Commission Sharing and Search Agents*

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Preliminary and in Progress: Sep 29 2009

Abstract

When a principal hires an agent to do searching, she needs to motivate the agent to pay effort as well as to deliver a suitable result. Since different principals have different taste, there is an opportunity for the agents to cooperate among themselves and use commission sharing to better match search results to principals. This paper studies how such fee-sharing arrangement affects the agents' incentive when exerting effort and the principals' incentive when offering contracts. I show that fee-sharing arrangement introduces a public good problem between the principals. In the case of linear contracts, contracts would have lower piece-rates and the agents would exert lower effort in searching when fee-sharing arrangement is allowed. However, efficiency may increase because the search results would be better matched to the principals.

1 Introduction

Often agents are hired to perform the function of searching. After exerting efforts, the agents may find that the result of the search does not match the principal's taste perfectly. When different principals have different taste, an outcome bad for one principal may be good for another, so opportunities exist for agents to trade among themselves based on the realized values of

*Thanks to valuable comments from Yuk-fai Fong, Johannes Horner and Jin Li. Thanks to participants at IO workshop at HKUST. All errors remain my own. This project began when I was affiliated with Cornerstone Research.

the result of their efforts to different principals. Therefore, principals have to simultaneously motivate the agents to exert effort and also to cooperate among themselves to better serve the principals' taste.

Such principal and agent relationship exists in the real estate and rental markets. Brokers pay effort to look for potential buyers for the seller, but in the process they may find buyers that are not entirely suitable to his contracted seller. To solve this problem, brokers cooperate with each other through the MLS ("Multiple Listing Service"), a cooperative arrangement among brokers in a particular area to share their property listings with each other. Through the MLS, one broker can deliver a buyer found by other brokers by sharing the commission with that broker if the seller eventually makes a deal with that buyer.

Such cooperative commission sharing arrangement may also be found desirable among other agents who are hired to do searching, such as third party recruiters, also called headhunters. For instance, an industry newsletter, Executive Recruiter News, argues that acting in the client's best interest demanded that if the retained firm could not complete the assignment, it should partner with a firm that could finish the job on a fee-sharing basis. Guidebooks for recruiter have sections about how to write fee-sharing contracts and various websites provide sample fee sharing agreements for recruiters to use.

In these situations, the suitability of a search result to the principal's taste is subject to some uncertainty that is out of the agent's control, but the agents have access to information such as other agents' identity and search results and thus have the freedom to get together with other agents to reach a mutually beneficial outcome. The principal cannot directly control the agent's ability in "cooperating" with others. This freedom obviously increases the efficiency in terms of matching search results to principals. However, this raises interesting questions such as how the possibility of inter-agent trading affects the incentive of the agents when exerting efforts, as well as the incentive of the principals when offering contracts.

From a more theoretic point of view, it is also interesting to study a multi-principal multi-agent model where principals are restricted to contract only with one agent and the agents have multiple tasks. We will elaborate later on how such a model differs from the existing literature.

More specifically, this paper studies a two-principal and two-agent model where each principal can contract with only one agent by offering him a linear contract. Agents first exert effort and then consider whether or not to swap the outcome with the other agent accompanied with monetary transfers. The fee-sharing arrangment is determined through Nash Bargaining. Then the principals pay according to the project received. In this model, the search effort only determines the vertical quality of the result, and the horizontal aspect (taste component) is random, i.e. not controlled by the agents' effort.

In this setting, we show that when inter-agent trading is possible, on the unique symmetric equilibrium, the principals offer a lower-powered incentive contract (i.e., with a lower piece-rate) than when inter-agent trading is not possible. These are socially sub-optimal in the sense that they are too low-powered to make agents exert the efficient effort in providing the vertical quality of the good. The result that first-best effect level can not be achieved on equilibrium holds even when we allow for general contracts, instead of just linear contracts. Linear contracts allow for an easy characterization of the "power" of the contract, namely, the piece-rate or so-called "commission rate".

The intuition is that the principals face a public good/free-rider problem as soon as the fee sharing arrangement links the two problems of the principals: each principal fails to consider the benefit to the other principal in motivating more effort. When fee-sharing is not allowed, the two principals' problems are separate and they can achieve the first-best level of effort. When fee-sharing is allowed, each principal only internalizes one agent's payoff through the individual rationality constraint, while this principal's contract, if accepted, affects both agents' incentives to trade. When a principal unilaterally decreases the power of her contract from the first-best level, the resulted loss of the surplus is born by both agents, and thus by the other principal as well. A slightly lower-powered contract than the first-best (given that the other principal uses the first-best contract) does not change the total pie much, but gives this principal a bigger share of the pie. This creates an incentive for both principals to shirk in motivating agents' effort. When the contract space consists only of linear contract, the result is that the equilibrium piece-rate is lower than the level when fee-sharing is not allowed (the first-best level). However, the total welfare may increase because of the fee-sharing arrangement because results are better matched with the principals' taste. Intuitively, when the gain from better matching taste is big enough, allowing fee-sharing strictly improved welfare (all of which is extracted by the principals). In fact, when the gain from better matching taste is small enough, the welfare is also always improved. We characterize the welfare comparison for quadratic cost function of effort.

By focusing on the linear contract, this paper highlights several effects of the commission rate, i.e., the piece-rate. First, a higher commission rate not only makes one's own agent work harder, it also transfers surplus to the other agent (and ultimately to the other principal) because trading surplus are shared between the agents before they are extracted away by the principals. Second, different commission rates by principals will cause inefficiency in agents' trading decisions because they make agents as a whole put non-equal weights on the welfare of the different principals.

This model also highlights the role of the restriction that a principal cannot contract with all agents whose action affects this principal's payoff. This is the source of the inefficiency in the equilibrium. If we instead we allow each principal to offer two bilateral contracts, one to each agents, then the non-cooperative equilibrium will be efficient, even when the principal can only contract on very limited information: her own received outcome. We consider this restriction is very natural. In real life, contracting with multiple agents can be very costly. The principals may not even be able to know the identity of the agents. Some hold-up problems may also lead to exclusive contracts with agents.

This paper relates to the literature on multi-task agency, where the principal wants to motivate the agent to do two or more tasks.[1] This paper differs from previous multi-task agency problems in at least two ways. First, the tasks are carried out in sequence instead of simultaneously and before doing the second task, the agent receives new information (through the bargaining process), such as the realization of exogenous factors and the other agent's previous action in the first task. (However, this model does not contain adverse selection because when accepting the contract from the principal the agents have no private information.) Second, the tension between the two tasks arises from the fact that there are two principals. In other words, if the two principals can cooperate then both tasks can be achieved with first-best outcomes in this model. Agents in my model do not face the trade-off between working on one task versus the other.

The paper is also related to the common agency models in the sense that one agent's effort affect multiple principals' payoff. Common Agency models

[1] Holmstrom and Milgrom (1991) looked at the tension between allocating risks and rewarding productive work. Itoh (1991) studied a multi-task problem with multiple agents, where the principal wants to motivate the agents to exert effort on their own tasks and also to help other agents.

however assumes that each principal can offer contracts to every agent.[2] In this paper, since a principal only contract with one agent and thus does not internalize the other agent's utility, the first-best outcome cannot be achieved even though the collusive outcome would be first-best. The fact that efficiency can be restored in this model if each principal can offer two contracts, one to each agent, echoes the result in Segal (1999), which showed that, when a principal can offer multiple bilateral public offers, if there is no externality on agents' reservation utilities, the equilibrium is efficient.

There is also a literature that focuses on agents that are hired to do search. Lewis and Ottaviani (2008) studied a general single-principal and single-agent model where the agent can gain private benefits over the cause of searching, and their focus was on the interaction of the agent's incentives to exert search effort with agent's incentives to report the private information the agent acquires during the search process. Other papers have modeled search agents specifically for the real estate market, but have not looked at the fee sharing arrangements in a principal-agent framework. Several papers look at the conflict between the effort (search) dimension and the informational (suggest a reservation price) dimension [3] In comparison, this paper looks at the tension between the dimension of effort (search) and the dimension of cooperating with other agents (share commission).

There is also a literature on referral, another form of interaction among agents. It has been studied in a non-principal-agent setting in Garicano and Santos (2004), which focused on matching opportunities with agents' talent, rather than on matching outcomes with principals' tastes. Agents in their model decide on who should supply the effort, while in my model, agents decide on to whom they should deliver the result of their effort.

The paper is organized as follows. Section 2 presents the model. Next, section 3 shows the benchmark when the agents are unable to trade with each other. Section 4 shows the main model and its results. Section 5 discusses the assumptions and robustness of the results. Section 6 concludes.

[2]Bernheim and Whinston (1986), Bernheim and Whinston (1985)

[3]See Geltner, Kluger and Miller (1991) and Arnold (1992).

2 Model

2.1 The players

There are two principals, 1 and 2. They each have an agent, 1 and 2. The principal-agent relationship is assumed to be fixed, meaning that principal 1 can only contract with agent 1 and principal 2 can only contract with agent 2.[4] We will refer to an agent as him and a principal as her.

An agent can exert effort to find an outcome. Denote the effort by $e_i \geq 0$, with $i = 1, 2$. Agents incur a cost of $C(e_i)$. Assume C is infinitely differentiable and $C(0) = 0$, $C'(0) = 0$, $C'(e) > 0$ for all $e > 0$, $C''(e) > 0$ for all e.

Principal 1 and principal 2 have differentiated taste. Whether or not agents exert effort, the taste element in the outcome is a random draw. More specifically, agent 1 gets a realization x_1 and agent 2 gets a realization x_2, both of which are independent random draws from a distribution on $[-\frac{1}{2}, \frac{1}{2}]$ and with a continuously differentiable density that is symmetric around 0.

The project from agent i is worth $v + e_i - x_i t$ to principal 1 and $v + e_i + x_i t$ to principal 2, where $t > 0$ and $i = 1, 2$.

To make the problem continuous and avoid considering the case of negative project value, we have assumed the agent can create quality v with no effort and he always does so. We also assume $v > \frac{1}{2}t$, so that there is enough base value to the project such that a project will always have some value to any of the two principals. We assume the agents have the same outside option if they do not get hired, and normalize it to be 0.

Denote the socially efficient level of effort by e^*, i.e., e^* is defined by $1 = C'(e^*)$. We assume $e^* - C(e^*) > 0$.

2.2 Information structure

The agents' actions (exerting effort and swapping outcomes) are not observable to the principals, the only thing that the principals can observe is the value of the outcome she receives. In particular, she does not observe the value of the outcome the other principal gets.

[4] We can relax this fixed pairing by adding the following stages at the beginning: let both principals offer one contract to all the agents, and then let the agents decide whether to accept each contract, with the restriction that he can only accept one contract at most. Then the results of this paper will not change.

Both principals can offer a contract to her own agent. We assume the contract to be linear[5]. In particular, principal 1 offers (F_1, k_1) and principal 2 offers (F_2, k_2) with $k_1 \geq 0$ and $k_2 \geq 0$ denoting the piece-rate. Denote the value of the project that principal 1 *receives* to be v_1 and that of the project that principal 2 *receives* to be v_2. The contracts oblige principal 1 to pay $k_1(v_1 - v) + F_1$ and principal 2 to pay $k_2(v_2 - v) + F_2$. We will omit the base project value v as it does not affect the analysis. The offered contracts[6] and whether agents accept or reject them are the common knowledge.

2.3 Timing

The timing of the game is as follows.

1. Principals 1 and 2 simultaneously make take-it-or-leave-it linear contract offers to their respective agents.

2. Agents 1 and 2 simultaneously decide whether to accept the offers or not.

3. Agents 1 and 2 simultaneously exert efforts.

4. Project values are realized.

5. Agents 1 and 2 Nash-bargain (with equal bargaining power) to determine the transfer between them and whether there will be swapping of outcomes.

6. Agents 1 and 2 hand projects to their respective principals and receive the promised payments from their principals.

7. Agents 1 and 2 pay each other transfers as promised. (The timing of this one step can be right after Nash Bargaining.)

[5]In the Appendix, we have a section that shows the main results hold if we allow non-linear contracts.

[6]We will show that this observability of the other agent's contract is not essential.

2.4 Equilibrium concept

All players are risk-neutral and they maximized their expected payoff. We assume there is no discounting.

Despite that there is incomplete information in this model, since the principals only act once at the beginning of the game by offering contracts, their beliefs will not play a role. Principals' strategies are simply the contracts they offer. The agents' behavioral strategies are, 1) a mapping to the acceptance decision given the two contracts offered and 2) a mapping to efforts given what contracts are accepted. (After the realization of outcomes, they are assumed to do Nash Bargaining.) We define the equilibrium as a strategy profile such that each player has no unilateral incentive to deviate to a different strategy at any stage of the game.

3 No-trading Benchmark

When the two agents cannot trade with each other[7] , the two principal-agent pairs are not related in this game, so we can just look at one pair. The analysis is very standard. Since the principal can extract all the surplus from the agent through the fixed part of the linear contract, the principal is effectively maximizing the total expected surplus of the principal-agent pair. WLOG, we consider the pair 1-1. Principal 1 solves:

$$\begin{aligned} \max_{k_1} \quad & e_1 - E[x_1]t - C(e_1) = e_1 - C(e_1) \\ s.t. \quad & e_1 = argmax_e\{k_1(e - E[x_1]t) - C(e)\} = argmax_e\{k_1 e - C(e)\} \end{aligned}$$

The unique solution clearly is $k_1 = 1$. The resulting effort level is efficient and in this benchmark $F_1 = -(e^* - C(e^*))$.

Lemma 1. *In the no-trading-allowed benchmark, the unique equilibrium outcome is that both principals offer* $(-(e^* - C(e^*)), 1)$*, both agents accept the contract and they both exert the efficient effort level.*

[7]If one generalizes the game such that principals first choose to whether or not to ban trading (maybe through banning the communications between agents) and trading can only occur if both principles allow it, then this benchmark of no trading will exist as an equilibrium, because given that one principal bans trading, the other weakly prefers to ban trading as well.

4 Trading Allowed

Despite that the effort levels are efficient in the no-trading Benchmark, there exists a source of inefficiency there: the projects are not efficiently matched to principals. principal 1 always receives the project created by agent 1 and principal 2 always receives the project created by agent 2 in the Benchmark, while the most efficient pairing between projects and principals depends on the realization of x_1 and x_2. The symmetry of the setup implies the following two trading rules are efficient, and only they are efficient:

1. To trade if and only if $x_1 - x_2 \geq 0$.
2. To trade if and only if $x_1 - x_2 > 0$.

From now on, to simplify notation, we will simply use $x_1 - x_2 \geq 0$ as the efficient trading rule. Keep in mind that the analysis would be the same if we use $x_1 - x_2 > 0$ instead.

4.1 Trading incentive given efforts

Because of the nature of Nash Bargaining between the two agents, trading will occur whenever it is subgame-efficient to do so, i.e., efficient given the effort level e_1, e_2, the realized value of x_1 and x_2 and the contracts the agents have accepted. Consider the subgame just after x_1 and x_2 are realized. Therefore, trading happens if and only if[8]:

$$\begin{aligned} & [k_1(e_2 - x_2 t) + k_2(e_1 + x_1 t)] - [k_1(e_1 - x_1 t) + k_2(e_2 + x_2 t)] \geq 0 \\ \Leftrightarrow & -(k_1 - k_2)(e_1 - e_2) + (k_1 + k_2)(x_1 - x_2)t \geq 0 \end{aligned}$$

For any $k_1 + k_2 > 0$, the above trade-condition is equivalent to:

$$x_1 - x_2 \geq \frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)}$$

Let's contrast this with the efficient condition for trading. To trade is efficient if and only if,

[8]We assume the tie-breaking rule that they will trade if indifferent.

$x_1 - x_2 \geq 0$

This immediately implies the following lemma:

Lemma 2. *To trade is efficient if and only if one of the two conditions is true:*

1. *the two contracts have the same piece-rates:* $k_1 = k_2$*.*
2. *the two agents exerted the same effort:* $e_1 = e_2$*.*

To understand this lemma, notice that trading can increase efficiency in only two ways.

- Case 1. Benefit both principals. $\Rightarrow$ Agents will do it no matter what contracts they are given (as long as $k_1 \geq 0$ and $k_2 \geq 0$).
- Case 2. Benefit one principal while hurt the other. $\Rightarrow$ Agents may not agree to do the trading.

When $e_1 = e_2$, i.e., when there is no vertical difference in the quality of the outcomes, only Case 1 is possible. In other words, if trading is efficient, then trading benefits both principals, so agents trade.

When $k_1 = k_2$, then agents collectively view the two principals as a social planner would: equally, so through nash bargaining, they will trade whenever trading is efficient (even when it is Case 2). Notice that the efficiency criteria is simply a utilitarian criteria that gives the same weights to both principals' outcomes.

When $e_1 \neq e_2$ and $k_1 \neq k_2$, only Case 2 may happen and the agents collectively give different "weights" to the two principals, therefore, they do not act like a social planner and they do not trade efficiently.

Since to trade is efficient if $k_1 = k_2$, the first-best pair of contracts when trading is allowed is simply all contracts that satisfy $k_1 = k_2 = 1$.

We will denote the maximum gain of trading to match taste (for all players as a whole) as S:

$$S \equiv 2E[x_1 - x_2 | x_1 - x_2 \geq 0] Pr(x_1 - x_2 \geq 0) = 2t \int_0^1 s f(s) ds$$

where f denotes the the density of the distribution of $x_1 - x_2$, and $Pr()$ denotes the probability of the event in the bracket. S is the amount the players as a whole will gain if trading is fully efficient. It depends on the strength of taste t and the distribution of $x_1 - x_2$. S is an important exogenous variable in the model.

Given a pair of contracts and a set of realization of x_1, x_2, let's denote the agents' total gain of trade as $g(k_1, k_2, x_1, x_2)$:

$$g(k_1, k_2, x_1, x_2) \equiv -(k_1 - k_2)(e_1 - e_2) + (k_1 + k_2)(x_1 - x_2)t$$

For the sake of simplicity of notation, we will sometimes suppress the arguments of g. By Nash-Bargaining with equal bargaining power, this gain is equally shared between the two agents, so each gets $\frac{1}{2}g$.

4.2 Incentive to exert efforts

Now we go one step backward in the game. Agent 1's problem in choosing effort is (after dropping the fixed term in the contract):

$$\begin{aligned}
\max_{e_1} \quad & E[k_1(e_1 - x_1 t)|no\,trade]Pr(no\,trade) + \\
& E[k_1(e_1 - x_1 t) + \frac{1}{2}g|trade]Pr(trade) - C(e_1) = \\
& k_1(e_1 - E[x_1]t) - C(e_1) + \\
& E\left[\frac{1}{2}g|g \geq 0\right] Pr\left(g \geq 0\right)
\end{aligned}$$

Since the fall-back from breaking up of the Nash-Bargaining is to supply the project to one's own principal, the payoff from a trading is $k_1(e_1 - x_1 t) + \frac{1}{2}g$ for agent 1.

Notice that when $k_1 = k_2$ two things are true. First, the gain from trade, g, does not depend on e_1 (as explained in Subsection 4.1), and second (as a result of the first) the probability of trading does not depend on e_1 either. In other words, as long as $k_1 = k_2$, the trading decision in the subgame will be efficient.

The agent's problem highlights the three effects of a commission rate.

- The term $k_1 e_1 - C(e_1)$ shows that k_1 has the normal effect of motivating agents to work harder.

- If $k_1 = k_2$, then the rest of the agent 1's objective function becomes the following.

$$E[\frac{1}{2}g|g \geq 0]Pr(g \geq 0) = \frac{1}{4}(k_1 + k_2)E[x_1 - x_2|x_1 - x_2 \geq 0]$$

  By the same logic, there is a term in agent 2's objective function that is also

$$\frac{1}{4}(k_1 + k_2)E[x_1 - x_2|x_1 - x_2 \geq 0]$$

  Therefore, higher commission rate transfers surplus to both agents. This is a crucial effect because as we will see, principal 1 is only able to extract back the part given to agent 1 through individual rationality constraint but not the part given to agent 2.

- When $k_1 \neq k_2$, trading is inefficient because trading happens if and only if $x_1 - x_2 \geq \frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}$ (as explained in Subsection 4.1).

Recall that f denotes the the density of the distribution of $x_1 - x_2$. By definition, f is symmetric around 0 and is positive over $[-1, 1]$.

Lemma 3. *Given any k_1 and k_2, a pure strategy subgame equilibrium exists in the effort game. Let $(\tilde{e}_1(k_1, k_2), \tilde{e}_2(k_1, k_2))$ denote an equilibrium.*

On the equilibrium, the probability of trading is strictly less than one. If an equilibrium is such that the probability of trading is positive, then it is characterized by:

$$k_1 - C'(e_1) - \frac{1}{2}(k_1 - k_2)\int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\,ds = 0 \qquad (FOC_1)$$

$$k_2 - C'(e_2) - \frac{1}{2}(k_2 - k_1)\int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\,ds = 0 \qquad (FOC_2)$$

Proof. In the Appendix. □

Remark: The proof for existence of equilibrium is essentially an application of results from supermodular game. Notice that trading may not happen in the subgame equilibium.

The term $k_1 - C'(e_1)$ in the FOC condition is the same as that in the no-trading Benchmark. The rest in the FOC condition is the incentive provided by the possibility of trading. Notice that this term disappears if $k_1 = k_2$. In other words, if $k_1 = k_2$, the effort incentive is the same as in the Benchmark: agents exert efforts as if trading is banned. However when $k_1 \neq k_2$, there is "spill over" of incentive and 2) "free-riding" in efforts.

When principal 1 increases k_1 slightly above k_2, it also motivates agent 2 to work harder because now agent 2 gets a share of principal 1's commission through Nash-bargaining if agent 2's project is supplied to principal 1. However, at the same time, when $k_1 > k_2$, there is a free-riding problem. Agent 1 gains from agent 2's effort, so higher effort from agent 2 reduces agent 1's incentive to work. [9]

These "spill-over" and "free-riding" effects causes e_1 to go down when k_1 is increased if we *only* consider the effects coming from trading. However, the effort level is still higher for the agent who is promised a higher piece-rate, as shown in Lemma 5 below. This is because the direct motivating effect from the piece-rate dominates the indirect effects through trading.

The above lemma however does not establish that the subgame equilibrium to be unique.

Lemma 4. *When the cost function is quadratic and when the distribution of $x_1 - x_2$ is uniform, the subgame equilibrium given any non-negative k_1, k_2 is unique.*

Proof. In the Appendix. □

The next lemma shows that even though a commission rate can motivate both agents' effort. The effect is larger on one's own agent.

[9] Let $\mathbf{e_1}(e_2)$ denote a's reaction function as implied by Condition FOC_1, then around $(\tilde{e}_1, \tilde{e}_2)$, we have:

$$\frac{d\mathbf{e_1}(e_2)}{de_2} = \frac{\frac{1}{2}\frac{(k_1-k_2)^2}{t(k_1+k_2)} f\left(\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}\right)}{-C''(e_1) + \frac{1}{2}\frac{(k_1-k_2)^2}{t(k_1+k_2)} f\left(\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}\right)} < 0 \text{ when } k_1 \neq k_2$$

The inequality follows from the necessary condition that second order conditions must be satisfied at $(\tilde{e}_1, \tilde{e}_2)$.

Lemma 5. $k_1 > k_2 \Rightarrow \tilde{e}_1 > \tilde{e}_2$.

Proof. In the Appendix. □

Remark: This is intuitive. If trading is to happen for sure, then both agents are equally motivated by the commission rate k_1. Agent 1 is also motivated because higher effort raises his outside option in bargaining. If the trading is not to happen for sure, only agent 1 is motivated by k_1. Because there is positive probability that trading will not happen on equilibrium by Lemma 3.

4.3 Principal's choices of piece-rates

The equilibrium fixed rates are implied by the agents' individual rationality constraints, so an equilibrium is characterized by a pair of piece-rates, which we denote by $\tilde{k}_1$ and $\tilde{k}_2$. Recall that S denotes the total gain of trading under efficient trading, i.e., when agents trade if and only if $x_1 - x_2 \geq 0$.

Proposition 1. *(Necessary Condition)*

If a pure strategy symmetric equilibrium exists such that $\tilde{k}_1 = \tilde{k}_2 = k$, we have the following results.

First, we must have $k < 1$. In other words, the equilibrium piece-rates must be lower than those in the no-trade Benchmark and the first-best.

Second, k is unique and is decreasing in S.

For $S < \frac{1}{C''(0)}$, k is uniquely determined by the necessary condition:

$$\frac{1-k}{C''((C')^{-1}(k))} = S$$

For $S \geq \frac{1}{C''(0)}$, $k = 0$.

Proof. See the Appendix. □

Remark:

In the Benchmark, principal 1 fully internalizes the effect of her piece-rate: even though higher piece-rate gives her agent a higher cut of the result of his effort, she extracts all of the agent's cut back through her fixed rate. Now when there is trading, higher piece-rate by principal 1 increases the total gain from trading for the agents, but that gain is shared by agent 1 and 2, so the value accrued to agents from principal 1's sacrifice (in the sense of a higher piece-rate) cannot be fully recovered through principal 1's fixed rate. There is a "leakage". This gives principal 1 an incentive to reduce her piece-rate. In other words, decreasing k_1 helps principal 1 to grab a bigger share of the trading surplus. To see that, given any symmetric contracts:

Principal 1 gets

$$e_1 - C(e_1) + \frac{(1-k_1) + \frac{1}{2}(k_1+k_2)}{2} S$$

Principal 2 gets

$$e_2 - C(e_2) + \frac{(1-k_2) + \frac{1}{2}(k_1+k_2)}{2} S$$

The effects of decreasing k_1 (from $k_1 = k_2 = 1$) on e_1, e_2, and the gain of trading are second order, while the effect on the sharing of S is first order. Therefore, Principal 1 has a unilateral incentive to reduce k_1 from $k_1 = 1$.

The intuition for the comparative statics result is that higher S increases the incentive to lower the piece-rate because the principal wants to capture a larger share of the gain of trading. In other words, S is the force that depresses equilibrium effort. When S is as high as $\frac{1}{C''(0)}$, effort is reduced to 0 and thus it won't be reduced any further. Several things can increase S: higher level of heterogeneity in principal's taste, such as higher t; and higher chance of getting extreme characteristic projects, such as an f that is heavier on the two ends. When $S = 0$, we go back to the standard case where agents do not trade and the only equilibrium involves $k = 1$.

Sufficiency of the equilibrium has been proved for uniform f distribution and quadratic cost function, and the search for more general sufficient condition for the existence of the equilibrium is under progress.

Proposition 2. *(Welfare) When the gain from fully matching taste, S, is either small enough or big enough, allowing agent trading (through fee-sharing arrangement) always increases welfare.*

Proof. Let $W(S)$ denote the total social surplus on the symmetric equilibrium when trading is allowed for $S \in [0, \infty)$:

$$W(S) = 2[\tilde{e}(S) - C(\tilde{e}(S))] + S$$

where $\tilde{e}(S)$ denotes the equilibrium effort level given S when trading is allowed.

Proposition 1 implies that when $S = 0$, the piece-rate is first-best. Therefore, $W(0) = 2(e^* - C(e^*))$. When $S > \frac{1}{C''(0)}$, we know that $W(S) = S$. Therefore, when S is big enough. $W(S) > W(0)$.

$$
\begin{aligned}
W'(S) &= 2(1 - C'(\tilde{e}))\frac{d\tilde{e}}{dk}\frac{dk}{dS} + 1 \\
&= 2(1 - C'(\tilde{e}))\frac{1}{C''(\tilde{e})}\frac{C''(\tilde{e})}{(1-k)C'''(\tilde{e}) - 1} + 1 \\
&= 2\frac{1 - C'(\tilde{e})}{(1-k)C'''(\tilde{e}) - 1} + 1
\end{aligned}
$$

When $S \to 0$, we have $k \to 1$ and $C'(\tilde{e}) \to k$, which implies that $W'(S) \to 1 > 0$. Therefore, when S is small enough $W(S)$ is strictly increasing, i.e., $W(S) > W(0)$.

□

Remark: Trading increases the efficiency gain from matching the projects to principals, but reduces the surplus in the stage of exerting efforts because agents are under-incentivized. These two effects do not always interact in a monotonic way. Next we show how welfare changes with the parameters for the case of quadratic cost function.

For quadratic cost function, since $C''(e)$ is a constant, we will simply denote it by C''. Since $C''' = 0$, we get that for $S \leq \frac{1}{C''}$, $W'(S) = -2[1 - C'(\tilde{e}(S))] + 1$ and $W''(S) = -2C'' < 0$. Therefore, welfare is a concave function of S for $S \in [0, \frac{1}{C''}]$. Let $C(e) = \frac{(C'')^2}{2}e^2 + \gamma e$. We will plot the welfare function for two cases: $\gamma > 0$ and $\gamma < 0$.

[Insert Figures]

5 Discussion

We will first discuss some essential assumptions and how changing them would affect the result and then we will talk about some non-essential assumptions.

5.1 Only contracting with one agent

This assumption is important. The following proposition shows that when each principal can offer contracts to both agents, there exist a continuum of equilibria that are efficient, where the principals use linear contracts.

Proposition 3. *Let* (F_i, k_i) *denote a linear contract offered to agent* i *by principal* i*, and let* $(\hat{F}_i, \hat{k}_i)$ *denote the contract offer to agent* $-i$ *by principal* i*. There exist efficient equilibria characterized by:*

- $k_1 + \hat{k}_1 = 1$ *and* $k_2 + \hat{k}_2 = 1$
- $F_1 + \hat{F}_2 = \hat{F}_1 + F_2 = e^* - C(e^*) + \frac{1}{2}S$.

Proof. See the Appendix. □

First-best is achieved because the principal internalizes both agents' payoff. However, there are many real world applications where principals do not have the ability to offer contracts to many agents.

5.2 Private contracts

This assumption is not essential to the intuition.

This paper assumes that the contracts are observable, i.e. Principal 1's offer to agent 1 is observable to agent 2. A natural extension is when the principal 1's offer is not observable to agent 2 and vice versa. Then we need to use Perfect Bayesian Equilibrium. Let b_1 be agent 2's belief of principal 1's piece-rate and b_2 be agent 1's belief of principal 2's piece-rate. Now since the contract is private, k_1 lost its ability to motivate agent 2's effort, as a result, the equilibrium piece-rate will be even lower. The proof is a straightforward modification of Proposition 1. A natural next step is to allow the principals to choose whether or not to disclose the private contract to the market.

5.3 Fixed fee sharing arrangement

This paper assumes that fee sharing is determined though Nash Bargaining. In reality however, often it is already a industry consensus that fee should always be shared half and half between two "cooperating" agents. This alternative will not change the qualitative results. The main driving force still remains the same in the sense that trading surplus will be shared between agents and thus there is a free-riding problem between the principals on contributing to the public good of "trading". However, trading cannot be efficient in any equilibrium. To see that, given that any fee has to be shared half and half, trading only happens if both agents find it better to get the half of the total fee after swapping projects:

$$\frac{1}{2}k_1(e_2 - x_2 t) + \frac{1}{2}k_2(e_1 + x_1 t) \geq \max\{k_1(e_1 - x_1 t), k_2(e_2 + x_2 t)\}$$

$$\Leftrightarrow x_1 \geq \frac{(k_1 - k_2)e_1}{t(e_1 + e_2)} \text{ and } x_2 \leq \frac{(k_1 - k_2)e_2}{t(e_1 + e_2)}$$

This means that on a symmetric equilibrium (if exists), the agents trades only when $x_1 \geq 0$ and $x_2 \leq 0$, while complete trading efficiency calls for trading whenever $x_1 - x_2 \geq 0$.

5.4 Uncertainty in vertical quality

This paper also assumes that uncertainty only lies in the horizontal aspect of the search results. That is, there is no uncertainty in the vertical quality of the search results. If the vertical quality is also uncertain, then it creates a competition between the principals as they can offer higher piece-rate to influence the direction of the inter-agent trading so that they can get the better of the two results. This will be a countervailing force that pushes up the equilibrium piece-rate. That implies this consideration will pressure a home seller not to lower her commission rate below other sellers', not because it will reduce cooperation from other agents, but because she would not want bad buyers be sent her way.

6 Conclusion

This paper shows that when inter-agent trading is possible, on the unique symmetric equilibrium, the principals offer a lower-powered incentive contract (i.e., with a lower piece-rate) than when inter-agent trading is not possible. These are socially sub-optimal piece-rates in the sense that they are too low to make agents exert the efficient effort in searching. Here the principals need to motivate the agents to pay effort in searching and also to cooperate with other agents to deliver the most suitable result to the principals.

Motivating effort calls for paying the agents a high piece-rate. However, principals face negative externality when motivating the agents to do efficient inter-agent trading. Note that the principals internalize the agents' payoff through the individual rationality constraints. When a principal unilaterally decreases her piece-rate, the loss to the surplus from trading is born by both agents, and thus by the other principal as well. This creates an incentive for both principals to shirk in motivating efficient inter-agent trading. The result is that the equilibrium piece-rate is lower than the efficient level. When the cost function is quadratic, the principals are always better-off overall if the agents can swap search outcomes. This is because the loss from lower effort is more than compensated by the gain from better matched outcomes.

7 Appendix

Proof of Lemma 3.

Proof. A game is a supermodular game if the strategy set is bounded, the payoff is upper-semi continuous and the payoff has increasing difference between strategies.

To be able to use results from supermodular game, we need to prove that the choice set for efforts is effectively bounded for both players.

Fix any $k_1 > k_2 \geq 0$.

Fix $\forall e_2 \geq 0$. Define two functions $\underline{e}$ and $\bar{e}$ as follows:

$$\frac{(k_1 - k_2)(\underline{e}(e_2) - e_2)}{t(k_1 + k_2)} = -1 \quad \frac{(k_1 - k_2)(\bar{e}(e_2) - e_2)}{t(k_1 + k_2)} = 1$$

$$k_1 - k_2 > 0 \Rightarrow \underline{e}(e_2) < e_2 < \bar{e}(e_2)$$

Note that the payoff to agent 1 depends on which segment e_1 is in. For $e_1 \leq \underline{e}(e_2)$, the probability of trading is 1 and thus the payoff is $u_1(e_1, e_2) = k_1 e_1 - C(e_1) - \frac{1}{2}(k_1 - k_2)(e_1 - e_2) = \frac{1}{2}(k_1 + k_2)e_1 - C(e_1) + \frac{1}{2}(k_1 - k_2)e_2 \geq k_1 e_1 - C(e_1) + \frac{1}{2}t(k_1 + k_2)$. For $e_1 \geq \bar{e}(e_2)$, the probability of trading is 0 and thus the payoff is just $u_1(e_1, e_2) = k_1 e_1 - C(e_1)$.

Let $e^*(k)$ be the solution to $\max_e \quad ke - C(e)$.

Case 1. The argmax $\mathbf{e_1}(e_2) \in [0, \underline{e}(e_2))$, then we have $\mathbf{e_1}(e_2) = e^*(\frac{1}{2}(k_1 + k_2)) < e^*(k_1)$.

Case 2. If $\mathbf{e_1}(e_2) > \bar{e}(e_2)$, then we have $(e_1) = e^*(k_1)$.

Case 3. If $\mathbf{e_1}(e_2) \in [\underline{e}(e_2), \bar{e}(e_2)]$, then it solves the following problem:

$$\begin{aligned}\max_{e_1} \quad & k_1 e_1 - C(e_1) + \\ & \frac{1}{2}[(k_1 + k_2)t \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} sf(s)\, ds - \\ & (k_1 - k_2)(e_1 - e_2) \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\, ds] \\ s.t. \quad & -1 \geq \frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)} \leq 1\end{aligned}$$

The first order derivative of the objective function is:

$$k_1 - C'(e_1) - \frac{1}{2}(k_1 - k_2) \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\, ds$$

For $e_1 \in [e^*(k_1), \bar{e}(e_2))$, the derivative is strictly negative. Therefore again, $\mathbf{e_1}(e_2) \leq e^*(k_1)$.

This proves that $\mathbf{e_1}(e_2) \leq e^*(k_1)$.

We do the same analysis for agent 2's problem. The above defined function $\bar{e}$ and $\underline{e}$ implies:

$$\frac{(k_1 - k_2)(e_1 - \underline{e}(e_1))}{t(k_1 + k_2)} = 1 \quad \frac{(k_1 - k_2)(e_1 - \bar{e}(e_1))}{t(k_1 + k_2)} = -1$$

The payoff to agent 2 also depends on which segment e_2 is in. For $e_2 \leq \underline{e}(e_1)$, the probability of trading is 0, thus the payoff is just $u_2(e_1, e_2) = k_2 e_2 - C(e_2)$. For $e_2 \geq \bar{e}(e_1)$, the probability of trading is 1, so $u_2(e_1, e_2) = k_2 e_2 - C(e_2) - \frac{1}{2}(k_1 - k_2)(e_1 - e_2) = \frac{1}{2}(k_1 + k_2)e_2 - C(e_2) - \frac{1}{2}(k_1 - k_2)e_1 \leq k_2 e_2 - C(e_2) - \frac{1}{2}t(k_1 + k_2)$.

Case 1. The argmax $\mathbf{e_2}(e_1) \in (\bar{e}(e_1), +\infty)$, then trading happens with probability 1, which implies $\mathbf{e_2}(e_1) = e^*(\frac{1}{2}(k_1 + k_2))$.

Case 2. If $\mathbf{e_2}(e_1) < \underline{e}(e_1)$, then trading happens with probability 0, which implies that $\mathbf{e_2}(e_1) = e^*(k_2)$.

Case 3. $\mathbf{e_2}(e_1) \in [\underline{e}(e_1), \bar{e}(e_1)]$.

Notice that the following inequalities hold:

$$\bar{e}(e^*(k_1)) > e^*(k_1) > e^*(\frac{1}{2}(k_1 + k_2)) > e^*(k_2)$$

Therefore, when e_1 is bounded by $e^*(k_1)$, agent 2's best response is bounded above by $\bar{e}(e^*(k_1))$.

By symmetry of the setup, the case for $k_1 < k_2$ will yield the same result. Therefore, given any non-negative k_1, k_2, we can find two bounded choice sets for e_1 and e_2 that are without loss of generality.

Note that the payoff functions are continuous in e_1 and e_2.

Now look at the cross derivative of agent 1's payoff when $\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)} \in (-1, 1)$ (elsewhere the cross derivative trivially equals zero):

$$\frac{\partial^2 u_1}{\partial(-e_2)\partial(e_1)} = \frac{1}{2}\frac{(k_1 - k_2)^2}{t(k_1 k_2)} f\left(\frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)}\right) \geq 0$$

Therefore, u_1 has increasing difference in e_1 and $-e_2$. Similarly, for u_2.

Now we can conclude that the subgame given k_1 and k_2 is a supermodular game. Therefore, we can apply the result that a pure-strategy equilibrium exists.

□

Proof of Lemma 4

Proof. There are only two cases: either the probability of trading is zero, or it is positive. (We will use notations defined in proof of Lemma 3.

Step 1. We show that there are at most only one equilibrium with trading probability of zero. Let $(\tilde{e}_1, \tilde{e}_2)$ be an equilibrium. WLOG, suppose $k_1 > k_2$. Then we must have $e^*(k_1) \geq \bar{e}(\tilde{e}_2)$ because otherwise agent 1's best response is not in a range that makes trading probability zero. It also implies that $\tilde{e}_1 = e^*(k_1)$. Similarly, we have $\tilde{e}_2 = e^*(k_2)$. Therefore, there can at most be only one equilibrium that has trading probability equals zero.

Step 2. We show that if an equilibrium with trading probability zero exists then there does not exist any equilibrium with positive trading probability. WOLG, suppose $k_1 > k_2$. Step 1 already implies that if an equilibrium with trading probability zero exists, then $e^*(k_1) \geq \bar{e}(e^*(k_2))$ and $e^*(k_2) \leq \underline{e}(e^*(k_1))$. Suppose another equilibrium with positive trading probability exists. Then $\bar{e}(\tilde{e}_2) > \bar{e}(e^*(k_2)) \Rightarrow \tilde{e}_2 > e^*(k_2)$ because otherwise agent 1's best response is one that makes trading probability zero. However, this forms a contradiction, because agent 2's payoff is decreasing over $[e^*(k_2), \bar{e}(\tilde{e}_1)]$.

Step 3. We show that there exists at most one equilibrium that has strictly positive trading probability using quadratic cost function and uniform distribution of $x_1 - x_2$.

Quadratic cost function implies that $C'(e)$ to e is a linear function. Let it be $C'(e) = pe + q$ where $p \neq 0$.

Then we can solve for the equilibrium efforts through the following equations:

$$k_1 - pe_1 - q = \frac{1}{4}(k_1 - k_2)(1 - \frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)})$$
$$k_2 - pe_2 - q = -\frac{1}{4}(k_1 - k_2)(1 - \frac{(k_1 - k_2)(e_1 - e_2)}{t(k_1 + k_2)})$$

Since both equations are linear, there are at most one solution.

□

Proof of Lemma 5

Proof. WLOG, let $k_1 > k_2$. There are two possible cases. Either there is positive probability of trading on the equilibrium or there is not.

Case 1, positive probability of trading.

Let $\Delta = k_1 - k_2 > 0$. Let $Prob(\Delta) = \int_{\frac{(k_1-k_2)(e_1-e_2)}{t(k_1+k_2)}}^{1} f(s)\,ds$, then FOCs imply that:

$$\begin{aligned} C'(\tilde{e}_1) - C'(\tilde{e}_2) &= (k_2 + \Delta - \frac{1}{2}\Delta Prob(\Delta)) - (k_2 + \frac{1}{2}\Delta Prob(\Delta)) \\ &= \Delta - \Delta Prob(\Delta) \\ &\geq 0 \end{aligned}$$

This implies that $\tilde{e}_1 \geq \tilde{e}_2$ [10]. Next, we prove the inequality by contradiction. Suppose $\tilde{e}_1 = \tilde{e}_2$, then we have:

$$\Delta - \Delta Prob(\Delta) = 0 \Rightarrow Prob(\Delta) = 1$$

However, because of the symmetry of the density function f around 0, we know that $Prob(\Delta) = \int_0^1 f(s)\,ds = \frac{1}{2}$ when $\tilde{e}_1 = \tilde{e}_2$. Therefore, it is a contradiction, and we know that $\tilde{e}_1 \neq \tilde{e}_2$.

Case 2, zero probability of trading.

This implies that $\tilde{e}_1 = (C')^{-1}(k_1)$ and $\tilde{e}_2 = (C')^{-1}(k_2)$. Therefore, $\tilde{e}_1 > \tilde{e}_2$. □

Proof of Proposition 1.

Proof. A solves the following problem:

$$\begin{aligned}\max_{k_1} \quad U_1(k_1) \equiv & E[(1-k_1)(\tilde{e}_1(k_1,k_2) - x_1 t)|no\,trade]Pr(no\,trade) \\ & + E[(1-k_1)(\tilde{e}_2(k_1,k_2) - x_2 t)|trade]Pr(trade) \\ & + E[k_1(\tilde{e}_1(k_1,k_2) - x_1 t)|no\,trade]Pr(no\,trade) \\ & + E[k_1(\tilde{e}_1(k_1,k_2) - x_1 t) + \frac{1}{2}g|trade]Pr(trade) - C(\tilde{e}_1(k_1,k_2)) \\ = & \tilde{e}_1(k_1,k_2) - C(\tilde{e}_1(k_1,k_2)) \\ & - (\tilde{e}_1(k_1,k_2) - \tilde{e}_2(k_1,k_2))[(1-k_1) + \frac{1}{2}(k_1 - k_2)]Pr(trade) \\ & + [(1-k_1) + \frac{1}{2}(k_1 + k_2)]E[x_1 - x_2|trade]tPr(trade)\end{aligned}$$

Take derivative with respect to k_1 and evaluate the derivative at $k_1 = k_2 = k$, which also implies that $\tilde{e}_1 = \tilde{e}_2 \equiv \tilde{e} = (C')^{-1}(k)$. [11]

[10] We will suppress the arguments of $\tilde{e}_1(k_1,k_2), \tilde{e}_2(k_1,k_2)$ when there is no ambiguity.

[11] Note that $\frac{\partial \tilde{e}_1}{\partial k_1}\Big|_{k_1=k_2=k}$ and $\frac{\partial \tilde{e}_2}{\partial k_1}\Big|_{k_1=k_2=k}$ exists. This is because when k_1 and k_2 are close enough, the objective function of agents in the subgame is strictly concave, which implies that the subgame equilibrium of efforts is unique and continuously differentiable in the "parameters" k_1 and k_2.

$$
\begin{aligned}
\left.\frac{dU_1}{dk_1}\right|_{k_1=k_2=k} &= (1-C'(\tilde{e}_1))\left.\frac{\partial \tilde{e}_1}{\partial k_1}\right|_{k_1=k_2=k} \\
&-(1-k)\left.\frac{\partial(\tilde{e}_1-\tilde{e}_2)Pr(trade)}{\partial k_1}\right|_{k_1=k_2=k} \\
&+\left.\frac{\partial E[x_1-x_2|trade]tPr(trade)}{\partial k_1}\right|_{k_1=k_2=k} \\
&+(-\frac{1}{2})t\int_0^1 sf(s)ds \\
&= (1-k)\left.\frac{\partial \tilde{e}_1}{\partial k_1}\right|_{k_1=k_2=k} - \frac{1}{2}(1-k)\left.\frac{\partial(\tilde{e}_1-\tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} \\
&-\frac{1}{2}t\int_0^1 sf(s)ds \\
&=\frac{1}{2}(1-k)\left.\frac{\partial(\tilde{e}_1+\tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} - \frac{1}{2}t\int_0^1 sf(s)ds
\end{aligned}
$$

If $k \geq 1$, then we have and $\frac{1}{2}(1-k)\left.\frac{\partial(\tilde{e}_1+\tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} \leq 0$. Therefore, the above derivative is strictly negative when $k \geq 1$. This shows that principal 1 has unilateral incentive to deviate downward when $k \geq 1$, therefore we must have $k < 1$

From Conditions FOC_1 and FOC_2, we get:

$$
\left.\frac{\partial \tilde{e}_1}{\partial k_1}\right|_{k_1=k_2=k} = \frac{1-\frac{1}{2}\int_0^1 f(s)ds}{C''(\tilde{e})} = \frac{\frac{3}{4}}{C''(\tilde{e})}
$$

$$
\left.\frac{\partial \tilde{e}_2}{\partial k_1}\right|_{k_1=k_2=k} = \frac{\frac{1}{2}\int_0^1 f(s)ds}{C''(\tilde{e})} = \frac{\frac{1}{4}}{C''(\tilde{e})}
$$

Since a necessary condition for a symmetric equilibrium where $k > 0$ is that $\left.\frac{dU_1}{dk_1}\right|_{k_1=k_2=k} = 0$, k should satisfy:

$$(1-k)\left.\frac{\partial(\tilde{e}_1+\tilde{e}_2)}{\partial k_1}\right|_{k_1=k_2=k} = t\int_0^1 sf(s)ds \Rightarrow$$

$$\frac{1-k}{C''((C')^{-1}(k))} = t\int_0^1 sf(s)ds$$

Denote the left-hand-side of the necessary condition as $h(k)$, and let $\tilde{e} \equiv \tilde{e}_1(k,k)$. Recall that $S = t\int_0^1 sf(s)ds$, so we can rewrite the necessary condition for k as:

$$h(k) = S \quad \text{if } k > 0$$

The following shows that $h(k)$ is decreasing in k:

$$\frac{dh(k)}{dk} = -\frac{C''(\tilde{e}) + (1-k)\frac{C'''(\tilde{e})}{C''(\tilde{e})}}{(C''(\tilde{e}))^2} = -\frac{C'' + \frac{S}{2}C'''}{(C'')^2} < 0$$

Since $h(1) = 0 < S$, for S small enough ($< h(0)$), there exists a unique $k >$ such that $h(k) = S$.

When $S > h(0) = \frac{1}{C''(0)}$, for any $k < 1$, we have $h(k) < S$. This implies that $\left.\frac{dU_1}{dk_1}\right|_{k_1=k_2=k} = h(k) - S < 0$ for any $k < 1$, therefore, if a symmetric pure strategy exists it must be $k = 0$. □

Proof of Proposition 3

Proof. It is easy to see if the four contracts form an equilibrium, then the equilibrium is efficient, because both projects were "sold out" to the agents and the weights on both projects are the same. We will just prove that it is an equilibrium.

Let (e_1, e_2) denote the effort equilibrium as functions of contracts accepted: $(F_1, k_1), (\hat{F}_1, \hat{k}_1), (F_2, k_2), (\hat{F}_2, \hat{k}_2)$. We suppress the arguments of the functions. Binding rationality constraint of agent 1 implies:

$$\begin{aligned}
-F_1 - \hat{F}_2 = & E[k_1(e_1 - x_1t)|\text{no trade})Pr(\text{no trade}) \\
& + E[\hat{k}_2(e_2 + x_2t)|\text{no trade})Pr(\text{no trade}) \\
& + E[k_2(e_2 + x_2t)|\text{trade})Pr(\text{trade}) \\
& + E[\hat{k}_1(e_1 - x_1t)|\text{trade})Pr(\text{trade}) - C(e_1)
\end{aligned}$$

Binding rationality constraint of agent 2 implies a similar equality with 1 and 2 flipped.

Principal 1 maximizes:

$$\begin{aligned}&E[(1-k_1-\hat{k}_1)(e_1-x_1t)|\text{no trade})Pr(\text{no trade})\\&+E[(1-k_1-\hat{k}_1)(e_2-x_2t)|\text{trade})Pr(\text{trade})-F_1-\hat{F}_1\end{aligned}$$

Some substitution and reorganizing yield that it is equivalent to maximize the following, given that $k_2+\hat{k}_2=1$:

$$\begin{aligned}&E[(e_1-x_1t)|\text{no trade})Pr(\text{no trade})+E[(k_2+\hat{k}_2)(e_2+x_2t)|\text{no trade})Pr(\text{no trade})\\&+E[(e_2-x_2t)|\text{trade})Pr(\text{trade})+E[(k_2+\hat{k}_2)(e_1+x_1t)|\text{trade})Pr(\text{trade})-C(e_1)-C(e_2)\end{aligned}$$

Since $k_2+\hat{k}_2=1$, principal 1 is actually maximizing the total surplus of the all four players. Therefore, any $k_1+\hat{k}_1=1$ is a best-response. □

7.1 General contracts

In this Appendix, we show that the result that the level of effort when trading is not allowed (the first-best level) cannot be achieved on equilibrium holds when we allow the principals to use general contract.

We still assume that the principals can only observe the outcome delivered to them v_1 and v_2, but they can choose $f_1(v_1-v)$ and $f_2(v_2-v)$, where f_1 and f_2 are any differentiable functions that do not have to be linear.

Denote the effort equilibrium given f_1 and f_2 as $(\tilde{e}_1(f_1,f_2),\tilde{e}_2(f_1,f_2))$.

Proposition 4. *There does not exist a symmetric equilibrium (f_1,f_2) such that the associated agents' level of effort is equal to the level when no trading is allowed.*

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