{1}
\]

s.t. \( (1 + \tau)c_t = w_t l_t + \pi^F_t + T_t \tag{2} \]

where \( \tau \) is consumption tax rate assumed to be constant.

The optimality condition of the household is as follows:

\[
\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{w_t}{1 + \tau} \tag{3}
\]

4.2 Firm’s Problem

A representative firm chooses labor demand \( l_t \) and public capital stock \( k^p_t \) for goods production given both exogenous productivity shock \( a_t \) and fixed private capital stock \( \bar{k}p(=1) \). The production function is Cobb-Douglas shown as:

\[
y_t = a_t(l_t)^{\alpha_l}(k^p_t)^{\alpha_k}(\bar{k}p)^{1-\alpha_l-\alpha_k} \tag{4}
\]

\(^{20}\)Though the household lacks access to the external market as in conventional sovereign debt models, there are still three methods available for him to smooth utility: (i) private consumption through public transfers, (ii) public consumption, and (iii) the choice of labor supply.
where $0 < \alpha_l, \alpha_k < 1$.

The firm’s optimization problem is written as follows:

$$\max_{l_t} \pi_t^F = a_t(l_t)\alpha_l (k_t^g)^{\alpha_l} (\bar{k}^p)^{1-\alpha_l - \alpha_k} - w_t l_t$$  \hspace{1cm} (5)$$

The optimality condition of the firm is as follows:

$$w_t = \alpha_l a_t(l_t)^{\alpha_l-1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1-\alpha_l - \alpha_k}$$  \hspace{1cm} (6)$$

### 4.3 Sovereign Government’s Problem

The sovereign maximizes its expected lifetime utility, and its value function is denoted by $V(b_t, k_{t+1}^g, h_t, a_t)$. First, we start with its problem when the sovereign has a good credit record ($h_t = 0$).

For $b_t \geq 0$ ($h_t = 0$) where the sovereign has savings, it receives tax revenues from the household and debt repayments from the creditors, and determines public consumption, capital and transfers, and next-period assets.\(^{21}\)

\[
V(b_t, k_{t+1}^g, 0, a_t) = \max_{g_t, b_{t+1}, k_{t+1}^g, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t)
\]  \hspace{1cm} (7)$$

\[s.t. \quad g_t + k_{t+1}^g + T_t + q(b_{t+1}, k_{t+1}^g, 0, a_t)b_{t+1} = \tau c_t + (1 - \delta^g)k_{t+1}^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g + b_t \]  \hspace{1cm} (8)$$

\[T_t \geq 0 \]  \hspace{1cm} (9)$$

\[
\frac{u_t(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t(l_t)^{\alpha_l-1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1-\alpha_l - \alpha_k}}{1 + \tau}
\]  \hspace{1cm} (10)$$

\[ (1 + \tau)c_t = y_t + T_t \]  \hspace{1cm} (11)$$

where equation (8) is the budget constraint for the sovereign government where it receives consumption tax revenues $\tau c_t$, post-adjustment costs capital stocks $(1 - \delta^g)k_{t+1}^g - \frac{\Omega}{2}(\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g$—non-linear adjustment costs are assumed\(^{22}\) and $\delta^g$ is the depreciation rate of capital—and savings $b_t$, and allocates to public consumption $g_t$, capital $k_{t+1}^g$, transfers $T_t$ and next-period assets/debt $q(b_{t+1}, k_{t+1}^g, 0, a_t)b_{t+1}$. Equation (9) is the “non-lump sum taxation constraint”—which corresponds to “fiscal constraint” in Arellano and Bai (2017)—indicating a limitation of the gov-

\(^{21}\)In this case, two assets—external bonds with risk-free returns and investment with state-dependent returns—co-exist due to a state-dependent difference in returns, and the sovereign optimally allocates its total savings according to their returns.

\(^{22}\)Non-linear adjustment costs are assumed in order to replicate smooth investment dynamics. Replacing with quadratic adjustment costs provides the same qualitative results.
government from transferring resources from the private sector. Mechanically, the government can freely transfer positive net borrowing through transfers, but cannot extract more resources from the private sector beyond the distortionary consumption tax revenues. Equations (10) and (11) denote combined optimality condition and budget constraint for both the household and the firm, respectively.

For \( b_t < 0 \) (\( h_t = 0 \)), where the sovereign has debt, it decides whether to repay or to default after observing its productivity shock. If the sovereign decides to repay its debt, it determines public consumption, capital, transfers, and next-period assets/debt.

In contrast, if the sovereign chooses to default, it will be excluded from the international capital market and its credit record deteriorates to \( h_{t+1} = 1 \), with debt in arrears \( b_{t+1} = (1 + r^*)b_t \) in the next period where \( r^* \) is a constant risk-free interest rate. After suffering the direct productivity loss, the sovereign determines public expenditure composition (consumption, capital, and transfers).

\[
V(b_t, k^g_t, 0, a_t) = \max \left[ V^R(b_t, k^g_t, 0, a_t), V^D(b_t, k^g_t, 0, a_t) \right]
\]

(12)

where \( V^R(b_t, k^g_t, 0, a_t) \) is its value associated with repaying debt:

\[
V^R(b_t, k^g_t, 0, a_t) = \max_{g_t, b_{t+1}, k^g_{t+1}, T_t} \left( 1 - \lambda \right) u(c_t, l_t) + \lambda v(g_t) + \beta \int_A V(b_{t+1}, k^g_{t+1}, 0, a_{t+1}) d\mu(a_{t+1}|a_t)
\]

(7')

s.t. \( g_t + k^g_{t+1} + T_t + q(b_{t+1}, k^g_{t+1}, 0, a_t)b_{t+1} = \tau c_t + (1 - \delta^g)k^g_t - \Omega \frac{k^g_{t+1} - k^g_t}{k^g_t} + b_t \)

(8)

\[
T_t \geq 0
\]

(9)

\[
\frac{u_t(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t(l_t)^{\alpha_l-1}(k^g_t)^{\alpha_k}(\bar{k}^g)^{1-\alpha_l-\alpha_k}}{1 + \tau}
\]

(10)

\[
(1 + \tau)c_t = \bar{y}_t + T_t
\]

(11)

and \( V^D(b_t, k^g_t, 0, a_t) \) is its value associated with default

\[
V^D(b_t, k^g_t, 0, a_t) = \max_{g_t, b_{t+1}, k^g_{t+1}, T_t} \left( 1 - \lambda \right) u(c_t, l_t) + \lambda v(g_t) + \beta \int_A V((1 + r^*)b_t, k^g_{t+1}, 1, a_{t+1}) d\mu(a_{t+1}|a_t)
\]

(13)

s.t. \( g_t + k^g_{t+1} + T_t = \tau c_t + (1 - \delta^g)k^g_t - \frac{\Omega}{2} \frac{k^g_{t+1} - k^g_t}{k^g_t}^2 k^g_t \)

(8')

\[
T_t \geq 0
\]

(9')

\[
\frac{u_t(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t(l_t)^{\alpha_l-1}(k^g_t)^{\alpha_k}(\bar{k}^g)^{1-\alpha_l-\alpha_k}}{1 + \tau}
\]

(10')

\[
(1 + \tau)c_t = \bar{y}_t + T_t
\]

(11')
where $\bar{a}_t$ and $\bar{y}_t = \bar{a}_t(l_t)^{\alpha_l}(k^g_t)^{\alpha_k}(\bar{k}^p)^{1-\alpha_l-\alpha_k}$ denote the direct productivity loss and its associated output.

Next comes the sovereign’s problem with a bad credit record with unpaid debt arrears ($h_t = 1$ & $b_t < 0$). The sovereign is currently excluded from the international market, suffering the direct productivity loss, and may settle on recovery rates through renegotiations with the creditors. The renegotiation process determines endogenously both recovery rates and length of financial autarky. Its value of remaining in financial autarky and continuing renegotiations is denoted by $V(b_t, k^g_t, 1, a_t)$, equivalent to an expected payoff that the debtor obtains from the bargaining game starting from period $t$:

$$V(b_t, k^g_t, 1, a_t) = \Gamma(b_t, k^g_t, a_t)$$

The sovereign’s default policy can be characterized by default set $D(b_t, k^g_t, 0)$, defined as a set of productivity shocks $a_t$, which default is optimal given the sovereign’s debt $b_t$, public capital $k^g_t$ and its good credit record ($h_t = 0$):

$$D(b_t, k^g_t, 0) = \{a_t \in A : V_R(b_t, k^g_t, 0, a_t) < V^D(b_t, k^g_t, 0, a_t)\}$$

### 4.4 Debt Renegotiation

The debt renegotiation takes the form of a two-player stochastic bargaining game with complete information as in Merlo and Wilson (1995). It is a multi-round stochastic bargaining game in that both the productivity process of the sovereign debtor and the identity of the proposer are stochastic, as in previous studies on a multi-round bargaining game (Benjamin and Wright 2013; Bi 2008). The creditors’ incentive to delay the settlement is identical to that in previous studies on multi-round renegotiations (Benjamin and Wright 2013; Bi 2008): the risk-neutral creditors (with constant discount rate), who care only about recovery rates in present value terms, prefer to wait for the sovereign’s willingness to repay high recovered debt payments.

More importantly, however, the sovereign’s incentive to delay the renegotiation clearly differentiates our model from that in these previous papers: in their models, the sovereign is willing to wait for recovery of capacity of repayment, i.e., output which follows exogenous process. In contrast, in our model, what determines the sovereign’s choice of delay and settlement are state-dependent benefits and costs of public investments, i.e., marginal product of public capital. The sovereign opts to postpone the settlement because it prioritizes public capital accumulation over debt settlement—benefits of public investment outweigh costs—until public capital reaches high level.

In every round, a state is realized and the proposer is randomly selected. For simplicity, each player has a constant probability of being selected as the proposer in each round of the

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23 While the bargaining game between two parties could be modeled in other different forms, we follow the conventional bargaining game in Merlo and Wilson (1995) for their simplicity and tractability.

24 Asunuma and Joo (2017) consider the risk averse creditor whose consumption-smoothing motive is state-dependent. In their framework, the creditor’s income process influences not only outcomes, but also equally importantly, the timing of the debt settlement.
negotiation. That is, the identity of the proposer is independent of the sovereign’s productivity process. Let \( \phi \) denote the probability that the borrower, \( B \), can propose and \( 1 - \phi \) the probability that the lender, \( L \), can propose. The probability, which one of the players is selected as the proposer is a parsimonious way to reflect the bargaining power obtained through one’s ability to enjoy the first-mover advantage. The proposer may either propose recovery rates or pass. If he proposes, then the other player chooses to accept or to reject the proposal.\(^{25} \) If the proposal is accepted, then the sovereign debtor immediately repays its reduced debt arrears, and then resumes access to the international capital market in the next period (\( h_{t+1} = 0 \)) with no outstanding debt. If the proposal is rejected, both parties repeat the bargaining game in the next period. If the proposer opts to pass, both parties proceed to the next period and continue the bargaining game.

First, we define some basic concepts of the game. A stochastic bargaining game is denoted by \( (C, \beta, 1/(1 + r^*)) \), where for each productivity process \( a \in A \), \( C(a) \) is the set of feasible utility vectors that may be materialized upon in the state. \( \beta \) and \( 1/(1 + r^*) \) are the discount factors for \( B \) and \( L \), respectively.\(^{26} \) A payoff function is an element \( \Delta(a) \in C(a) \), where \( \Delta_i(a) \) is the utility to player \( i \) for \( i = B, L \).

As in Merlo and Wilson (1995), we focus on a game with stationary strategies, that is, the players’ actions depend only on the current state \( (b_t, k_t^q, a_t) \) where \( h_t = 1 \) and the current offer. In equilibrium, the proposer’s strategy is to propose when the other player would accept for sure and to pass otherwise. In contrast, the other player’s strategy is to accept when the proposal is made and to reject otherwise. Therefore, we can denote the proposer \( i \)'s and the other player \( j \)'s equilibrium strategies as follows: (a) \( \theta_i(b_t, k_t^q, 1, a_t) = 1 \) (propose) when the proposer \( i \) proposes and \( \theta_j(b_t, k_t^q, 1, a_t) = 1 \) (accept) when the other player \( j \) accepts the offer, and (b) \( \theta_i(b_t, k_t^q, 1, a_t) = 0 \) (pass) when the proposer \( i \) passes and \( \theta_j(b_t, k_t^q, 1, a_t) = 1 \) (reject) when the other player \( j \) rejects the offer.\(^{27} \)

A stationary subgame perfect (SP) equilibrium is defined as the players’ equilibrium stationary strategies \( \theta \) and \( \theta^* \), and the payoff functions, \( \Gamma \) and \( \Gamma^* \) associated with these strategies for player \( B \) and \( L \). The expected payoff for the borrower \( B \) and lender \( L \) in period \( t \), are shown as:

\[
\Gamma(b_t, k_t^q, a_t) = \phi \Gamma^B(b_t, k_t^q, a_t) + (1 - \phi) \Gamma^L(b_t, k_t^q, a_t) \tag{16}
\]

\[
\Gamma^*(b_t, k_t^q, a_t) = \phi \Gamma^{*B}(b_t, k_t^q, a_t) + (1 - \phi) \Gamma^{*L}(b_t, k_t^q, a_t) \tag{17}
\]

\(^{25} \)We assume that the proposer makes an offer that the respondent accepts when the value of proposing is higher or equal to the value of passing, and chooses to pass otherwise. This assumption can get rid of trivial sources of multiplicity. See Merlo and Wilson (1995) and Ortner (2013) for the same treatment.

\(^{26} \)Merlo and Wilson (1995) assume that the players have the same discount factor. But they also explain that “there is no real restriction implied by the assumption that players discount utility at a common constant rate. So long as the discounted size of the cake converges uniformly to 0.— player-dependent discount factors can always be represented by a discount case process with a common fixed discount factor”. There, in our model, we assume that the borrower and the lender have different discount factors.

\(^{27} \)Benjamin and Wright (2013) theoretically prove both existence and uniqueness of the equilibrium in multi-round bargaining game over defaulted debt.
Here, the superscript denotes the identity of the proposer: $\Gamma^B(\Gamma^B)$ represents the borrower’s (lender’s) payoff when the borrower is the proposer and $\Gamma^L(\Gamma^L)$ refers to the borrower’s (lender’s) payoff when the lender is the proposer.

First, we start from the case when the borrower $B$ is the proposer. We denote the proposed debt recovery rates as $\delta^B_t$, the borrower’s values of proposing and passing as $V^{PRO}$ and $V^{PASS}$, and the creditors’ values of accepting offer and rejecting as $V^{ACT}$ and $V^{REJ}$, respectively. When the borrower $B$ proposes and the proposal is accepted, the sovereign immediately pays recovered debt payments $-\delta^B_t b_t$ and resumes access to the international capital market in the next period with no outstanding debt as in Bi (2008).

$$V^{PRO}(b_t, k^g_t, a_t) = \max_{g_t, k^g_{t+1}, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) + \beta \int_A V(0, k^g_{t+1}, 0, a_{t+1})d\mu(a_{t+1}|a_t)$$  \hspace{1cm} (18)

$$\text{s.t.} \quad g_t + k^g_{t+1} + T_t = \tau c_t + (1 - \delta^g_t)k^g_t - \frac{\Omega}{2} \left(\frac{k^g_{t+1} - k^g_t}{k^g_t}\right)^2 k^g_t + \delta^B_t b_t$$  \hspace{1cm} (8')

$$T_t \geq 0$$  \hspace{1cm} (9)

$$\frac{u_c(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_t a_t(l_t)^{\alpha_t-1}(k^g_t)^{\alpha_k}(\bar{\delta}^g_t)^{1-\alpha_l-\alpha_k}}{1 + \tau}$$  \hspace{1cm} (10')

$$(1 + \tau)c_t = \bar{y} + T_t$$  \hspace{1cm} (11')

$$V^{ACT}(b_t, k^g_t, a_t) = -\delta^B_t b_t$$  \hspace{1cm} (19)

When the borrower $B$ passes, both parties proceed to the next period with accumulated arrears.

$$V^{PASS}(b_t, k^g_t, a_t) = \max_{g_t, k^g_{t+1}, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) + \beta \int_A V((1 + r^*)b_t, k^g_{t+1}, 1, a_{t+1})d\mu(a_{t+1}|a_t)$$  \hspace{1cm} (20)

$$\text{s.t.} \quad g_t + k^g_{t+1} + T_t = \tau c_t + (1 - \delta^g_t)k^g_t - \frac{\Omega}{2} \left(\frac{k^g_{t+1} - k^g_t}{k^g_t}\right)^2 k^g_t$$  \hspace{1cm} (8')

$$T_t \geq 0$$  \hspace{1cm} (9)

$$\frac{u_c(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_t a_t(l_t)^{\alpha_t-1}(k^g_t)^{\alpha_k}(\bar{\delta}^g_t)^{1-\alpha_l-\alpha_k}}{1 + \tau}$$  \hspace{1cm} (10')

$$(1 + \tau)c_t = \bar{y} + T_t$$  \hspace{1cm} (11')

$$V^{REJ}(b_t, k^g_t, a_t) = \frac{1}{1 + r^*} \int_A \Gamma^*((1 + r^*)b_t, k^g_{t+1}, 1, a_{t+1})d\mu(a_{t+1}|a_t)$$  \hspace{1cm} (21)

In equilibrium, the agreed recovery rates $\delta^B_t$ satisfy the following.\(^{28}\)

\(^{28}\)Off-equilibrium paths are eliminated in equilibrium.
\[ \delta^B_t = \arg\max V^{PRO}(b_t, k^g_t, a_t) \]
\[ s.t. \quad V^{PRO}(b_t, k^g_t, a_t) \geq V^{PASS}(b_t, k^g_t, a_t) \]
\[ V^{*ACT}(b_t, k^g_t, a_t) \geq V^{*REJ}(b_t, k^g_t, a_t) \]  \hspace{1cm} (22)

If both parties reach an agreement, the two parties’ payoffs are as follows:
\[ \Gamma^B(b_t, k^g_t, a_t) = V^{PRO}(b_t, k^g_t, a_t) \]  \hspace{1cm} (23)
\[ \Gamma^{B*}(b_t, k^g_t, a_t) = V^{*ACT}(b_t, k^g_t, a_t) \]  \hspace{1cm} (24)

Otherwise,
\[ \Gamma^B(b_t, k^g_t, a_t) = V^{PASS}(b_t, k^g_t, a_t) \]  \hspace{1cm} (23')
\[ \Gamma^{B*}(b_t, k^g_t, a_t) = V^{*REJ}(b_t, k^g_t, a_t) \]  \hspace{1cm} (24')

The renegotiation settlement can be characterized by settlement set \( R^B(b_t, k^g_t) \), defined as the set of productivity shocks \( a_t \), which both parties agree on settlements when the borrower is the proposer given the sovereign’s debt \( b_t \) and public capital \( k^g_t \).
\[ R^B(b_t, k^g_t) = \left\{ a_t \in A : V^{PRO}(b_t, k^g_t, a_t) \geq V^{PASS}(b_t, k^g_t, a_t), V^{*ACT}(b_t, k^g_t, a_t) \geq V^{*REJ}(b_t, k^g_t, a_t) \right\}. \]  \hspace{1cm} (25)

Similarly, when the lender is the proposer, we denote the proposed debt recovery rates as \( \delta^L_t \), the borrower’s value as \( V^{ACT} \) and \( V^{REJ} \), and the lender’s values as \( V^{*PRO} \) and \( V^{*PASS} \), respectively. When the lender L proposes,
\[ V^{*PRO}(b_t, k^g_t, a_t) = -\delta^L_t b_t \]  \hspace{1cm} (26)
\[ V^{ACT}(b_t, k^g_t, a_t) = \max_{c_t, k^g_{t+1}, T_t} (1 - \lambda)u(c_t, l_t) + v(g_t) + \beta \int_A V(0, k^g_{t+1}, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \]  \hspace{1cm} (27)
\[ s.t. \quad g_t + k^g_{t+1} + T_t = \tau c_t + (1 - \delta^g)k^g_t - \frac{\Omega}{2} \left( \frac{k^g_{t+1} - k^g_t}{k^g_t} \right)^2 k^g_t - \delta^L_t b_t \]  \hspace{1cm} (8'')
\[ T_t \geq 0 \]  \hspace{1cm} (9)
\[ \frac{u_t(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_t \tilde{a}_t(l_t)^{\alpha - 1}(k^g_t)^{\alpha_k}(\tilde{k}^p)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \]  \hspace{1cm} (10')
\[ (1 + \tau)c_t = \tilde{y}_t + T_t \]  \hspace{1cm} (11')
When the lender passes,
\[
V^{*\text{PASS}}(b_t, k_t^g, a_t) = \frac{1}{1 + r^*} \int_A \Gamma^*((1 + r^*)b_t, k_{t+1}^g, a_{t+1})d\mu(a_{t+1}|a_t)
\]  
(28)

\[
V^{\text{REJ}}(b_t, k_t^g, a_t) = \max_{g_t, k_{t+1}, T_t} (1 - \lambda)u(c_t, l_t) + \lambda v(g_t) + \beta \int_A V((1 + r^*)b_t, k_{t+1}^g, 1, a_{t+1})d\mu(a_{t+1}|a_t)
\]  
(29)

s.t. \(g_t + k_{t+1}^g + T_t = \tau c_t + (1 - \delta^g)k_t^g - \frac{\Omega}{2} (\frac{k_{t+1}^g - k_t^g}{k_t^g})^2 k_t^g\) \(\tag{8'}\)

\(T_t \geq 0\) \(\tag{9}\)

\[
u(c_t, l_t) = \frac{\alpha_t \bar{a}_t(l_t)^{\alpha_t - 1} (k_t^g)^{\alpha_k} (k_t^p)^{1 - \alpha_t - \alpha_k}}{1 + \tau}
\]  
(10’)

\((1 + \tau)c_t = \tilde{y}_t + T_t\) \(\tag{11'}\)

In equilibrium, the agreed recovery rates \(\delta^L_t\) satisfy the following:

\[
\delta^L_t = \arg\max V^{*\text{PRO}}(b_t, k_t^g, a_t)
\]

s.t. \(V^{*\text{PRO}}(b_t, k_t^g, a_t) \geq V^{*\text{PASS}}(b_t, k_t^g, a_t)\)

\(V^{\text{ACT}}(b_t, k_t^g, a_t) \geq V^{\text{REJ}}(b_t, k_t^g, a_t)\) \(\tag{30}\)

If both parties reach an agreement, the two parties’ payoffs are as follows:

\[
\Gamma^*(b_t, k_t^g, a_t) = V^{*\text{PRO}}(b_t, k_t^g, a_t)
\]  
(31)

\[
\Gamma^L(b_t, k_t^g, a_t) = V^{\text{ACT}}(b_t, k_t^g, a_t)
\]  
(32)

Otherwise,

\[
\Gamma^*(b_t, k_t^g, a_t) = V^{*\text{PASS}}(b_t, k_t^g, a_t)
\]  
(31’)

\[
\Gamma^L(b_t, k_t^g, a_t) = V^{\text{REJ}}(b_t, k_t^g, a_t)
\]  
(32’)

The renegotiation settlement can be characterized by settlement set \(R^L(b_t, k_t^g)\), defined as the set of productivity shocks \(a_t\) at which both parties agree on the settlements.

\[
R^L(b_t, k_t^g) = \left\{ a_t \in A : V^{*\text{PRO}}(b_t, k_t^g, a_t) \geq V^{*\text{PASS}}(b_t, k_t^g, a_t) \right. \\
\left. V^{\text{ACT}}(b_t, k_t^g, a_t) \geq V^{\text{REJ}}(b_t, k_t^g, a_t) \right\}.
\]  
(33)
4.5 Foreign Creditors’ Problem

With the sovereign’s good credit record \( h_t = 0 \), taking the bond price function as given, the foreign creditors who can borrow from the international capital market with the risk-free rate \( (r^*) \) choose the amount of next-period assets/debt \( (b_{t+1}) \) that maximizes their expected profit, shown as

\[
\pi^c(b_{t+1}, k_{t+1}^g, 0, a_t) = \begin{cases} 
q(b_{t+1}, k_{t+1}^g, 0, a_t)b_{t+1} - \frac{1}{1+r^*}b_{t+1}, & \text{if } b_{t+1} \geq 0 \\
\left[ 1 - p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \right] + \frac{p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1} | a_t) \right] (-b_{t+1}) & \text{if } b_{t+1} < 0
\end{cases}
\]  

where \( p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \) and \( \gamma(b_{t+1}, k_{t+1}^g, 0, a_t) \) are the expected default probability and expected recovery rates, respectively. Since we assume that the market for new sovereign bonds is completely competitive, foreign creditors’ expected profit is zero in equilibrium. Using the zero expected profit condition, we get

\[
q(b_{t+1}, k_{t+1}^g, 0, a_t) = \begin{cases} 
\frac{1}{1+r^*}, & \text{if } b_{t+1} \geq 0 \\
\left[ 1 - p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \right] + \frac{p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1} | a_t) \right] (-b_{t+1}) & \text{if } b_{t+1} < 0
\end{cases}
\]  

When the sovereign buys bonds from the foreign creditors \( b_{t+1} \geq 0 \), the sovereign bond price is equal to the price of risk-free bond, \( \frac{1}{1+r^*} \). When the sovereign issues bonds to the foreign creditors \( b_{t+1} \leq 0 \), there is default risk and the bonds are priced to compensate the foreign creditors for this. Since \( 0 \leq p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \leq 1 \) and \( 0 \leq \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) \leq 1 \), the bond price \( q(b_{t+1}, k_{t+1}^g, 0, a_t) \) lies in \([0, \frac{1}{1+r^*}]\).

4.6 Equilibrium

A recursive equilibrium is defined as a set of functions for (a) the sovereign government’s value function, public consumption, capital, transfers, assets/debt position, and default set, (b) the household’s consumption and labor supply, (c) the firm’s labor demand, (d) the sovereign’s and the foreign creditor’s decision functions, payoffs, and two sets of recovery rates (depending on who is the proposer), and (e) sovereign bond prices such that
[1]. the sovereign government’s value function, public consumption, capital, transfers, assets/debt position, and default set satisfy its optimization problem (7)–(15);
[2]. the household’s consumption and labor supply satisfy his optimization problem (1)–(3);
[3]. the firm’s labor demand satisfies its optimization problem (4)–(6);
[4]. both parties’ decisions, payoffs and recovery rates solve the multi-round debt renegotiation problem (16)–(33);
[5]. sovereign prices satisfy the foreign creditors’ optimization problem (34)–(35).

In equilibrium, default probability is defined by using the sovereign’s default set:

\[ p^D(b_{t+1}, k_{t+1}^g, 0, a_t) = \int_{D(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t), \]

Similarly, probability of settlement is defined by using the two settlement sets:

\[ p^R(b_{t+1}, k_{t+1}^g, 0, a_t) = \phi \int_{R^B(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t) + (1 - \phi) \int_{R^L(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t), \]

Expected recovery rates conditional on default choice is shown as:

\[ \gamma(b_{t+1}, k_{t+1}^g, 1, a_t) \]

\[ = \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1}|a_t) \]

\[ = \int_A \left[ \phi \mathbb{1}_{a_{t+1} \in R^B(b_{t+1}, k_{t+1}^g)} \delta_{\text{true}}^B(b_{t+1}, k_{t+1}^g, a_{t+1}) \\
+ (1 - \phi) \mathbb{1}_{a_{t+1} \in R^L(b_{t+1}, k_{t+1}^g)} \delta_{\text{true}}^L(b_{t+1}, k_{t+1}^g, a_{t+1}) \right] d\mu(a_{t+1}|a_t) \]

\[ + \left[ \phi \mathbb{1}_{a_{t+1} \notin R^B(b_{t+1}, k_{t+1}^g)} \delta_{\text{true}}^B(b_{t+1}, k_{t+1}^g, a_{t+1}) \\
+ (1 - \phi) \mathbb{1}_{a_{t+1} \notin R^L(b_{t+1}, k_{t+1}^g)} \delta_{\text{true}}^L(b_{t+1}, k_{t+1}^g, a_{t+1}) \right] \gamma(b_{t+2}, k_{t+2}^g, 1, a_{t+1}) \]

The sovereign’s total spread, i.e., the difference between the sovereign’s interest rate and the risk-free rate, is defined as

\[ s(b_{t+1}, k_{t+1}^g, 0, a_t) = \frac{1}{q(b_{t+1}, k_{t+1}^g, 0, a_t)} - (1 + r^*) \]
5 Quantitative Analysis

This section provides the quantitative analysis of our model with three main findings. First, our model numerically shows not only the conventional phase, a “run-up to default period” but also one new phase of sovereign debt overhang, a “restructuring (run-up to debt settlement) period”. Second, we explain the role of public capital in the process of debt restructurings, in particular debt settlements. Third, our simulation exercise successfully replicates the four stylized facts: (i) a severe decline and slow recovery of public investment, (ii) a short-lived decline and quick recovery of public consumption and transfers, (iii) public expenditure tilting towards consumption and transfers, (iv) association between public investment declines and restructuring duration.

5.1 Parameters and Functional Forms

All the parameter values and functional forms follow closely those in previous studies on sovereign debt and fiscal policy. We assume the following constant relative risk aversion (CRRA) functions for private consumption/labor and public consumption:

\[ u(c_t, l_t) = \frac{(c_t - \frac{l^{1+\psi}}{1+\psi})^{1-\sigma}}{1-\sigma}, \quad v(g_t) = \frac{g^{1-\sigma_g}}{1-\sigma_g} \]  

As in conventional sovereign debt models (e.g., Mendoza and Yue 2012; Cuadra et al. 2010), \( u(\cdot) \) follows Greenwood et al. (1988)'s specification, which provides the marginal rate of substitution between private consumption and labor orthogonal to the level of private consumption. Thus, this implies no wealth effects on labor supply. We set both risk aversion for private and public consumption as \( \sigma = \sigma_g = 3 \), as in previous studies on sovereign debt and Hatchondo et al. (2017) to maintain the same degree of consumption smoothing between two types of consumption.\(^{29}\)

The risk-free interest rate is \( r^* = 0.01 \) from the US Treasury bills. Labor elasticity \( \psi \) is set to 0.48 following Mendoza (1991). Labor and public capital income shares are 0.64 and 0.058, respectively, based on Gordon and Guerron-Quintana (2018) and Argentine public and private capital share in 1993–2005. Public capital depreciation rate is set at 0.04 following US BEA (1999). Effective consumption tax rate \( \tau = 0.33 \) is from Argentine tax revenues in 1993–2005.

The productivity process is calibrated to match quarterly seasonally adjusted GDP data from the Ministry of Economy and Production in Argentina (MECON). As in previous studies (Gordon and Guerron-Quintana 2018), we model the productivity shock as an AR (1) process:

\[ \log(a_t) = \rho \log(a_{t-1}) + \epsilon_{a,t}, \]  

where productivity shock \( \epsilon_{a,t} \) is i.i.d \( N(0, \sigma^{a,2}) \). We obtain auto-correlation and standard devi-

\(^{29}\)Hatchondo et al. (2017) assume asymmetric risk aversion between two types of consumption (\( \sigma = 2, \sigma_g = 3 \)) because there are no public transfers in their paper. However, with public transfers included in our model, the same degree of risk aversion to smooth the household’s utility is necessary to have both fiscal instruments available for the sovereign government.
ation of productivity shock: $\rho = 0.85$ and $\sigma^a = 0.017$. We approximate the stochastic process as a discrete Markov chain of equally spaced grids by using the quadrature method in Tauchen (1986).

The direct productivity loss due to default follows the functional form in Arellano and Bai (2017)—originally from Arellano (2008)’s asymmetric output costs:

$$\tilde{a}_t = \begin{cases} (1 - \lambda_d)E(a_t) & \text{if } a_t \geq (1 - \lambda_d)E(a_t) \\ a_t & \text{otherwise} \end{cases} \quad (42)$$

where $\lambda_d$ is set to 0.02 to generate average GDP deviation from the trend during debt restructurings of -4.45%. The weight on public consumption in the household’s utility and public capital adjustment costs are set as $\gamma = 0.8$ and $\Omega = 10$ to replicate average public consumption/transfers-to-GDP ratio of 20.0% and standard deviation of public investment relative to that of output of 5.1 for Argentina in 1993–2005, respectively.

Sturzenegger and Zettelmeyer (2006) report that Argentina experienced 6 defaults in 1820–2004. We specify the sovereign’s discount factor $\beta = 0.90$—similar to those in Gordon and Guerron-Quintana (2018)—and bargaining power $\phi = 0.9$ (the debtor - Argentina) to replicate the average annual default frequency of 3.26 percent and a recovery rate of 25.0 percent (Argentina 2001–05 restructuring). Table 4 summarizes the model parameters and our computation algorithm is reported in Appendix C.

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Preceding studies (Yue 2010; Bi 2008; D’Erasmo 2010; Asonuma and Trebesch 2016) use different parameter values for bargaining power. This is because there is no well-established proxy indicator for bargaining power and these parameter values are set to replicate recovery rates specifically in their models with different assumptions (income, bargaining game, etc.).
Table 4: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion for private consumption</td>
<td>$\sigma = 3$</td>
<td>Previous studies</td>
</tr>
<tr>
<td>Risk aversion for public consumption</td>
<td>$\sigma_g = 3$</td>
<td>Hatchondo et al. (2017)</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r^* = 0.01$</td>
<td>US Treasury Bill</td>
</tr>
<tr>
<td>Labor elasticity</td>
<td>$\psi = 0.48$</td>
<td>Mendoza (1991)</td>
</tr>
<tr>
<td>Labor income share</td>
<td>$\alpha^L = 0.64$</td>
<td>Gordon and Guerron-Quintana (2018)</td>
</tr>
<tr>
<td>Public capital income share</td>
<td>$\alpha^K = 0.058$</td>
<td>Argentine public and private capital (1993–2005)</td>
</tr>
<tr>
<td>Public capital depreciation rate</td>
<td>$\delta = 0.04$</td>
<td>US BEA (1999)</td>
</tr>
<tr>
<td>Effective consumption tax rate</td>
<td>$\tau = 0.33$</td>
<td>Argentine tax revenues (1993–2005)</td>
</tr>
<tr>
<td>Auto-correlation of productivity shock</td>
<td>$\rho = 0.85$</td>
<td>Computed Argentine GDP- MECON</td>
</tr>
<tr>
<td>Standard deviation of productivity shock</td>
<td>$\sigma^a = 0.017$</td>
<td>Computed Argentine GDP- MECON</td>
</tr>
<tr>
<td>Direct productivity loss</td>
<td>$\lambda_d = 0.02$</td>
<td>Computed</td>
</tr>
<tr>
<td>Weight on public consumption</td>
<td>$\lambda = 0.8$</td>
<td>Computed</td>
</tr>
<tr>
<td>Public capital adjustment costs</td>
<td>$\Omega = 10$</td>
<td>Computed</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\beta = 0.90$</td>
<td>Computed</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\phi = 0.9$</td>
<td>Computed</td>
</tr>
</tbody>
</table>
5.2 Numerical Results on Equilibrium Properties

Figure 6 reports the sovereign’s repayment, default/settlement, and default/delay choice. In all panel charts, the horizontal axis is the sovereign’s debt, and the vertical axis is its productivity level. To demonstrate our new findings, three panel charts are classified according to the public capital level: (i) mean (benchmark), (ii) low, and (iii) high.

Figure 6: Repayment, Default/Settlement, and Default/Delay Choice

(i) Mean Public Capital

(ii) Low Public Capital

(iii) High Public Capital

In our benchmark case (mean public capital in Panel i), we observe the common trend of the sovereign’s choice as in previous studies on multi-round negotiations (Benjamin and Wright 2013; Bi 2008). When the sovereign’s debt is low, it repays its debt in full and maintains access to the international capital market shown by the green region (Repayment). Otherwise, the sovereign opts to default and proceed to post-default renegotiations. During renegotiations, when productivity is low, it refrains using limited resources for recovered debt repayments,
or the proposed low recovery rates do not satisfy the creditors’ demand. As a result, the sovereign chooses to postpone the settlement corresponding to the red region (Delay). When its productivity is high, the sovereign chooses to use sufficient resources for recovered debt payments. It proposes high recovery rates—shown in Figure D1 in Appendix D—which attract the creditors and settles the deal highlighted in the blue region (Settlement).

More importantly, what our model explains newly, in addition to the aforementioned trend in the literature, is that the sovereign opts to delay (settle the deal), ceteris paribus, when public capital is low (high). A new driver—a choice between investment in public capital and use of resources for debt settlement—determines the sovereign’s settlement and delay decisions in our model differentiating it from previous studies. In the case of low public capital (Panel ii), the sovereign opts to invest in public capital and refrains from using resources for recovered debt repayments given high marginal product of public capital (equivalent to high shadow value of public capital). The sovereign’s willingness to delay is reflected in the enlarged “Delay” region in red and shrunk (not existing) “Settlement” region in blue, respectively.

In contrast, in the case of high public capital, the sovereign hesitates to invest in public capital and chooses to use resources for recovered debt payments given low marginal product of public capital (corresponding to low shadow value of public capital). The sovereign’s willingness to settle at the renegotiations is highlighted in the shrunk “Delay” region in red and the enlarged “Settlement” region in blue, respectively. Moreover, we find that unconditional on default, i.e., ex-ante, the influence of public capital on default/repayment choice is multi-faceted. On the one hand, when public capital is low, costs of default—both lengthy renegotiations and productivity losses—are high. Simultaneously, benefits of default associated with low repayment capacity (i.e., output) are also high. On the other hand, when public capital is high, costs of default—both short renegotiations and productivity losses—are small. At the same time, benefits of default accompanied by high repayment capacity are also limited. These are reflected in the “Repayment” region broadly unchanged in three Panels (i), (ii), and (iii).
We next explore the sovereign’s choice of public investment and consumption/transfers in Figures 7 and 8. Both Figures 7 and 8 report the sovereign’s choice of public investment and consumption/transfers at mean public capital, and Figures D2 and D3 in Appendix D reports that at low and high public capital, respectively. We differentiate the public investment and consumption/transfers choice in two phases: that in the “Repayment” region (Panel i in Figures 7 and 8) and that in the “Delay” region (Panel ii in Figures 7 and 8).

Panels (i) and (ii) in Figure 7 show that public investment increases proportionally to the level of productivity. What drives this is marginal product of public capital: when marginal product of capital is low in the case of low productivity (red dashed line), the sovereign chooses to reduce public investment severely. Moreover, public investment decreases when the level of external debt increases in the “Repayment” region (Panel i in Figure 7): high external debt payments limit the resources allocated to public investment. On the contrary, public investment is irrelevant to the level of external debt in the “Delay” region (Panel ii in Figure 7): the sovereign government opts to delay the settlement corresponding to no external debt payments.

In contrast, Panel (i) in Figure 8 show that public consumption and transfers do not differ significantly depending on the level of productivity in the “Repayment” region. This is driven by the sovereign government’s willingness to smooth the household’s utility: despite limited resources due to low productivity, the sovereign government is willing to spend more on public consumption and transfers to smooth the household’s utility. Moreover, in parallel with public investment, public consumption and transfers decrease when the level of external debt increases in the “Repayment” region (Panel i in Figure 8): high external debt payments limit the resources allocated to public consumption and transfers. On the contrary, public consumption and transfers stay constant independently to the level of external debt in the “Delay” region (Panel ii in Figure 8) because of no external debt payments.
5.3 Simulation Exercise

Next, we provide simulation results to show how precisely our theoretical model predicts the Argentine default and restructuring in 2001–05. Following a conventional approach, this subsection applies 1000 rounds of simulations, with 2000 periods per round and extracts the last 200 observations. In the last 200 samples, we withdraw 40 observations before and observations during the last default/restructuring event at the stationary distribution to compute moment statistics.\footnote{See also Arellano (2008) and Yue (2010) for this treatment of simulation.}

For private sector data for Argentina, output, consumption, and the trade balance are all seasonally adjusted from the MECON for 1993Q1–2001Q4 (prior to default/restructuring) and 2002Q1–2005Q2 (during default/restructuring). The trade balance is measured in terms of percentage of GDP. For public sector data for Argentina, consumption, investment, transfers and capital are at annual frequency from our dataset for 1993–2001 (prior to default) and 2002–05 (during restructuring). Argentine external debt data are from the IMF World Economic Outlook (WEO) for 1993–2001 (prior to default) and 2002–05 (during default/restructuring). Average external debt is also measured in terms of percentage of GDP. Bond spreads are from the J.P. Morgan’s Emerging Markets Bond Index Global (EMBIG) for 1997Q1–2001Q4 (prior to default). We compare our non-target statistics with those in (i) model with fixed public capital (Arellano and Bai 2017; Cuadra et al. 2010; Hatchondo et al. 2017) and (ii) model without separation between private and public sectors (Gordon and Guerron-Quintana 2018)—we add specific features, respectively in our model of multi-round debt renegotiations keeping the same parameter values.

Panel (i) in Table 5 reports business cycle statistics for both private and public sectors. For private sector statistics, our model matches closely with the data. We replicate three prominent emerging market private sector features as in conventional models of sovereign debt with/without debt restructurings (Aguiar and Gopinath 2006; Arellano 2008; Yue 2010): volatile consumption and volatile and countercyclical trade balance.\footnote{Models with multi-round debt renegotiations, exogenous income process and symmetric output costs (Bi 2008; Asonuma and Joo 2017) fail to generate a negative correlation between trade balance and output. This is because when output is high, the sovereign is still willing to default, taking merits of low default costs due to short duration of restructurings. This results in an increase in borrowing costs (high spreads) and a reduction in external borrowing equivalent to an improvement in trade balance. However, in our model with multi-round debt renegotiations, endogenous output dynamics and asymmetric productivity loss, we account for the negative correlation between trade balance and output. The assumption of asymmetric productivity loss generates high default costs when output is high, and this in turn, results in low borrowing costs (low spreads) and high external borrowing equivalent to a deterioration in trade balance.} One caveat applies to balanced (constant) trade account due to the financial autarky assumption during debt restructurings in our model.

For public sector statistics, our simulated moments also fit the data well. Our model successfully replicates notable emerging market public sector characteristics: procyclical and volatile public consumption and transfers. This is in line with previous models of sovereign debt with fiscal policy as Arellano and Bai (2017), Cuadra et al. (2010), and Hatchondo et al. (2017).

Most importantly, our calibration results provide four novelties contributing to the literature.
First of all, our model successfully replicates lower public average investment during restructurings than that in the pre-default period (1.5 vs. 1.6 percent in the model and 1.2 vs 1.3 percent in the data). More specifically, we explain both downward and upward trends of public investment during restructurings as we observe in the data. Moreover, our model accounts for sizable public capital accumulation (1.7 percent) during restructurings consistent with the data (2.3 percent). The model with fixed public capital (Arellano and Bai 2017; Cuadra et al. 2010) replicates none of these features. Though the model without separation of private and public sectors (Gordon and Guerron-Quintana 2018) replicates lower average “total” investment during restructurings than that in the pre-default period, it accounts for neither downward and upward trends of “total” investment (1.4 and 0.9 percent) nor public capital accumulation (0.5 percent).

Second, our model replicates lower investment share in public expenditure during restructurings than that in the pre-default periods (5.8 vs. 6.4 percent in the model and 5.7 and 6.2 percent in the data). While public consumption and transfers-to-GDP ratio is marginally higher in the restructuring periods than the pre-default periods (23.3 and 23.1 percent), public investment-to-GDP ratio is lower during restructurings than in the pre-default periods (1.47 and 1.60 percent). In contrast, the model with fixed public capital (Arellano and Bai 2017; Cuadra et al. 2010) generates higher investment share in public expenditure during restructurings than that in the pre-default periods (9.5 and 8.0 percent) because of both fixed investment level and endogenous GDP.

Third, the model replicates average restructuring duration of 11.1 quarters close to the data (14.0 quarters). Contrary to conventional models of multi-round renegotiations with exogenous income process, what generates long duration of restructurings are both endogenous public capital accumulation (both downward and upward trends) and distortional consumption tax. Previous models—the model with fixed public capital (Arellano and Bai 2017; Cuadra et al. 2010) and the model without separation of private and public sectors (Gordon and Guerron-Quintana 2018) only result in shorter duration (8.7 and 8.9 quarters, respectively). This is because these models include only either distortional consumption tax or endogenous public capital accumulation, respectively.

Lastly, we generate a negative correlation between public investment declines and restructuring duration as observed in the data (-0.12 vs. and -0.08 in the data). Lower public capital following sharp public investment declines around the start of restructurings results in restructuring delays as explained in Section 5.2. Despite the different dynamics of “total” investment, the negative correlation can be generated by the model without separation of private and public sectors—Gordon and Guerrion-Quitana (2018) does not embed endogenous debt restructurings.
Table 5: Simulated Business and Non-business Cycle Statistics of Model

(i) Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Model with Fixed Public Capital</th>
<th>Model without Separation of Private/Public Sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Target statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pre-default periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average public consumption &amp; transfers/GDP ratio (%)</td>
<td>20.0</td>
<td>23.1</td>
<td>22.5</td>
<td>-</td>
</tr>
<tr>
<td>Public investment (std dev.)/output (std dev.) (%)</td>
<td>5.1</td>
<td>5.2</td>
<td>-</td>
<td>5.1</td>
</tr>
<tr>
<td><strong>Non-target statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pre-default periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private consumption (std dev.)/output (std dev.)</td>
<td>1.11</td>
<td>1.01</td>
<td>1.01</td>
<td>1.03</td>
</tr>
<tr>
<td>Trade balance/output: std dev. (%)</td>
<td>1.28</td>
<td>0.86</td>
<td>0.48</td>
<td>1.01</td>
</tr>
<tr>
<td>Corr.(trade balance, output)</td>
<td>-0.87</td>
<td>-0.18</td>
<td>-0.07</td>
<td>-0.23</td>
</tr>
<tr>
<td>Public sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public consumption &amp; transfers (std dev.)/output (std dev.)</td>
<td>1.26</td>
<td>1.23</td>
<td>1.22</td>
<td>-</td>
</tr>
<tr>
<td>Corr.(public consumption &amp; transfers, output)</td>
<td>0.52</td>
<td>0.86</td>
<td>0.94</td>
<td>-</td>
</tr>
<tr>
<td>Average public investment/GDP ratio</td>
<td>1.31</td>
<td>1.60</td>
<td>2.01</td>
<td>1.58</td>
</tr>
<tr>
<td>Average public investment/public expenditure ratio</td>
<td>6.20</td>
<td>6.40</td>
<td>8.04</td>
<td></td>
</tr>
<tr>
<td>Corr.(public investment, output)</td>
<td>0.51</td>
<td>0.63</td>
<td>-</td>
<td>0.66</td>
</tr>
<tr>
<td><strong>Renegotiation periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private consumption (std dev.)/output (std dev.)</td>
<td>1.17</td>
<td>1.01</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>Trade balance/output: std dev. (%)</td>
<td>0.45</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Corr.(trade balance, output)</td>
<td>-0.97</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Debtor output deviation (diff. btw start and end, %)</td>
<td>12.6</td>
<td>21.2</td>
<td>22.58</td>
<td>21.47</td>
</tr>
<tr>
<td>Public sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public consumption &amp; transfers (std dev.)/output (std dev.)</td>
<td>0.99</td>
<td>2.23</td>
<td>1.07</td>
<td>-</td>
</tr>
<tr>
<td>Corr.(public consumption &amp; transfers, Output)</td>
<td>0.99</td>
<td>0.77</td>
<td>0.67</td>
<td>-</td>
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<tr>
<td>Average public consumption &amp; transfers/GDP ratio (%)</td>
<td>20.23</td>
<td>23.3</td>
<td>22.4</td>
<td>-</td>
</tr>
<tr>
<td>Average public investment/GDP ratio</td>
<td>1.19</td>
<td>1.47</td>
<td>2.36</td>
<td>1.24</td>
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<tr>
<td>Average public investment/GDP ratio (downward trend)</td>
<td>0.73</td>
<td>0.76</td>
<td>-</td>
<td>1.41</td>
</tr>
<tr>
<td>Average public investment/GDP ratio (upward trend)</td>
<td>1.64</td>
<td>1.65</td>
<td>-</td>
<td>0.91</td>
</tr>
<tr>
<td>Average public investment/public expenditure ratio</td>
<td>5.7</td>
<td>5.8</td>
<td>9.53</td>
<td>-</td>
</tr>
<tr>
<td>Corr.(public investment, output)</td>
<td>0.99</td>
<td>0.84</td>
<td>-</td>
<td>0.93</td>
</tr>
<tr>
<td>Public capital (percent change from the trough to the end)</td>
<td>2.31</td>
<td>1.7</td>
<td>-</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(ii) Non-business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Model with Fixed Public Capital</th>
<th>Model without Separation of Public/Private Sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Target statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default probability (%)</td>
<td>3.26</td>
<td>3.10</td>
<td>2.7</td>
<td>3.29</td>
</tr>
<tr>
<td>Average debt/GDP ratio (%)</td>
<td>25.0</td>
<td>27.1</td>
<td>22.5</td>
<td>35.2</td>
</tr>
<tr>
<td>Average debtor output deviation during debt renegotiation (%)</td>
<td>-4.45</td>
<td>-3.73</td>
<td>-4.5</td>
<td>-4.23</td>
</tr>
<tr>
<td><strong>Pre-default periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average debt/GDP ratio (%)</td>
<td>45.4</td>
<td>43.9</td>
<td>45.6</td>
<td>50.8</td>
</tr>
<tr>
<td>Bond spreads: average (%)</td>
<td>9.4</td>
<td>0.2</td>
<td>0.17</td>
<td>0.5</td>
</tr>
<tr>
<td>Bond spreads: std dev. (%)</td>
<td>7.6</td>
<td>0.3</td>
<td>0.34</td>
<td>0.9</td>
</tr>
<tr>
<td>Corr.(spreads, output)</td>
<td>-0.88</td>
<td>-0.1</td>
<td>-0.30</td>
<td>-0.13</td>
</tr>
<tr>
<td>Corr.(debt/GDP, spreads)</td>
<td>0.92</td>
<td>0.21</td>
<td>0.36</td>
<td>0.26</td>
</tr>
<tr>
<td>Corr.(debt/GDP, output)</td>
<td>-0.97</td>
<td>-0.69</td>
<td>-0.69</td>
<td>-0.65</td>
</tr>
<tr>
<td><strong>Renegotiation periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average debt/GDP ratio (%)</td>
<td>130.5</td>
<td>50.6</td>
<td>53.6</td>
<td>59.8</td>
</tr>
<tr>
<td>Corr.(debt/GDP, output)</td>
<td>-0.95</td>
<td>-0.99</td>
<td>-0.99</td>
<td>-0.99</td>
</tr>
<tr>
<td>Duration of renegotiation/exclusion (quarters)</td>
<td>14.0</td>
<td>11.1</td>
<td>8.7</td>
<td>8.8</td>
</tr>
<tr>
<td>Corr.(cumulative change in public investment to GDP, duration)</td>
<td>-0.12</td>
<td>-0.08</td>
<td>-</td>
<td>-0.15</td>
</tr>
<tr>
<td>Corr.(cumulative percent change in public investment, duration)</td>
<td>-0.16</td>
<td>-0.1</td>
<td>-</td>
<td>-0.14</td>
</tr>
</tbody>
</table>
To emphasize the aforementioned novelties of our model, Figures 9 and 10 report the dynamics of public investment, and consumption and transfers, and public expenditure composition and restructuring duration in the Argentine debt restructuring in 2001–05, respectively. For Figure 9, we follow the same presentation approach as in Figures 1 and 2 in terms of both time...

Figure 9: Public Investment, Consumptions, and Transfers around Debt Restructuring

(i) Public Investment

(ii) Public Consumption and Transfers
horizon, timing of events—both the start and end of debt crisis denoted as year 0 and 3.5 (14 quarters) and marked by gray and pink (light orange and green) vertical bars—, scale (real and level), and normalization of the series at the pre-default levels (-1). Blue solid, red dashed, and green dotted lines show the Argentine data, our baseline model, and a model with exogenous re-entry (Arellano 2008), respectively. For Panels (i) and (ii) in Figure 10, we follow the same presentation approaches in Figure 3 and 4, respectively in terms of both time horizon, periods (pre-restructuring and restructuring), scale (percent of public expenditure and percent GDP).

First and most importantly, Panel (i) in Figure 9 shows that our baseline model (red dashed line) replicates both downward and upward trends of public investment—a sharp decline in the run up to the restructuring and a gradual recovery of public investment to the pre-restructuring level in the subsequent years—as observed in the data (blue solid line). This is the main driver of longer duration of renegotiations in our baseline model (11.1 quarters), which matches closely with the data (14 quarter). On the contrary, the model with fixed public capital (Arellano and Bai 2017; Cuadra et al. 2010; Hatchondo et al. 2017) does not replicate the dynamics of public investment. This is because, the sovereign fixes public investment to maintain the constant level of public capital. Duration of renegotiations is 8 quarters, shorter than that in our baseline model—delays are driven by only recovery of capacity of repayment (Benjamin and Wright 2013; Bi 2008).

Second, Panel (ii) in Figure 9 shows that our baseline model (red dashed line) replicates a small decline and quick recovery in public consumption and transfers as observed in the data (blue solid line). The dynamics of public consumption and transfers differ significantly from that of public investment in Panel (i) in Figure 9. Moreover, the model with fixed public capital (green dotted line) also generates the same dynamics of public consumption and transfers with our model until the debt settlement in year 2 (8 quarters).

Third, Panel (i) in Figure 10 shows that our baseline model (center panel chart) generates a tilting of public expenditure towards consumption and transfers as observed in the data (left panel chart). On the contrary, the model with fixed public capital (right panel chart) shows a tilting of public expenditure towards investment. This is because the sovereign mildly reduces public consumption and transfers, while maintains public investment constant.

Fourth, Panel (ii) in Figure 10 shows that our baseline model replicates a negative correlation between restructuring duration and public investment decline. This is consistent with what we observe in the sample of post-default restructurings in Figure 4. In contrast, the model with fixed public capital does not replicate this feature due to fixed public investment and capital. There is no decline in public investment due to constant public investment and debt settlement is driven purely by recovery of repayment capacity, independent of the constant level of public capital.

Lastly, we use simulated data series obtained from our baseline model and apply a logit regression on debt settlements (binary). To be consistent with a logit regression on post-default restructuring episodes reported in Table 10, our main explanatory variables are either public investment or capital—measured as lagged public investment in percent of mean TFP or lagged
growth rate of public capital—, and external debt-to-GDP ratio. Logit regression results reported in Table 6 show that both low lagged public investment and lagged public capital growth rate significantly decrease the likelihood of settlements. Therefore, our theoretical model shows that slow recovery of public investment delays the settlement of restructurings.

Figure 10: Public Expenditure Composition and Duration of Restructurings

(i) Public Expenditure Composition

Argentina 2001–05

Baseline

Fixed public capital

(ii) Public Investment Decline and Duration of Restructurings
### Table 6: Public Investment and Capital, and Debt Settlements

<table>
<thead>
<tr>
<th></th>
<th>Debt settlement (binary, current)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>coef/</td>
</tr>
<tr>
<td></td>
<td>se</td>
</tr>
<tr>
<td>Public investment (lagged, percent of mean TFP)</td>
<td>3.833***</td>
</tr>
<tr>
<td></td>
<td>(0.452)</td>
</tr>
<tr>
<td>Public capital growth, annualized (lagged, percent)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>External debt (lagged, percent of GDP)</td>
<td>-0.085***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>Constant</td>
<td>-</td>
</tr>
</tbody>
</table>

| Episode-specific Fixed effect | No | No |
| Number of episodes | 76 | 76 |
| Number of observations | 831 | 831 |
| Wald $\chi^2$ | 327.9 | 470.8 |
| Prob.$>\chi^2$ | 0.000 | 0.000 |

Notes: The table shows results from random effects multinomial logit regressions. The dependent variable is debt settlement in the current year (binary choice). The main explanatory variables are public investment and public capital growth rate. Public investment, public capital growth rate, and external debt (percent of GDP) are lagged by one year. Significance levels are denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. Robust standard errors (Delta-method standard errors) are in parentheses.
5.4 Comparison with Models of Multi-round Negotiations

Table 7: Simulation Results of Model - Non-business Cycle Statistics

<table>
<thead>
<tr>
<th>Target statistics</th>
<th>Data Baseline Model</th>
<th>Model with fixed public capital (case i)</th>
<th>Model with endogenous public capital (case ii)</th>
<th>Model with fixed public capital and no distortionary tax (case iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default probability (%)</td>
<td>3.26</td>
<td>3.10</td>
<td>2.71</td>
<td>3.20</td>
</tr>
<tr>
<td>Average recovery rate (%)</td>
<td>25.0</td>
<td>27.1</td>
<td>32.4</td>
<td>28.0</td>
</tr>
<tr>
<td>Average debtor output deviation during debt renegotiation (%)</td>
<td>-4.45</td>
<td>-3.73</td>
<td>-4.50</td>
<td>-4.6</td>
</tr>
</tbody>
</table>

**Pre-default periods**

| Average debt/GDP ratio (%) | 45.4 | 42.9 | 45.6 | 62.5 | 40.0 | 24.5 | 52.5 |
| Bond spreads: average (%) | 9.4 | 0.2 | 0.2 | 0.1 | 0.2 | 0.4 | 0.15 |
| Bond spreads: std dev (%) | 7.8 | 0.3 | 0.34 | 0.35 | 0.2 | 0.6 | 0.2 |
| Cor (spreads, output) | -0.88 | -0.1 | -0.31 | -0.18 | -0.17 | -0.05 | -0.32 |
| Cor (spreads, GDP, spreads) | 0.92 | 0.25 | 0.37 | 0.29 | 0.24 | 0.78 | 0.39 |
| Cor (debt/GDP, output) | -0.97 | -0.69 | -0.70 | -0.68 | -0.6 | -0.1 | -0.72 |

**Renegotiation periods**

| Average debt/GDP ratio (%) | 130.5 | 50.6 | 53.7 | 73.5 | 47.1 | 30.0 | 63.7 |
| Cor (debt/GDP, output) | -0.95 | -0.38 | -0.09 | -0.09 | -0.06 | -0.09 | -0.39 |
| Duration of renegotiation (quarters) | 14.0 | 11.1 | 11.5 | 8.8 | 8.9 | 8.3 | 6.4 |
| Debt output deviation (diff. low start and end, %) | 12.6 | 21.2 | 22.7 | 22.1 | 22.2 | 17.1 | 21.6 |
| Public capital (percent change from the trough to the end) | 2.31 | 0.7 | 0.7 | 0.3 | 0.5 | 0 | 0.1 |
| Cor (cumulative change in public investment to GDP, duration) | -0.12 | -0.08 | - | -0.01 | -0.17 | -0.17 | - |
| Cor (cumulative percent change in public investment, duration) | -0.10 | -0.1 | - | -0.15 | -0.1 | - |

**Sources:** Datastream, IMF WEO, MECON.

**Notes:**
1/ Model with fixed public capital corresponds to our model (with the same parameter values) with public capital fixed at the average (under the baseline model).
2/ Model with endogenous public capital corresponds to our model (with the same parameter values) without separation of public and private sectors or with lump-sum taxation on income.
3/ Model with fixed public capital and no distortionary tax corresponds to our model (with the same parameter values) with public capital fixed at the average (under the baseline model) and without distortionary consumption tax.
4/ A difference in the sovereign’s output deviation at the the end and at the start of restructurings.

Table 7 contrasts non-business cycle statistics in our baseline model with those in previous models of multi-round negotiations. We consider three cases: (i) a model with fixed public capital, (ii) a model with endogenous public capital, and (iii) a model with fixed public capital and no distortionary tax as in Benjamin and Wright (2013). To generate moments comparable to ours, we embed an assumption of exogenously fixed public capital for the case (i), remove separation of public and private sectors or add a lump-sum taxation assumption for the case (ii), and apply both assumptions of fixed public capital and no distortionary taxation for the case (iii) in our model, respectively, leaving all other parameters unchanged.

Comparing our model with case (i)—with distortionary tax (case i-a)—, the most striking result is a sizable difference in the average restructuring duration between our model and the model with fixed public capital (11.1 quarters in our model vs. 8.3 quarters). Distortionary consumption taxation, generates delays in renegotiations in both models. However, slow recovery of public investment followed by a severe decline is present only in our model generating further delays. Moreover, when the government has two methods of taxation, i.e., distortionary consumption taxation and lump-sum taxation on income (case i-b), a constraint on resource allocations between private and public sectors is relaxed—the government can also extract resources through lump-sum taxation without generating further distortion—and this, in turn results in both shorter delays (7.8 quarters) and higher debt-to-GDP ratio (62.5 percent and 73.5 percent).
Next, when we compare our model to case (ii)—without separation of public and private sectors (case ii-a)—, we also see a sizable difference in the average restructuring duration between our model and the model without separation of public and private sectors (11.1 quarters in our model vs. 8.9 quarters). Slow recovery of public investment followed by a severe decline generates delays in renegotiations in both models. On the contrary, fiscal constraint, limiting resource allocation across public and private sectors, is present only in our model generating further delays. Furthermore, if we allow the government have an additional method of taxation, i.e., lump-sum taxation on income (case ii-b), the impact of distortional taxation on resource allocation is relaxed. The model generates the same length of restructuring duration (8.9 quarters), while lower debt-to-GDP ratio (24.5 percent and 30 percent)—most due to an interaction between adjustments in public investment and the two methods of taxation.

Lastly, when we contrast our model and case (iii), a difference in the average restructuring duration between our model and a model with fixed public capital and no distortionary tax becomes even more larger (11.1 quarters in our model vs. 6.4 quarters). The large difference is generated because neither slow recovery of public investment after a severe decline nor distortionary taxation limiting resource allocation across two sectors is playing the role to generate further delays in the case (iii). The delays in renegotiation are only due to recovery in the debtor’s repayment capacity driven by TFP shocks and an associated response of elastic labor—these correspond to recovery of “exogenous income process” in Benjamin and Wright (2013) and Bi (2008).
5.5 Robustness Checks

Table 8: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>Adjustment costs</th>
<th>Depreciation rate</th>
<th>Weight on public cons</th>
<th>Risk aversion</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default probability (%)</td>
<td>4.22 3.1 1.9</td>
<td>5.2 3.1 1.9</td>
<td>3.7 3.1 1.9</td>
<td>5.7 3.1 1.9</td>
<td>3.3 3.1 1.9</td>
</tr>
<tr>
<td>Average recovery rate (%)</td>
<td>31.9 27.1 25.9</td>
<td>29.7 27.1 25.8</td>
<td>26.9 27.1 25.4</td>
<td>49.1 27.1 33.5</td>
<td>28.5 27.1 31.4</td>
</tr>
<tr>
<td>Public investment (std dev.)/output (std dev. %)</td>
<td>7.9 6.0 3.8</td>
<td>8.47 6.0 2.7</td>
<td>11.7 6.0 5.2</td>
<td>4.5 6.0 7.6</td>
<td>12.6 6.0 5.8</td>
</tr>
</tbody>
</table>

Non-target statistics

Pre-default periods

<table>
<thead>
<tr>
<th></th>
<th>Public consumption &amp; transfers (std dev.)/output (std dev.)</th>
<th>Corr(public consumption &amp; transfers, output)</th>
<th>Average public investment/GDP ratio</th>
<th>Corr(public investment, output)</th>
<th>Average debt/GDP ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.30 1.23 1.18</td>
<td>0.83 0.89 0.92</td>
<td>1.6 1.6 1.6</td>
<td>0.62 0.63 0.69</td>
<td>40.2 43.9 41.6</td>
</tr>
<tr>
<td></td>
<td>1.42 1.23 1.50</td>
<td>0.78 0.89 0.84</td>
<td>1.18 1.60 2.58</td>
<td>0.61 0.63 0.42</td>
<td>40.2 43.9 41.6</td>
</tr>
<tr>
<td></td>
<td>1.35 1.23 1.5</td>
<td>0.87 0.89 0.81</td>
<td>1.39 1.6 1.91</td>
<td>0.44 0.63 0.76</td>
<td>42.4 43.9 41.9</td>
</tr>
<tr>
<td></td>
<td>1.80 1.23 1.5</td>
<td>0.78 0.89 0.87</td>
<td>1.97 1.60 1.49</td>
<td>0.62 0.63 0.52</td>
<td>25.9 43.9 23.4</td>
</tr>
<tr>
<td></td>
<td>1.46 1.23 1.23</td>
<td>0.85 0.89 0.84</td>
<td>1.35 1.60 1.92</td>
<td>0.35 0.63 0.61</td>
<td>42.7 43.9 36.4</td>
</tr>
</tbody>
</table>

Renegotiation periods

<table>
<thead>
<tr>
<th></th>
<th>Public consumption &amp; transfers (std dev.)/output (std dev.)</th>
<th>Corr(public consumption &amp; transfers, output)</th>
<th>Average public investment/GDP ratio</th>
<th>Corr(public investment, output)</th>
<th>Duration of restructurings &amp; exclusion (quarters)</th>
<th>Public capital (percent change from the trough to the end, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.28 2.23 2.30</td>
<td>0.77 0.76 0.53</td>
<td>1.58 1.47 1.46</td>
<td>0.80 0.84 0.81</td>
<td>11.5 11.1 10.6</td>
<td>0.56 1.7 0.4</td>
</tr>
<tr>
<td></td>
<td>2.30 2.23 2.20</td>
<td>0.87 0.76 0.86</td>
<td>0.85 1.47 2.95</td>
<td>0.83 0.84 0.91</td>
<td>11.7 11.1 13.2</td>
<td>6.4 1.7 0.11</td>
</tr>
<tr>
<td></td>
<td>2.0 2.23 3.54</td>
<td>0.80 0.76 0.88</td>
<td>1.58 1.47 1.39</td>
<td>0.83 0.84 0.9</td>
<td>12.7 11.1 9.5</td>
<td>6.4 1.7 0.11</td>
</tr>
<tr>
<td></td>
<td>3.85 2.23 1.30</td>
<td>0.88 0.76 0.86</td>
<td>1.60 1.47 1.51</td>
<td>0.85 0.94 0.67</td>
<td>9.40 11.1 17.6</td>
<td>4.21 1.7 7.9</td>
</tr>
<tr>
<td></td>
<td>2.19 2.23 2.14</td>
<td>0.72 0.76 0.80</td>
<td>1.61 1.47 1.78</td>
<td>0.73 0.94 0.80</td>
<td>13.2 11.1 10.5</td>
<td>7.1 1.7 0.8</td>
</tr>
</tbody>
</table>

Source: Authors’ computation

Adjustment costs and depreciation rates on public capital, together with the household’s utility weight on public consumption and risk aversion, are key parameters pinning down the dynamics of public investment. Table 8 reports how changes in these parameter values (keeping other parameter values constant) influence the main moment statistics. An increase in adjustment costs on public capital reduces restructuring duration (10.6 quarters). In this case, the sovereign government is more hesitant to cut public investment severely due to high adjustment costs and attain quicker recovery of public investment to its pre-crisis level. Moreover, higher depreciation rates on public capital leads to larger demand for public investment (higher public investment in both pre-default and renegotiation periods than that in our baseline model), which in turn results in longer restructuring duration (13.2 quarters).

When the household assigns higher weight on public consumption in his utility, the sovereign government requires high public consumption and leads to lower public investment in ex-ante. This, in turn, results in high default probability, lower debt to GDP, and shorter renegotiation duration (9.5 quarters). On the contrary, when the household becomes more risk averse, the sovereign government opts to smooth its utility of household by allocating more on public consumption and transfers and less on pubic investment. This, in turn, ends up with a longer restructuring duration (17.6 quarters).
6 Testing the Theoretical Predictions

We also explore how slow recovery of public investment influences the likelihood of debt settlement. For this purpose, we assess determinants of debt settlements using a multinomial logit model as in conventional empirical studies on debt restructurings (Asonuma and Joo 2017). Our dataset is an unbalanced panel comprised of 111 post-default restructuring episodes in 60 countries (both emerging market and low-income countries) over the duration for each episode i.e., from the start of restructurings to the completion of exchanges. As in previous studies (Cruces and Trebesch 2013; Asonuma and Trebesch 2016), we treat each restructuring as an independent event when both debt instruments subject to exchange and dates of announcement and of settlement in one restructuring differ from those in other restructurings—there are overlapping observations included in our panel.

Following the conventions in the literature (e.g., Struzenegger 2004, Asonuma and Trebesch 2016), our data are at an annual frequency due to data availability of public investment and capital and external debt for the restructuring countries. The dependent variable captures whether restructurings are settled or not in the current year: 1 for completion of exchanges and 0 otherwise. Our main explanatory variables are either public investment or capital—measured as a deviation from the HP-filtered trend or a cumulative growth rate from the start of restructurings in the previous year—, public and publicly guaranteed (PPG) external debt-to-GDP ratio, and a deviation from and growth rate of the HP-filtered GDP trend for sovereign countries, which proxy productivity shocks. We also include world GDP growth rate, London Interbank Offered Rate (LIBOR) and the announcement of IMF-supported programs to control for growth and liquidity of the world economy and for a catalytic role of official financing.

Table 9 shows the logit regression results. We show that recovery of public investment or capital accumulation in the previous year increases the likelihood of completion of restructurings in the current year—quantitatively, a 1-percent increase in investment (from trend) or cumulative public capital growth rate increases the probability of settlements by 8.6 and 0.1 percent, respectively (columns 1’ and 2’). Both results are consistent with our theoretical findings. Moreover, the sovereign countries are more likely to agree on settlements when external debt is low, and growth and liquidity of the world economy is high and ample, respectively (columns 1–1’ and 2–2’). Neither a deviation from nor growth rate of the HP-filtered GDP trend enters as significant, possibly due to high correlation with public capital accumulation or investment, as discussed in Section 2 (columns 1–1’ and 2–2’).
Table 9: Public Investment and Capital, and Debt Settlements

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(1')</th>
<th>(2)</th>
<th>(2')</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coef/ dy/dx</td>
<td>Delta-method se</td>
<td>coef/ dy/dx</td>
<td>Delta-method se</td>
</tr>
<tr>
<td>Public investment, deviation from the trend (lagged, percent)$^{2/}$</td>
<td>0.347* 0.086*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.195)</td>
<td>(0.048)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public capital, cumulative growth rate (lagged, percent)$^{1/}$</td>
<td>-</td>
<td>-</td>
<td>0.005*</td>
<td>0.001*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.003)</td>
<td>(0.0007)</td>
</tr>
<tr>
<td>PPG external debt (lagged, percent of GDP)$^{3/}$</td>
<td>-0.003** -0.001**</td>
<td>-0.003*</td>
<td>-0.0007***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>GDP, deviation from the trend (current, percent)$^{2/}$</td>
<td>-0.017 0.004</td>
<td>0.561</td>
<td>0.139</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.951)</td>
<td>(0.236)</td>
<td>(0.864)</td>
<td>(0.215)</td>
</tr>
<tr>
<td>GDP, trend growth rate (current, percent)$^{2/}$</td>
<td>-0.045 0.011</td>
<td>-0.073</td>
<td>-0.018</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.010)</td>
<td>(0.045)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>LIBOR, 12-month average (current, percent)</td>
<td>-0.061*** -0.051***</td>
<td>-0.058***</td>
<td>-0.014***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.006)</td>
<td>(0.024)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>World GDP, growth rate (current, percent)</td>
<td>0.207*** 0.051***</td>
<td>0.206***</td>
<td>0.051***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td>(0.017)</td>
<td>(0.069)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.705*</td>
<td>-</td>
<td>-0.722*</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.369)</td>
<td></td>
<td>(0.372)</td>
<td></td>
</tr>
</tbody>
</table>

episode-specific Fixed effect: No

Number of episodes: 90

Number of observations: 496

Wald $\chi^2$: 21.01

Prob. $> \chi^2$: 0.002

Notes: The table shows results from random effects multinomial logit regressions. The dependent variable is debt settlement (binary choice). The main explanatory variables are public capital cumulative growth rate and public investment deviation from the trend. Public capital cumulative growth rate, public investment deviation from the trend, and PPG external debt (percent of GDP) are lagged by one year. The other explanatory variables are in the current year. Significance levels denoted by $*** p < 0.01$, $** p < 0.05$, $* p < 0.10$, respectively. Robust standard errors (Delta-method standard errors) are in parentheses.

$^{2/}$ Cumulative growth of public capital since the start of restructurings.

$^{2/}$ A deviation from the trend and trend growth rate are a percentage deviation from the trend and annual change in the trend obtained by applying a Hodrick-Prescott (HP) filter to annual series with filter of 6.25.

$^{3/}$ Public and publicly guaranteed external debt. Lagged level in terms of GDP.
7 Conclusion

The current paper explores the role of public capital on sovereign defaults and debt restructurings. We newly code a dataset on public expenditure composition around restructurings with private external creditors. We find four new stylized facts on sovereign debt overhang and public expenditure composition around post-default restructurings. To explain these facts, we embed both public expenditure composition and capital accumulation in a conventional theoretical model of sovereign debt with default and renegotiations. Our model quantitatively replicates these stylized facts and shows both severe decline and slow recovery in public investment delay debt settlement. Our theoretical predictions are supported by data.

Having a firm grasp of how public capital plays an important role in sovereign defaults and debt restructurings, we will be able to potentially explore the effectiveness of different types of fiscal rule (e.g., debt limit rule, primary balance rule, expenditure rule, and tax rule) depending on purposes (e.g., avoiding default, achieving quick debt settlements). Current policy debate centers only on the desirable rule (in particular size) the sovereign should implement before default, i.e., ex ante—targeting to avoid default. A future research project with enriched implications would potentially contribute to discussion on desirable timing of introduction of the rule (i.e., ex ante or ex post) and shift of the rule upon default—targeting quick debt settlement.
References


Appendix A  Dataset: Details of Coding

Coding details for government consumption, transfers and investment following US BEA (2005) are as follows:

- Government consumption = (i) Compensation of employees +
  = (ii) Purchases/ uses of goods and services +
  = (iii) Expense not elsewhere classified
- Government transfers = (iv) Social benefits
- Government investment = (v) Net acquisition of non-financial assets

Government consumption & transfers = Government consumption + Government transfers

Note that interest is not included in primary expenditure.

Definitions of public sector and references for the first 18 restructurings (by alphabet) are as follows:

- **Argentina 1982–85**: Public sector / Federal government. IMF (1986 - EBS/86/39),


• **Brazil 1989–92**: Not available

• **Brazil 1989–94**: Not available


Appendix B  Further Empirical Analysis

B.1  Comparison of Investment and Capital in Public and Private Sectors

Figure B1 and B2 show the dynamics of private and public investment and capital around restructurings. We follow the same presentation approach as in Figure 1 in terms of both time horizon, timing of events (both start and end of debt crisis), scale (real and level) and normalization of two series at the level at the two events.

Both Panels (i) and (ii) in Figure B1 show that the dynamics of private investment are identical to those of public investment. Both private and public investment declines sharply at the onset of debt crisis (year 0) and stays below the pre-crisis level in the subsequent years (Panel i). Private and public investment only recovers to the pre-crisis level in year 4, leading to debt settlements in year 5. After the settlements (year 0), both private and public investment follows a steady trend (Panel ii).

Figure B1: Private and Public Investment around Restructurings
(real, level - start=100)

(i) Around Start of Restructurings
(ii) Around End of Restructurings

Figure B2 shows that private capital experiences similar dynamics with public capital. Growth rate of private and public capital is significantly lower during debt restructurings than in the pre-restructuring periods. Actual level of both private and public capital is far below the projected level along the pre-crisis trend.
Figure B2: Private and Public Capital around Restructurings
(real, level - start=100)

(i) Around Start of Restructurings
B.2 Public Expenditure Composition around Restructurings

Figure B3: Public External Debt, and Public Consumption, Investment and Transfers during Restructurings

(i) Public Consumption (percent of GDP)

(ii) Public Investment (percent of GDP)

(iii) Public Transfers (percent of GDP)
### B.3 Public Investment Declines and Duration of Restructurings

Table B1: Correlation between Public Investment Declines and Duration of Restructurings

<table>
<thead>
<tr>
<th>Duration of restructurings (years)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coef/se</td>
<td>coef/se</td>
<td>coef/se</td>
<td>coef/se</td>
</tr>
<tr>
<td>Public investment/GDP, cumulative difference over first 2 years (percentage point)</td>
<td>-0.47*</td>
<td>-0.73**</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.35)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public investment/GDP, difference over first 1 year (percentage point)</td>
<td>-</td>
<td>-</td>
<td>-0.53*</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.29)</td>
<td></td>
</tr>
<tr>
<td>Public investment, cumulative change over first 2 years (%)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.03*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.01)</td>
</tr>
<tr>
<td>Public capital/GDP, pre-restructuring (%)</td>
<td>-0.006</td>
<td>-</td>
<td>0.905</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP, deviation from trend, end (%)(^2)</td>
<td>0.11*</td>
<td>0.11*</td>
<td>0.12**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td></td>
</tr>
<tr>
<td>External debt, end (percent of GDP)</td>
<td>0.016</td>
<td>0.018*</td>
<td>0.02*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Export-to-debt service ratio, end (%)</td>
<td>0.13</td>
<td>0.12</td>
<td>0.13</td>
<td></td>
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<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>LIBOR 12-month, end (%)</td>
<td>-0.69***</td>
<td>-0.67***</td>
<td>-0.64***</td>
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<tr>
<td></td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.16)</td>
<td></td>
</tr>
<tr>
<td>IMF-supported program, end (dummy)(^3)</td>
<td>-2.64***</td>
<td>-2.75***</td>
<td>-2.61***</td>
<td></td>
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<tr>
<td></td>
<td>(0.97)</td>
<td>(0.98)</td>
<td>(0.96)</td>
<td></td>
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<tr>
<td>Bond restructurings (dummy)(^4)</td>
<td>-4.33**</td>
<td>-4.53***</td>
<td>-4.47***</td>
<td></td>
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<td></td>
<td>(1.64)</td>
<td>(1.64)</td>
<td>(1.63)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>4.83***</td>
<td>10.66***</td>
<td>10.42***</td>
<td>9.55***</td>
</tr>
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<td></td>
<td>(0.535)</td>
<td>(1.91)</td>
<td>(1.88)</td>
<td>(1.91)</td>
</tr>
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<td>Number of observations</td>
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<td>86</td>
<td>86</td>
<td>87</td>
</tr>
<tr>
<td>Adjusted-(R^2)</td>
<td>0.027</td>
<td>0.36</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Root MSE</td>
<td>4.79</td>
<td>4.05</td>
<td>4.08</td>
<td>4.06</td>
</tr>
</tbody>
</table>

Notes: The table shows results from ordinary least square (OLS) regressions. The dependent variable is duration of restructurings (years). The main explanatory variables are cumulative change in public investment-to-GDP ratio and cumulative change in public investment. Significance levels are denoted by *** \(p < 0.01\), ** \(p < 0.05\), * \(p < 0.10\), respectively. Standard errors are in parentheses.

\(^1\) Public capital-to-GDP ratio one year before the start of restructurings.

\(^2\) GDP deviation from the trend is a percentage deviation from the trend, obtained by applying a Hodrick-Prescott (HP) filter to annual GDP series with filter 6.25.

\(^3\) A dummy for an IMF-supported program is set to 1 when an IMF-supported program is approved at the year of completion of debt restructurings and 0 otherwise.

\(^4\) A dummy for bond restructurings is set to 1 if a restructuring is a bond exchange.
Appendix C  Computation Algorithm

The procedure to compute the equilibrium distribution of the model is the following:

1. First, we set finite grids on the space of asset holdings, public capital and productivity as by $B = [b_{min}, 0]$, $K^g = [k^g_{min}, k^g_{max}]$, and $A = [a_{min}, a_{max}]$. The limits of the asset space and public capital are set to ensure that limits do not bind in equilibrium. Limits on productivity shocks are large enough to include large deviations from mean value of shocks. We approximate the stochastic productivity process of the sovereign shown by equation (40) using a discrete Markov chain of 21 equally spaced grids as in Tauchen (1986). Moreover, we compute the transition matrix based on the probability distribution $\mu(a_{t+1}|a_t)$.

2. Second, we set finite grids on the space of recovery rates ($\delta_t$). Limits of both sets of recovery rates are to ensure that they do not bind in equilibrium.

3. Third, we set the initial values for equilibrium sovereign bond prices, payoffs of debt renegotiation for the sovereign and the creditors, and the sovereign’s value function. We use the risk-free bond price ($q^0 = (1 + r^*)^{-1}$) for the baseline equilibrium bond price. We set $\Delta_t^B, 0 = \Delta_t^L, 0 = 0$ and $V^0 = V^R, 0 = V^D, 0 = 0$ for the baseline payoffs and value function.

4. Fourth, given the baseline equilibrium bond price, payoffs, and value function, we solve for the household’s and the firm’s maximization problems to get private consumption and labor.

5. Fifth, given the baseline equilibrium sovereign bond prices, payoffs, and the private sector’s equilibrium policy functions, we solve for the sovereign’s optimization problem for good and bad credit records ($h_t = 0, 1$). This procedure finds the value function as well as the default decisions. We first guess the value function ($V^0, V^R, 0, V^D, 0$) and iterate it using the Bellman equation to find the fixed value ($V^1, V^R, 1, V^D, 1$). By iterating the Bellman function, we also derive the optimal asset policy function ($b^1, b^R, 1, b^D, 1$) and public capital policy function ($k^g, 1, k^g, R, 1, k^g, D, 1$) for each value. We obtain the default choice, which requires a comparison of the values of defaulting and non-defaulting. By comparing these two values, we calculate the corresponding default set. Based on the default set, we also evaluate the default probability using the transition matrix.

6. Sixth, using the default set in step (5), and the zero profit condition for the foreign creditors, we compute the new prices of sovereign bonds ($q^1$).

7. Seventh, given the value function for the sovereign, we solve the bargaining problem and compute the new payoffs for two cases either the borrower or the lender is the proposer ($\Delta_t^B, 1, \Delta_t^L, 1$).
8. We iterate steps (4), (5), (6) and (7) to have the fixed optimal value function, payoffs, and bond prices.

Appendix D  Additional Equilibrium Properties

We provide additional equilibrium properties of our theoretical model. Figure D1 reports agreed recovery rates when the sovereign proposes. Panels (i) and (ii) report recovery rates at mean (benchmark) and high public capital, respectively. As there is no settlement when public capital is low (corresponding to no “Settlement” region in Panel (i) in Figure 6), there are no agreed recovery rates. Our benchmark case of mean public capital (Panel i) replicates general patterns of recovery rates as in previous work on debt restructurings (Yue 2010; Bi 2008; Asonuma and Trebesch 2016). Recovery rates are increasing respect to both the sovereign’s assets and productivity: as the size of debt becomes smaller, the sovereign can pay higher portion of defaulted debt. When the sovereign has more resources associated with high productivity, it repays high recovered debt payments. The region where there are no agreed recovery rates corresponds to either the region the sovereign opts to repay debt in full (“Repayment” region), or the region where the sovereign does not settle on the deal with the creditors (“Delay” region) (Panel i in Figure 6).

More importantly, our model shows one new feature of recovery rates: agreed recovery rates do not change depending on the level of public capital where both parties agree on debt settlements. There are two driving forces of public capital working in opposite directions and resulting in this outcome. On the one hand, high public capital increases the sovereign’s value of “proposing” (shown in equation 18)—corresponding to the role of public capital as one part of resources. Thus, the sovereign is more willing to settle resulting in high recovery rates. On the other hand, high public capital increases the sovereign’s value of “passing” (shown in equation 20). As the sovereign has high savings, it is not costly for the sovereign to remain in financial autarky. The sovereign is less willing to settle which, in turn results in low recovery rates. These two effects offset each other. As a result, recovery rates do not change depending on the level of public capital.
Figure D1: Agreed Recovery Rates

(i) Mean Public Capital

(ii) High Public Capital

Figure D2: Public Consumption/Transfers

(i) Low Public Capital

(ii) High Public Capital
Figure D3: Public Investment

(i) Low Public Capital

(ii) High Public Capital
### Appendix E  Further Quantitative Analyses

#### Table E1: Non-business Cycle Statistics

<table>
<thead>
<tr>
<th>Target statistics</th>
<th>Data</th>
<th>Baseline Model</th>
<th>Model with no public capital</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With distortion tax</td>
<td>With distortion tax and lump-sum tax</td>
<td>No distortion tax</td>
</tr>
<tr>
<td>Default probability (%)</td>
<td>3.26</td>
<td>3.10</td>
<td>2.8</td>
</tr>
<tr>
<td>Average recovery rate (%)</td>
<td>25.0</td>
<td>27.1</td>
<td>22.3</td>
</tr>
<tr>
<td>Average debtor output deviation during debt renegotiation (%)</td>
<td>-4.45</td>
<td>-3.73</td>
<td>-4.35</td>
</tr>
<tr>
<td>Bond spreads: average (%)</td>
<td>45.4</td>
<td>43.9</td>
<td>51.3</td>
</tr>
<tr>
<td>Bond spreads: std dev. (%)</td>
<td>9.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Corr.(spreads, output)</td>
<td>0.92</td>
<td>0.21</td>
<td>0.35</td>
</tr>
<tr>
<td>Corr.(debt/GDP, spreads)</td>
<td>-0.97</td>
<td>-0.69</td>
<td>-0.71</td>
</tr>
</tbody>
</table>

#### Pre-default periods

| Renegotiation periods                          | Data | Baseline Model | Model with no public capital |
|                                                | With distortion tax | With distortion tax and lump-sum tax | No distortion tax |
| Average debt/GDP ratio (%)                    | 130.5 | 50.6           | 60.2                        | 62.9              |
| Corr.(debt/GDP, output)                       | -0.95 | -0.98           | -0.99                       | -0.99             |
| Duration of renegotiation/ exclusion (quarters) | 14.0  | 11.1           | 8.8                         | 7.7               |
| Debtor output deviation (diff. btw start and end, %) | 12.6  | 21.2           | 22.4                        | 22.0              |
| Public capital (percent change from the trough to the end) | 2.31  | 1.7            | -                           | -                 |
| Corr.(cumulative change in public investment to GDP, duration) | -0.12 | -0.08           | -                           | -                 |
| Corr.(cumulative percent change in public investment, duration) | -0.16 | -0.1            | -                           | -                 |

Sources: Datastream, IMF WEO, MECON.

Notes:
1/ Model with fixed public capital corresponds to our model (with the same parameter values) with public capital fixed at the average (under the baseline model).
2/ Model with endogenous public capital corresponds to our model (with the same parameter values) without separation of public and private sectors or with lump-sum taxation on income.
3/ Model with fixed public capital and no distortionary tax corresponds to our model (with the same parameter values) with public capital fixed at the average (under the baseline model) and without distortionary consumption tax.
4/ A difference in the sovereign’s output deviation at the the end and at the start of restructurings.
Table E2: Non-business Cycle Statistics of Model

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Yue (2010)$^{1/}$</th>
<th>Arellano (2008)$^{2/}$</th>
</tr>
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<tbody>
<tr>
<td><strong>Target statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default probability (%)</td>
<td>3.26</td>
<td>3.10</td>
<td>4.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Average recovery rate (%)</td>
<td>25.0</td>
<td>27.1</td>
<td>29.5</td>
<td>-</td>
</tr>
<tr>
<td>Average debtor output deviation during debt renegotiation (%)</td>
<td>-4.45</td>
<td>-3.73</td>
<td>-7.1</td>
<td>-9.0</td>
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<tr>
<td><strong>Pre-default periods</strong></td>
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<td></td>
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<tr>
<td>Average debt/GDP ratio (%)</td>
<td>45.4</td>
<td>43.9</td>
<td>5.6</td>
<td>4.6</td>
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<tr>
<td>Bond spreads: average (%)</td>
<td>9.4</td>
<td>0.2</td>
<td>0.1</td>
<td>3.8</td>
</tr>
<tr>
<td>Bond spreads: std dev. (%)</td>
<td>7.6</td>
<td>0.3</td>
<td>0.1</td>
<td>6.3</td>
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<tr>
<td>Corr.(spreads, output)</td>
<td>-0.88</td>
<td>-0.1</td>
<td>0.30</td>
<td>-0.15</td>
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<tr>
<td>Corr.(debt/GDP, spreads)</td>
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<td>0.21</td>
<td>-0.58</td>
<td>0.32</td>
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<tr>
<td>Corr.(debt/GDP, output)</td>
<td>-0.97</td>
<td>-0.69</td>
<td>-0.50</td>
<td>0.24</td>
</tr>
<tr>
<td><strong>Renegotiation periods</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average debt/GDP ratio (%)</td>
<td>130.5</td>
<td>50.6</td>
<td>6.4</td>
<td>5.2</td>
</tr>
<tr>
<td>Corr.(debt/GDP, output)</td>
<td>-0.95</td>
<td>-0.99</td>
<td>-0.98</td>
<td>-0.89</td>
</tr>
<tr>
<td>Duration of renegotiation/exclusion (quarters)</td>
<td>14.0</td>
<td>11.1</td>
<td>2.0</td>
<td>4.2</td>
</tr>
<tr>
<td>Corr.(cumulative change in public investment to GDP, duration)$^{4/}$</td>
<td>-0.12</td>
<td>-0.08</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Corr.(cumulative percent change in public investment, duration)$^{5/}$</td>
<td>-0.16</td>
<td>-0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Debtor output deviation (diff. btw start and end, %)$^{6/}$</td>
<td>12.6</td>
<td>21.2</td>
<td>0.6</td>
<td>9.3</td>
</tr>
<tr>
<td>Public capital (percent change from the trough to the end)</td>
<td>2.31</td>
<td>1.7</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Sources: Datastream, IMF WEO, MECON.
Notes: $^{1/}$Yue (2010) model corresponds to our model (with the same parameter values) with one-round renegotiations (Nash bargaining game).

$^{2/}$Arellano (2008) model corresponds to our model (with the same parameter values) without debt renegotiations (e.g., exogenous re-entry).

$^{3/}$Average public investment/GDP in its downward trend since the start of restructurings and its upward trend since its trough.

$^{4/}$A correlation between cumulative percentage point change in public investment/GDP ratio from t-4 to t+4 and duration of restructurings.

$^{5/}$A correlation between cumulative percent change in public investment (level) from t-4 to t+4 and duration of restructurings.

$^{6/}$A difference in the sovereign’s output deviation at the the end and at the start of restructurings.